

## FACULTY OF ENGINEERING \& TECHNOLOGY

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## BOOLEAN ALGEBRA

## Canonical form of Boolean Expression (Standard form)

- In standard SOP and POS each term of Boolean expression must contain all the literals (with and without bar) that has been used in Boolean expression.
-If the above condition is satisfied by the Boolean expression, that expression is called Canonical form of Boolean expression.
-In Boolean expression $A B+A C$, the literal $C$ is not in the 1st term $A B$ and $B$ is not in $2 n d$ term $A C$. That is why $A B+A C$ is not a Canonical SOP.


## e.g. Convert AB+AC in Canonical SOP (Standard SOP).

## Sol.

$$
\begin{aligned}
& A B+A C \\
& =A B\left(C+C^{\prime}\right)+A C\left(B+B^{\prime}\right) \\
& =A B C+A B C^{\prime}+A B C+A B^{\prime} C \\
& =A B C+A B C^{\prime}+A B^{\prime} C
\end{aligned}
$$

## Minterm \& Maxterm:

-Each term of Canonical Sum of Products (SOP) is called Minterm. In otherwords minterm is a product of all the literals (with or without bar) within the Boolean expression.

- 1 ' means the variable is "Not Complemented" and ' 0 ' means the variable is "Complemented".
-Each term of Canonical Products of Sum (POS) is called Maxterm. In otherwords maxterm is a sum of all the literals (with or without bar) within the Boolean expression.
- 0 ' means the variable is "Not Complemented" and ' 1 ' means the variable is "Complemented".


## BOOLEAN ALGEBRA

Minterms \& Maxterms for 2 variables (Derivation of Boolean function from Truth Table)

| $\mathbf{x}$ | $\mathbf{y}$ | Index | Minterm | Maxterm |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | $m_{0}=x^{\prime} y^{\prime}$ | $M_{0}=x+y$ |
| 0 | 1 | 1 | $m_{1}=x^{\prime} y$ | $M_{1}=x+y^{\prime}$ |
| 1 | 0 | 2 | $m_{2}=x y^{\prime}$ | $M_{2}=x^{\prime}+y$ |
| 1 | 1 | 3 | $m_{3}=x y$ | $M_{3}=x^{\prime}+y^{\prime}$ |

-The minterm $\mathrm{m}_{i}$ should evaluate to 1 for each combination of x and y .
-The maxterm is the complement of the minterm

## BOOLEAN ALGEBRA

Karnaugh map (K - map)
Boolean functions can be simplified using the Boolean theorems but This method of simplification is not used in practice due to reduced expression is not minimal \& unique. For that reason Karnaugh map ( K - map) method is used most frequently.

- It is used when output is $0,1 \& x$ (don't care).
-In K-Map gray code representation is used.
-K-maps are graphical representations of Boolean functions.
-lt's similar to truth table; instead of being organized (i/p \& o/p) into columns and rows, the K-map is an array of cells in which each cell represents a binary value of the input variables.
-K-maps can be used for expressions with 2, 3, 4, and 5 variables.
Two-Variable Map


OR


- ordering of variables is IMPORTANT for $f\left(x_{1}, x_{2}\right)$..
- Cell 0 represents $x_{1}^{\prime} x_{2}^{\prime}$; Cell 1 represents $x_{1}{ }^{\prime} x_{2}$; etc. If a minterm is present in the function, then a 1 is placed in the corresponding cell.


## 2-Variable Map -- Example

- $f\left(x_{1}, x_{2}\right)=x_{1}{ }^{\prime} x_{2}{ }^{\prime}+x_{1}{ }^{\prime} x_{2}+x_{1} x_{2}{ }^{\prime}$

$$
\begin{aligned}
& =m_{0}+m_{1}+m_{2} \\
& =x_{1}^{\prime}+x_{2}^{\prime}
\end{aligned}
$$

- $\quad 1 \mathrm{~s}$ placed in K-map for specified minterms $\mathrm{m}_{0}, \mathrm{~m}_{1}, \mathrm{~m}_{2}$
- Grouping of 1 s allows simplification
- What (simpler) function is represented by each dashed rectangle?
$-\mathrm{x}_{1}{ }^{\prime}=\mathrm{m}_{0}+\mathrm{m}_{1}$
$-x_{2}{ }^{\prime}=m_{0}+m_{2}$
- Here $\mathrm{m}_{0}$ covered twice

