Faculty of Engineering and Technology

Discrete Structure
(BCA-309)

Somendra Tripathi
Assistant Professor
Computer Science and Engineering

Outlines

- **Set Operations**
- **The Union Operator**
- **The Intersection Operator**
-

Set Operations

- *The complement of a set A contains exactly those elements under consideration that are not in A: denoted* A^c *(or* $\overline{A}$ *as in the text)*
- $A^c = U - A$
- *Example:* $U = N$, $B = \{250, 251, 252, \ldots\}$
- $B^c = \{0, 1, 2, \ldots, 248, 249\}$

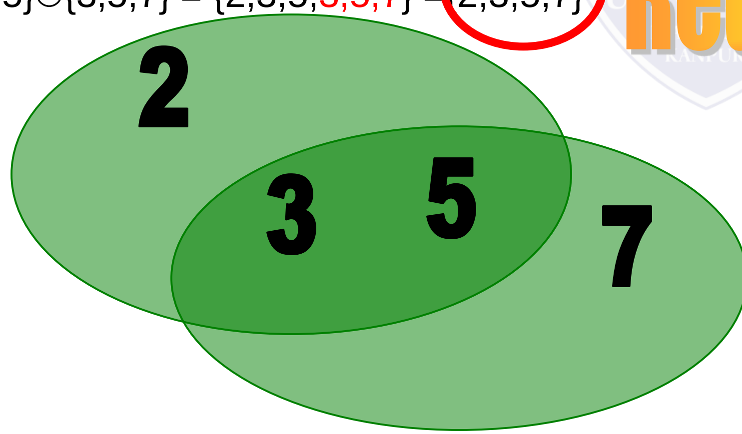
The Union Operator

- For sets A, B, their *union* $A \cup B$ is the set containing all elements that are either in A, **or** (“$\vee$”) in B (or, of course, in both).
- Formally, $\forall A,B$: $A \cup B = \{x \mid x \in A \vee x \in B\}$.
- Note that $A \cup B$ contains all the elements of A **and** it contains all the elements of B: $\forall A, B$: $(A \cup B \supseteq A) \wedge (A \cup B \supseteq B)$

Union Examples:

- $\{a, b, c\} \cup \{2,3\} = \{a,b,c,2,3\}$
- $\{2,3,5\} \cup \{3,5,7\} = \{2,3,5,3,5,7\} = \{2,3,5,7\}$

Required Form

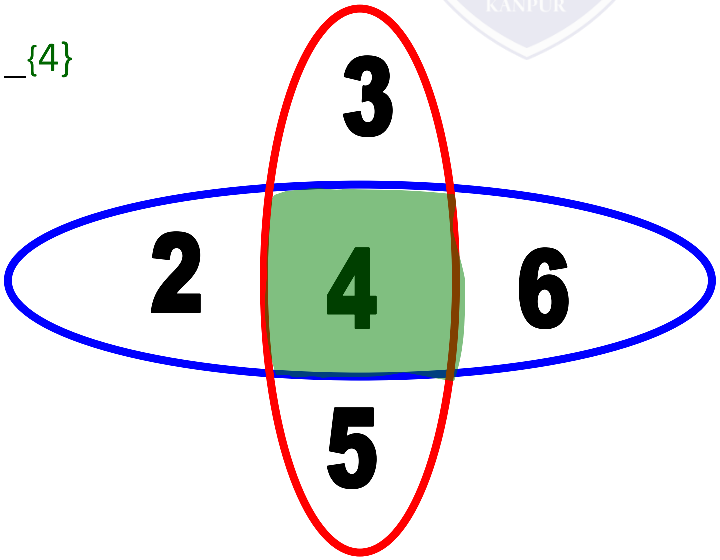


The Intersection Operator

- For sets A, B, their *intersection* $A \cap B$ is the set containing all elements that are simultaneously in A **and** ("$\land$") in B.
- Formally, $\forall A,B$: $A \cap B \equiv \{x \mid x \in A \land x \in B\}$.
- Note that $A \cap B$ is a subset of A **and** it is a subset of B:
  $\forall A, B$: $(A \cap B \subseteq A) \land (A \cap B \subseteq B)$

Intersection Examples:

- $\{a,b,c\} \cap \{2,3\}$ = __$\varnothing$
- $\{2,4,6\} \cap \{3,4,5\}$ = _$\{4\}$



The Power Set

- The *power set* P(S) of a set S is the set of all subsets of S. P(S) = {x | $x \subseteq S$}.
- *E.g.* P({a,b}) = {∅, {a}, {b}, {a,b}}.
- Sometimes P(S) is written $\mathbf{2}^{S}$.

  Note that for finite S, $|P(S)| = 2^{|S|}$.
- It turns out that |P(**N**)| > |**N**|.

  There are different sizes of infinite sets!

- P(A) "power set of A" (also written as 2^A)
- P(A) = {B | B $\subseteq$ A} (contains all subsets of A)

Examples:

- A = {x, y, z}
- P(A) = {∅, {x}, {y}, {z}, {x, y}, {x, z}, {y, z}, {x, y, z}}

- A = ∅
- P(A) = {∅}

  Note: |A| = 0, |P(A)| = 1

The Power Set

- Cardinality of power sets: $| P(A) | = 2^{|A|}$
- Imagine each element in A has an "on/off" switch
- Each possible switch configuration in A corresponds to one subset of A, thus one element in P(A)

A	1	2	3	4	5	6	7	8
x	x	x	x	x	x	x	x	x
y	y	y	y	y	y	y	y	y
z	z	z	z	z	z	z	z	z

- For 3 elements in A, there are $2 \times 2 \times 2 = 8$ elements in P(A)

Cartesian Products of Sets

- For sets A, B, their *Cartesian product*

  $A \times B :\equiv \{(a, b) \mid a \in A \wedge b \in B\}$.

- *E.g.* $\{a,b\} \times \{1,2\} = \{(a,1),(a,2),(b,1),(b,2)\}$

- Note that for finite A, B, $|A \times B| = |A||B|$.

- Note that the Cartesian product is ***not*** commutative: $\neg\forall AB: A \times B = B \times A$.

- Extends to $A_1 \times A_2 \times \ldots \times A_n$...

•The ordered n-tuple $(a_1, a_2, a_3, \ldots, a_n)$ is an ordered collection of n objects.

•Two ordered n-tuples $(a_1, a_2, a_3, \ldots, a_n)$ and

$(b_1, b_2, b_3, \ldots, b_n)$ are equal if and only if they contain exactly the same elements in the same order, i.e.

$a_i = b_i$ for $1 \leq i \leq n$.

•The Cartesian product of two sets is defined as:

•$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$

Cartesian Product

- Example:
- A = {good, bad}, B = {student, prof}
- A×B = {

(good, student), (good, prof), (bad, student), (bad, prof)}

B x A ={ (student, good), (prof, good), (student, bad), (prof, bad)}

Example: A = {x, y}, B = {a, b, c}

A x B = {(x, a), (x, b), (x, c), (y, a), (y, b), (y, c)}

Note that:

- $A\times\varnothing = \varnothing$
- $\varnothing\times A = \varnothing$
- For non-empty sets A and B: $A\neq B \Leftrightarrow A\times B \neq B\times A$
- $|A\times B| = |A|\cdot|B|$

- The Cartesian product of two or more sets is defined as:
- $A_1\times A_2\times\ldots\times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i\in A_i \text{ for } 1 \leq i \leq n\}$

Set Operations

- Union: $A \cup B = \{x \mid x \in A \vee x \in B\}$

Example: $A = \{a, b\}$, $B = \{b, c, d\}$
- $A \cup B = \{a, b, c, d\}$

- Intersection: $A \cap B = \{x \mid x \in A \wedge x \in B\}$

Example: $A = \{a, b\}$, $B = \{b, c, d\}$
- $A \cap B = \{b\}$
- Cardinality: $|A \cup B| = |A| + |B| - |A \cap B|$

•Two sets are called disjoint if their intersection is empty, that is, they share no elements:
•$A \cap B = \varnothing$

•The difference between two sets A and B contains exactly those elements of A that are not in B:
•$A - B = \{x \mid x \in A \wedge x \notin B\}$
Example: $A = \{a, b\}$, $B = \{b, c, d\}$, $A - B = \{a\}$

•Cardinality: $|A - B| = |A| - |A \cap B|$