From the graph, the different ranges of production volumes over which the best location to be selected are summarized.

Range of production volume	Best plant selected
$0 \le Q \le 400$	A
$400 \le Q \le 1000$	B
$1000 \le Q$	C

The same details can be worked out using a graph

From the graph one can visualize that the site c is desirable for lower volume of production. For higher volume production site B is desirable For moderate volumes of production site nA is desirable. In the increasing order of production volume the switch over from one site to another takes place as per the order below

Site C to site A to site B

Let Q be the volume at which we switch the site C to site A

Total cost of site C ≥ Total cost site A

$$5000000+4000Q \ge 6000000+1500*Q$$

$$2500Q \ge 1000000$$

$$Q \ge 400 \text{ Units}$$

Similarly the switch from site A to site B

Total cost of site A ≥ total cost of site B

$$6000000+1500Q \ge 7000000+500Q$$

$$1000Q \ge 1000000$$

$$Q \ge 1000 \text{ Units}$$

The cutoff production volume for different ranges of production may be obtained by using similar procedure.

4.5.2 GRAVITY LOCATION PROBLEM

Objective- The objective of the gravity location problem, the total material handling cost based on the squared Euclidian distance is minimized

Assumption:- If the same type of material handling equipment / vehicle is used for all the movements, then it is equivalent to minimize the total weighted squared Euclidian distance, since the cost per unit distance is minimized

a_i = x-co-ordinate of the existing facilities i

b_i = y- co-ordinate of the existing facilities i

x = x-co-ordinate of the new facilities

y= y-co-ordinate of the new facilities

w_i= weight associated with the existing facilities i. This is the quantum of materials moved between the new facility and existing facilities I per unit period

m= total no of existing facilities

the formula for the sum of the weighted squared Euclidian distance is given as:

$$f(x,y) = \sum_{i=1}^{m} w\,i[(x - ai)^2 + (y - bi)^2]$$

The objective is to minimize f(x,y)

This is quadratic in nature the optimal values for the x and y may be obtained by equating partial derivatives to zero

$$\frac{\delta f(x,y)}{\delta x} = o \ , \ \frac{\delta f(x,y)}{\delta y} = 0$$

$$x *= \frac{\sum_{i=1}^{m} wiai}{\sum_{i=1}^{m} wi} \quad ,y *= \frac{\sum_{i=1}^{m} wibi}{\sum_{i=1}^{m} wi}$$

Optimal location (x*,y*)=($\frac{\sum_{i=1}^{m} wiai}{\sum_{i=1}^{m} wi}$, $\frac{\sum_{i=1}^{m} wibi}{\sum_{i=1}^{m} wi}$)

These are weighted averages of the x-coordinate and y-co ordinates of the existing facilities.

Problem

There are five Existing facilities which are to be served by single new facilities are shown below in the table

Existing facility (i)	1	2	3	4	5
Co-ordinates (ai,bi)	(5,10) (15,20)	(20,5) (30,35)	(15,20) (25,40)	(30,25) (28,30)	(25,5) (32,40)
No of trips of loads/years (wi)	100 200	300 300	200 400	300 500	100 600

Find the optimal location of the new facilities based on giving location concept

SOLUTION

$$X^*=\frac{\sum_{i=1}^{5} wiai}{\sum_{i=1}^{5} wi}=\frac{(100*5+300*20+200*15+300*30+100*25)}{(100+300+200+300+100)}= 21$$

$$Y^*=\frac{\sum_{i=1}^{5} wibi}{\sum_{i=1}^{5} wi}=\frac{(100*10+300*5+200*20+300*25+100*5)}{(100+300+200+300+100)}=14.5$$

4.5.3 SINGLE FACILITY LOCATION PROBLEM

Objective – To determine the optimal location for the new facility by using the given set of existing facilities co-ordinates on X-Y plane and movement of materials from a new facility to all existing facilities.

Generally we follow rectilinear distance for such decision. The rectilinear distance between any two points whose co-ordinates are (X1,Y1)and(X2,Y2) is given by the following formula

$d_{12}=|X1-X2|+|Y1-Y2|$

some properties of an optimum solution to the rectilinear distance location problems are as follows:

1. The X-coordinate of the new facility will be same as the X-co-ordinate of some existing facility. Similarly the Y co-ordinate of the new facility will coincide with the Y coordinate of some existing facility. It is not necessary that both coordinates of the new facility
2. The optimum X or Y-co-ordinate location for new facility is a median location. A median location is defined to be a location such that no more than one half the item movement is to the left/below of the new facility location and no more than one half the item movement is to the right /above of the new facility location.

EXAMPLE

Consider the location of a new plant which will supply raw materials to a set of existing plants in a group of companies, let there are 5 existing plants which have a materials movement relationship with the new plant. Let the existing plants have locations of (400,200),(800,500),(1100,800),(200,900)and(1300,300). Furthermore suppose that the number of tons of materials transported per year from the new plant to various existing plants are 450,1200,300,800 and 1500, respectively the objective is to determine optimum location for the new plant such that the distance moved(cost)is minimized

SOLUTION

Let (X,Y) be the coordinate of the new plant

The optimum X-coordinate for the new plant is determined as follows

Existing plant	X coordinate	weight	Cumulative Weight
4	200	800	800
1	400	450	1250
2	800	1200	2450
3	1100	300	2750
5	1300	1500	4250
		Total	4250 tons

Thus the median location corresponds to a cumulative weight of 4250/2=2125 from above the table, the corresponding X-coordinate value is 800, since the cumulative weight first exceeds 2125 at X=800

Similarly, the determination of Y coordinate is shown below

Existing plant	Y coordinate	weight	Cumulative Weight
1	200	450	450
5	300	1500	1950
2	500	1200	3150
3	800	300	3450
4	900	800	4250
		Total	4250 tons

Thus the median location corresponds to a cumulative weight of 4250/2=2125 from above the table, the corresponding Y-coordinate value is 500, since the cumulative weight first exceeds 2125 at X=500

The optimal (X*,Y*)=(800,500)