Adjusted Exponential Smoothing

$$AF_{t+1} = F_{t+1} + T_{t+1}$$

where

T = an exponentially smoothed trend factor

$$T_{t+1} = \beta(F_{t+1} - F_t) + (1 - \beta)\, T_t$$

where

T_t = the last period trend factor

β = a smoothing constant for trend

$0 \le \beta \le 1$

$F_{t+1} = A_t + T_t$

Where,

$A_t = \alpha D_t + (1 - \alpha)(A_{t-1} + T_{t-1})$ and

T = an exponentially smoothed trend factor

$T_t = \beta(A_t - A_{t-1}) + (1 - \beta)T_{t-1}$

T = an exponentially smoothed trend factor

T_{t-1} = the last period trend factor

β = a smoothing constant for trend

$0 \le \beta \le 1$

Question

PM Computer Services assembles customized personal computers from generic parts. they need a good forecast of demand for their computers so that they will know how many parts to purchase and stock. They have compiled demand data for the last 12months. There is an upward trend in the demand. Use trend-adjusted exponential smoothing with smoothing parameter α= 0.5 and trend parameter β= 0.3 to compute the demand forecast for January (Period 13).

Period	Month	Demand	Period	Month	Demand
1	January	37	7	July	43
2	February	40	8	August	47
3	March	41	9	September	56
4	April	37	10	October	52
5	May	45	11	November	55
6	June	50	12	December	54

Solution:

For Period 2,

we have $F2 = A1 + T1$, so to get the process started, let $A0 = 37$ and $T0 = 0$.

$A1 = \alpha D1 + (1 - \alpha)(A0 + T0) = 0.5(37) + (1 - 0.5)(37 + 0) = 37$,

and $T1 = \beta(A1 - A0) + (1 - \beta)T0 = 0.3(37 - 37) + (1 - 0.3)(0) = 0$

$F_2 = A_1 + T_1 = 37 + 0 = 37$

For Period 3,

A2 = αD2 +(1−α)(A1+T1) = 0.5(40)+(1−0.5)(37+0) = 38.5, and

T2 = β(A2 −A1)+(1−β)T1 =0.3(38.5− 37)+ (1 − 0.3)(0) = 0.45.

F3 = A2 + T2 = 38.5+ 0.45 = 38.95.

			Expon. Smooth..	Trend-Adjusted Expon. Smooth. (α= 0.5, β = 0.3)		
Period	Month	Demand	α = 0.5	At	Tt	Ft
1	Jan	37	37.00	37.00	0.00	37.00
2	Feb	40	37.00	38.50	0.45	37.00
3	Mar	41	38.50	39.98	0.76	38.95
4	Apr.	37	39.75	38.87	0.20	40.73
5	May	45	38.38	42.03	1.09	39.06
6	Jun.	50	41.69	46.56	2.12	43.12
7	Jul.	43	45.84	45.84	1.27	48.68
8	Aug.	47	44.42	47.05	1.25	47.11
9	Sep.	56	45.71	52.15	2.41	48.31
10	Oct.	52	50.86	53.28	2.02	54.56
11	Nov.	55	51.43	55.15	1.98	55.30
12	Dec.	54	53.21	55.56	1.51	57.13
13	Jan	?	53.61			57.07

B. Casual or explanatory methods

Simple Linear Regression Model

$y = a + bx$

where

a = intercept

b = slope of the line

x = time period

y = forecast for demand for period x

$Nov_4 = WMA_3$ 0.5(720)+0.3(678)+0.2(650)=693.4

$$a = \bar{y} - b\bar{x}$$

$$b = \frac{\sum xy - n(\bar{y})(\bar{x})}{\sum x^2 - n(\bar{x})^2}$$

Question: Given the data below, what is the simple linear regression model that can be used to predict sales in future weeks?

Week	Sales
1	150
2	157
3	162
4	166
5	177

Solution: First, using the linear regression formulas, we can compute "a" and "b".

Week	Week*Week	Sales	Week*Sales
1	1	150	150
2	4	157	314
3	9	162	486
4	16	166	664
5	25	177	885
3	*55*	*162.4*	*2499*
Average	Sum	Average	Sum

$$b = \frac{\sum xy - n(\bar{y})(\bar{x})}{\sum x^2 - n(\bar{x})^2} = \frac{2499 - 5(162.4)(3)}{55 - 5(9)} = \frac{63}{10} = \mathbf{6.3}$$

$$a = \bar{y} - b\bar{x} = 162.4 - (6.3)(3) = \mathbf{143.5}$$

The resulting regression model is:

$$Y_t = 143.5 + 6.3x$$

Correlation Coefficient, *r*

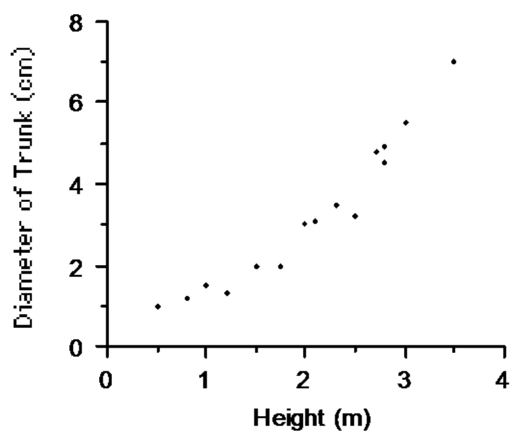
- ✓ The quantity *r*, called the *linear correlation coefficient*, measures the strength and the direction of a linear relationship between two variables. The linear correlation coefficient is sometimes referred to as the *Pearson product moment correlation coefficient* in honor of its developer Karl Pearson.
- ✓ The value of r is such that $-1 < r < +1$. The + and – signs are used for positive linear correlations and negative linear correlations, respectively.
- ✓ Positive correlation: If x and y have a strong positive linear correlation, r is close

to +1. An r value of exactly +1 indicates a perfect positive fit.   Positive values indicate a relationship between x and y variables such that as values for x increases, values for y also increase.

- ✓ Negative correlation: If x and y have a strong negative linear correlation, r is close to -1. An r value of exactly -1 indicates a perfect negative fit.   Negative values indicate a relationship between x and y such that as values for x increase, values for y decrease.
- ✓ No correlation: If there is no linear correlation or a weak linear correlation, r is Close to 0. A value near zero means that there is a random, nonlinear relationship between the two variables
- ✓ Note that r is a dimensionless quantity; that is, it does not depend on the units employed.
- ✓ A perfect correlation of ± 1 occurs only when the data points all lie exactly on a straight line. If r = +1, the slope of this line is positive. If r = -1, the slope of this line is negative.

Positive Correlation

Figure 1: Relationship between height and trunk diameter in Eastern White Pines



Notice that in this example as the heights increase, the diameters of the trunks also tend to increase. If this were a perfect positive correlation all of the points would fall on a straight line. The more linear the data points, the closer the relationship between the two variables.

Negative Correlation