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# Faculty of Engineering and Technology

# Discrete Structure
# (BCA-309)

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# Outlines

- Introduction of Set Theory
- Basic Notations for Sets
- Basic properties of sets
- Definition of Set Equality
- Examples for Sets
- Subsets
- Infinite Sets
- Venn Diagrams

# Introduction of Set Theory

## Introduction of Set Theory:

- A set is a structure, representing an unordered collection (group, plurality) of zero or more distinct (different) objects.
- Set theory deals with operations between, relations among, and statements about sets.
- Set: Collection of objects (called elements)
- $a \in A$ "a is an element of A"

  "a is a member of A"
- $A \notin A$ "a is not an element of A"
- A = {a1, a2, …, an} "A contains a1, …, an"
- Order of elements is insignificant
- It does not matter how often the same element is listed (repetition doesn't count).

# Basic Notations for Sets

- For sets, we'll use variables *S*, *T*, *U*, …
- We can denote a set *S* in writing by listing all of its elements in curly braces:
  - {a, b, c} is the set of whatever 3 objects are denoted by a, b, c.
- *Set builder notation*: For any proposition $P(x)$ over any universe of discourse,

  $\{x|P(x)\}$ is *the set of all x such that P(x).*

  e.g., $\{x \mid x$ is an integer where $x>0$ and $x<5\}$

# Basic properties of sets

- Sets are inherently ***unordered***:
  - No matter what objects a, b, and c denote,

    {a, b, c} = {a, c, b} = {b, a, c} =

    {b, c, a} = {c, a, b} = {c, b, a}.
- All elements are ***distinct*** (unequal);

  multiple listings make no difference!
  - {a, b, c} = {a, a, b, a, b, c, c, c, c}.
  - This set contains at most 3 elements!

# Definition of Set Equality

- Two sets are declared to be equal *if and only if* they contain exactly the same elements.
- In particular, it does not matter *how the set is defined or denoted.*
- For example: The set {1, 2, 3, 4} =

  {$x$ | $x$ is an integer where $x$>0 and $x$<5 } =

  {$x$ | $x$ is a positive integer whose square

  is >0 and <25}

•Sets A and B are equal if and only if they contain exactly the same elements.

•**Examples:**

- *A = {9, 2, 7, -3}, B = {7, 9, -3, 2} :* A = B
- A = {dog, cat, horse},
  B = {cat, horse, squirrel, dog} : A ≠ B
- A = {dog, cat, horse},
  B = {cat, horse, dog, dog} : A = B

# Examples for Sets

- "Standard" Sets:
- Natural numbers $\mathbf{N} = \{0, 1, 2, 3, \ldots\}$
- Integers $\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$
- Positive Integers $\mathbf{Z}^+ = \{1, 2, 3, 4, \ldots\}$
- Real Numbers $\mathbf{R} = \{47.3, -12, \pi, \ldots\}$
- Rational Numbers $\mathbf{Q} = \{1.5, 2.6, -3.8, 15, \ldots\}$

- (correct definitions will follow)

- $A = \emptyset$ "empty set/null set"
- $A = \{z\}$ Note: $z \in A$, but $z \neq \{z\}$
- $A = \{\{b, c\}, \{c, x, d\}\}$ set of sets
- $A = \{\{x, y\}\}$ Note: $\{x, y\} \in A$, but $\{x, y\} \neq \{\{x, y\}\}$
- $A = \{x \mid P(x)\}$ "set of all x such that P(x)"
- P(x) is the membership function of set A
- $\forall x\ (P(x) \rightarrow x \in A)$
- $A = \{x \mid x \in \mathbf{N} \land x > 7\} = \{8, 9, 10, \ldots\}$ "set builder notation"

## Subsets

- $A \subseteq B$ "A is a subset of B"
- $A \subseteq B$ if and only if every element of A is also an element of B.
- We can completely formalize this:
- $A \subseteq B \Leftrightarrow \forall x\ (x \in A \rightarrow x \in B)$
- Examples:

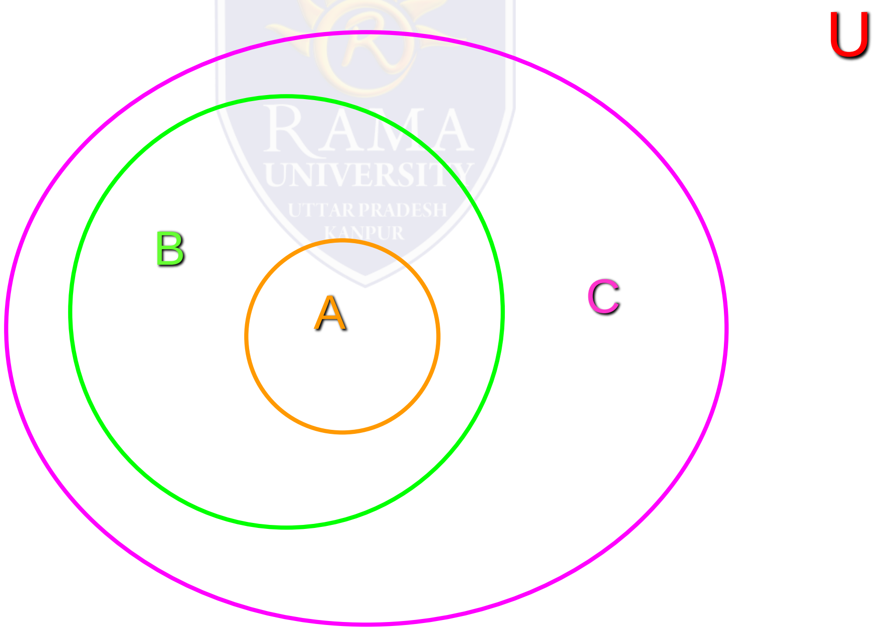
A = {3, 9}, B = {5, 9, 1, 3}, $A \subseteq B$ ? true

A = {3, 3, 3, 9}, B = {5, 9, 1, 3}, $A \subseteq B$ ? true

A = {1, 2, 3}, B = {2, 3, 4}, $A \subseteq B$ ? false

# Subsets

- Useful rules:
- $A = B \Leftrightarrow (A \subseteq B) \wedge (B \subseteq A)$
- $(A \subseteq B) \wedge (B \subseteq C) \Rightarrow A \subseteq C$ (see Venn Diagram)



# Subsets

- Useful rules:
- $\emptyset \subseteq A$ for any set A

  (but $\emptyset \in A$ may not hold for any set A)
- $A \subseteq A$ for any set A
- Proper subsets:
- $A \subset B$ "A is a proper subset of B"
- $A \subset B \Leftrightarrow \forall x\ (x \in A \rightarrow x \in B) \wedge \exists x\ (x \in B \wedge x \notin A)$

  or
- $A \subset B \Leftrightarrow \forall x\ (x \in A \rightarrow x \in B) \wedge \neg \forall x\ (x \in B \rightarrow x \in A)$

# Infinite Sets

- Conceptually, sets may be *infinite* (*i.e.,* not *finite*, without end, unending).
- Symbols for some special infinite sets:

  **N** = {0, 1, 2, …}    The **n**atural numbers.

  **Z** = {…, -2, -1, 0, 1, 2, …}  The integers.

  **R** = The "**r**eal" numbers, such as 374.1828471929498181917281943125…

- Infinite sets come in different sizes!

## Venn Diagrams