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Discrete Mathematics (CSPS-111)

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Outlines

- Introduction of Set Theory
- Basic Notations for Sets
- Basic properties of sets
- Definition of Set Equality
- Examples for Sets
- Subsets
- Infinite Sets
- Venn Diagrams

Introduction of Set Theory

Introduction of Set Theory:

- A set is a structure, representing an unordered collection (group, plurality) of zero or more distinct (different) objects.
- Set theory deals with operations between, relations among, and statements about sets.
- Set: Collection of objects (called elements)
- $a \in A$ "a is an element of A"

  "a is a member of A"
- $A \notin A$ "a is not an element of A"
- A = {a1, a2, …, an} "A contains a1, …, an"
- Order of elements is insignificant
- It does not matter how often the same element is listed (repetition doesn't count).

Basic Notations for Sets

- For sets, we'll use variables *S*, *T*, *U*, …
- We can denote a set *S* in writing by listing all of its elements in curly braces:
  - {a, b, c} is the set of whatever 3 objects are denoted by a, b, c.
- *Set builder notation*: For any proposition $P(x)$ over any universe of discourse,

  $\{x|P(x)\}$ is *the set of all x such that P(x).*

  e.g., $\{x \mid x$ is an integer where $x>0$ and $x<5\}$

Basic properties of sets

- Sets are inherently ***unordered***:
  - No matter what objects a, b, and c denote,

    {a, b, c} = {a, c, b} = {b, a, c} =

    {b, c, a} = {c, a, b} = {c, b, a}.
- All elements are ***distinct*** (unequal);

  multiple listings make no difference!
  - {a, b, c} = {a, a, b, a, b, c, c, c, c}.
  - This set contains at most 3 elements!

Definition of Set Equality

- Two sets are declared to be equal *if and only if* they contain exactly the same elements.
- In particular, it does not matter *how the set is defined or denoted.*
- For example: The set {1, 2, 3, 4} =

  {*x* | *x* is an integer where *x*>0 and *x*<5 } =

  {*x* | *x* is a positive integer whose square

  is >0 and <25}

•Sets A and B are equal if and only if they contain exactly the same elements.

•Examples:

- *A = {9, 2, 7, -3}, B = {7, 9, -3, 2} :* A = B
- A = {dog, cat, horse},
  B = {cat, horse, squirrel, dog} : A ≠ B
- A = {dog, cat, horse},
  B = {cat, horse, dog, dog} : A = B

Examples for Sets

- "Standard" Sets:
- Natural numbers **N** = {0, 1, 2, 3, …}
- Integers **Z** = {…, -2, -1, 0, 1, 2, …}
- Positive Integers **Z**$^+$ = {1, 2, 3, 4, …}
- Real Numbers **R** = {47.3, -12, π, …}
- Rational Numbers **Q** = {1.5, 2.6, -3.8, 15, …}

- (correct definitions will follow)

- A = $\varnothing$ "empty set/null set"
- A = {z} Note: $z \in A$, but $z \neq \{z\}$
- A = {{b, c}, {c, x, d}} set of sets
- A = {{x, y}} Note: $\{x, y\} \in A$, but $\{x, y\} \neq \{\{x, y\}\}$
- A = {x | P(x)} "set of all x such that P(x)"
- P(x) is the membership function of set A
- $\forall x\, (P(x) \rightarrow x \in A)$
- $A = \{x \mid x \in \mathbf{N} \wedge x > 7\} = \{8, 9, 10, \ldots\}$ "set builder notation"

Subsets

- $A \subseteq B$ “A is a subset of B”
- $A \subseteq B$ if and only if every element of A is also an element of B.
- We can completely formalize this:
- $A \subseteq B \Leftrightarrow \forall x\,(x \in A \rightarrow x \in B)$
- Examples:

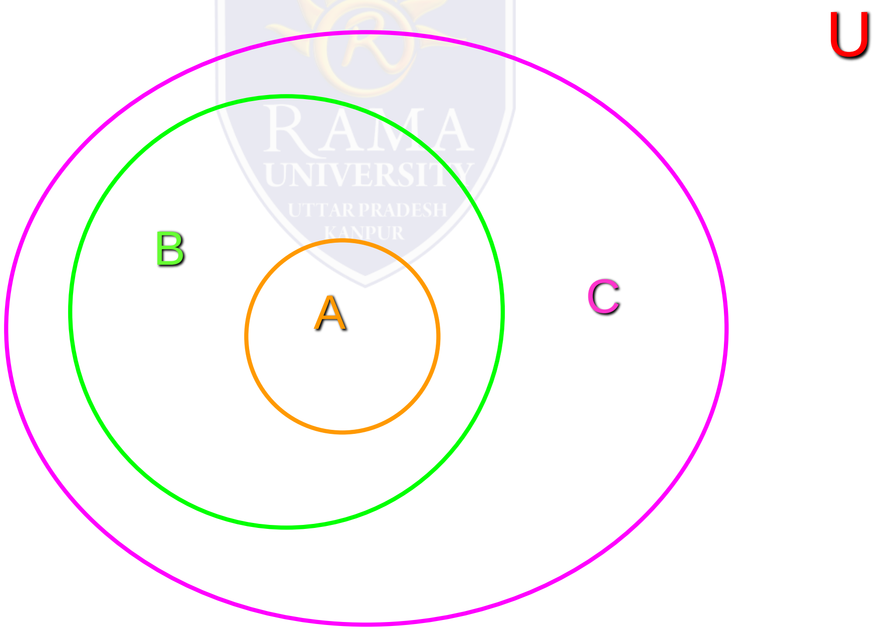
$A = \{3, 9\}$, $B = \{5, 9, 1, 3\}$, $A \subseteq B$? true

$A = \{3, 3, 3, 9\}$, $B = \{5, 9, 1, 3\}$, $A \subseteq B$? true

$A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $A \subseteq B$? false

Subsets

- Useful rules:
- $A = B \Leftrightarrow (A \subseteq B) \wedge (B \subseteq A)$
- $(A \subseteq B) \wedge (B \subseteq C) \Rightarrow A \subseteq C$ (see Venn Diagram)



Subsets

- Useful rules:
- $\varnothing \subseteq A$ for any set A

  (but $\varnothing \in A$ may not hold for any set A)
- $A \subseteq A$ for any set A
- Proper subsets:
- $A \subset B$ "A is a proper subset of B"
- $A \subset B \Leftrightarrow \forall x\ (x \in A \rightarrow x \in B) \wedge \exists x\ (x \in B \wedge x \notin A)$

  or
- $A \subset B \Leftrightarrow \forall x\ (x \in A \rightarrow x \in B) \wedge \neg \forall x\ (x \in B \rightarrow x \in A)$

Infinite Sets

- Conceptually, sets may be *infinite* (*i.e.,* not *finite*, without end, unending).
- Symbols for some special infinite sets:

  N = {0, 1, 2, …} The **n**atural numbers.

  Z = {…, -2, -1, 0, 1, 2, …} The integers.

  R = The "**r**eal" numbers, such as 374.1828471929498181917281943125…
- Infinite sets come in different sizes!

Venn Diagrams