



RAMA UNIVERSITY

www.ramauniversity.ac.in

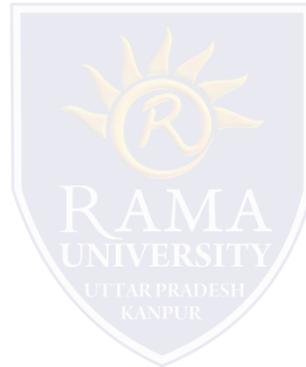
Faculty of Engineering and Technology

Discrete Mathematics (CSPS-111)

Somendra Tripathi
Assistant Professor
Computer Science and Engineering

Outlines

- **Basic Set Relations**
- **The Empty Set**
- **Sets Are Objects**
- **Cardinality of Sets**
- **The Power Set**
- **Cartesian Products of Sets**
- **Set Operations**



Basic Set Relations

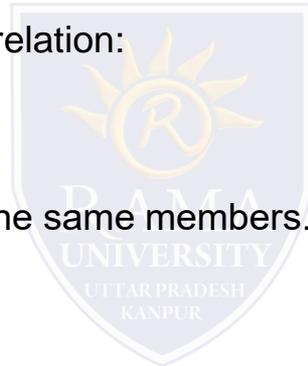
- $x \in S$ (“ x is in S ”) is the proposition that object x is an *element* or *member* of set S .
 - e.g. $3 \in \mathbf{N}$, “ a ” $\in \{x \mid x \text{ is a letter of the alphabet}\}$

- Can define set equality in terms of \in relation:

$$\forall S, T: S=T \leftrightarrow (\forall x: x \in S \leftrightarrow x \in T)$$

“Two sets are equal **iff** they have all the same members.”

- $x \notin S \equiv \neg(x \in S)$ “ x is not in S ”



The Empty Set

- \emptyset (“null”, “the empty set”) is the unique set that contains no elements whatsoever.

- $\emptyset = \{\} = \{x/\mathbf{False}\}$

- No matter the domain of discourse,

we have the axiom

$$\neg\exists x: x\in\emptyset.$$

Subset and Superset Relations

- $S\subseteq T$ (“S is a subset of T”) means that every element of S is also an element of T.

- $S\subseteq T \Leftrightarrow \forall x (x\in S \rightarrow x\in T)$

- $\emptyset\subseteq S, S\subseteq S.$

- $S\supseteq T$ (“S is a superset of T”) means $T\subseteq S.$

- Note $S=T \Leftrightarrow S\subseteq T \wedge S\supseteq T.$

- means $\neg(S\subseteq T),$ i.e. $\exists x(x\in S \wedge x\notin T)$



Sets Are Objects

- The objects that are elements of a set may *themselves* be sets.
- E.g. let $S = \{x \mid x \subseteq \{1,2,3\}\}$

then $S = \{\emptyset,$

$\{1\}, \{2\}, \{3\},$

$\{1,2\}, \{1,3\}, \{2,3\},$

$\{1,2,3\}\}$

- Note that $1 \neq \{1\} \neq \{\{1\}\} \text{ !!!!}$



Cardinality of Sets

- $|S|$ (read “the *cardinality* of S ”) is a measure of how many different elements S has.
- *E.g.*, $|\emptyset|=0$, $|\{1,2,3\}|=3$, $|\{a,b\}|=2$,
 $|\{\{1,2,3\},\{4,5\}\}|=$ 2
- We say S is *infinite* if it is not *finite*.
- What are some infinite sets we’ve seen?
- If a set S contains n distinct elements, $n \in \mathbb{N}$, we call S a finite set with cardinality n .



Examples:

- $A = \{\text{Mercedes, BMW, Porsche}\}$, $|A| = 3$
- $B = \{1, \{2, 3\}, \{4, 5\}, 6\}$ $|B| = 4$
- $C = \emptyset$ $|C| = 0$
- $E = \{x \in \mathbb{N} \mid x \geq 7000\}$ **E is infinite!**

The Power Set

- The *power set* $P(S)$ of a set S is the set of all subsets of S . $P(S) = \{x \mid x \subseteq S\}$.

- *E.g.* $P(\{a,b\}) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$.

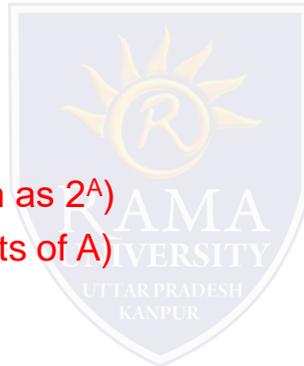
- Sometimes $P(S)$ is written 2^S .

Note that for finite S , $|P(S)| = 2^{|S|}$.

- It turns out that $|P(\mathbf{N})| > |\mathbf{N}|$.

There are different sizes of infinite sets!

- $P(A)$ “power set of A ” (also written as 2^A)
- $P(A) = \{B \mid B \subseteq A\}$ (contains all subsets of A)



Examples:

- $A = \{x, y, z\}$
- $P(A) = \{\emptyset, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}\}$

- $A = \emptyset$
- $P(A) = \{\emptyset\}$

Note: $|A| = 0$, $|P(A)| = 1$

The Power Set

- **Cardinality of power sets:** $|P(A)| = 2^{|A|}$
- Imagine each element in A has an “on/off” switch
- Each possible switch configuration in A corresponds to one subset of A , thus one element in $P(A)$

A	1	2	3	4	5	6	7	8
x	x	x	x	x	x	x	x	x
y	y	y	y	y	y	y	y	y
z	z	z	z	z	z	z	z	z

- For 3 elements in A , there are $2 \times 2 \times 2 = 8$ elements in $P(A)$

Cartesian Products of Sets

- For sets A, B , their *Cartesian product*

$$A \times B := \{(a, b) \mid a \in A \wedge b \in B\}.$$

- *E.g.* $\{a, b\} \times \{1, 2\} = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

- Note that for finite A, B , $|A \times B| = |A| |B|$.

- Note that the Cartesian product is **not** commutative: $\neg \forall A, B: A \times B = B \times A$.

- Extends to $A_1 \times A_2 \times \dots \times A_n \dots$

- The **ordered n-tuple** $(a_1, a_2, a_3, \dots, a_n)$ is an **ordered collection** of n objects.

- Two ordered n -tuples $(a_1, a_2, a_3, \dots, a_n)$ and

$(b_1, b_2, b_3, \dots, b_n)$ are equal if and only if they contain exactly the same elements **in the same order**, i.e.

$a_i = b_i$ for $1 \leq i \leq n$.

- The **Cartesian product** of two sets is defined as:

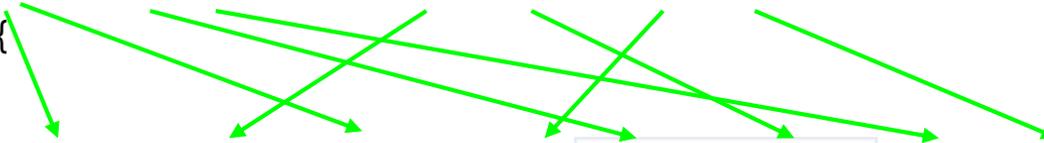
- $A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$

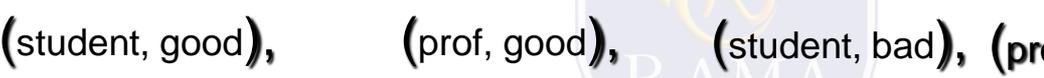


Cartesian Product

•Example:

• $A = \{\text{good, bad}\}$, $B = \{\text{student, prof}\}$

• $A \times B = \{$

 $(\text{good, student}), (\text{good, prof}), (\text{bad, student}), (\text{bad, prof})\}$

$B \times A = \{$

 $(\text{student, good}), (\text{prof, good}), (\text{student, bad}), (\text{prof, bad})\}$

Example: $A = \{x, y\}$, $B = \{a, b, c\}$

$A \times B = \{(x, a), (x, b), (x, c), (y, a), (y, b), (y, c)\}$

Note that:

- $A \times \emptyset = \emptyset$
- $\emptyset \times A = \emptyset$
- For non-empty sets A and B: $A \neq B \Leftrightarrow A \times B \neq B \times A$
- $|A \times B| = |A| \cdot |B|$
- The Cartesian product of **two or more sets** is defined as:
- $A_1 \times A_2 \times \dots \times A_n = \{(a_1, a_2, \dots, a_n) \mid a_i \in A_i \text{ for } 1 \leq i \leq n\}$

Set Operations

- Union: $A \cup B = \{x \mid x \in A \vee x \in B\}$

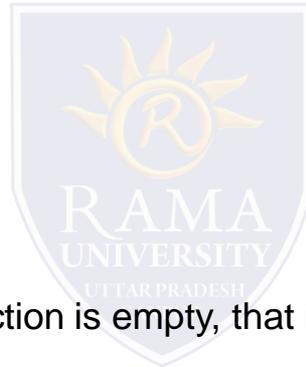
Example: $A = \{a, b\}$, $B = \{b, c, d\}$

- $A \cup B = \{a, b, c, d\}$

- Intersection: $A \cap B = \{x \mid x \in A \wedge x \in B\}$

Example: $A = \{a, b\}$, $B = \{b, c, d\}$

- $A \cap B = \{b\}$
- Cardinality: $|A \cup B| = |A| + |B| - |A \cap B|$



- Two sets are called **disjoint** if their intersection is empty, that is, they share no elements:

- $A \cap B = \emptyset$

- The **difference** between two sets A and B contains exactly those elements of A that are not in B:

- $A - B = \{x \mid x \in A \wedge x \notin B\}$

Example: $A = \{a, b\}$, $B = \{b, c, d\}$, $A - B = \{a\}$

- Cardinality: $|A - B| = |A| - |A \cap B|$