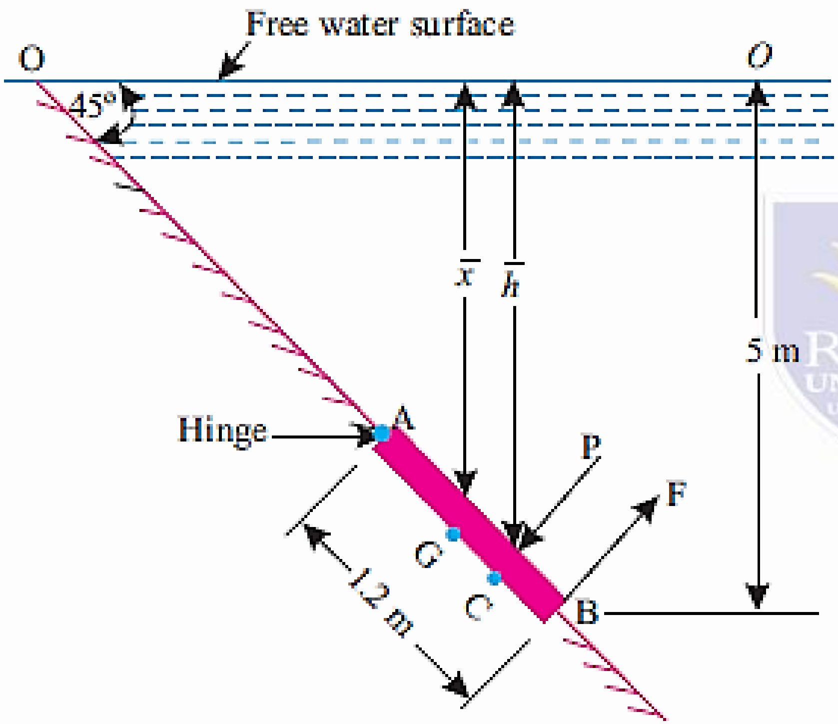
Fluid Properties

- An inclined rectangular
- sluice gate AB 1.2 m by 5 m size as shown in
- Fig. 3.33 is installed to control the discharge
- of water. The end A is hinged. Determine the
- force normal to the gate applied at B to open
- it.



- Solution. Size of the gate = 1.2 m × 5m
- Area of the gate = 1.2 × 5 = 6 m2
- Refer to Fig. Depth of c.g. of the gate from free water surface,
- xbar = 5 – BG sin 45° = 5 – 0.6 × 0.707 = 4.576 m
- The total pressure force (P) acting on the gate,
- P = wAx-bar = 9.81 × 6 × 4.576 = 269.3 kN
- This force acts at a depth h, given by the relation:

$$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$$

where, $$I_G = M.O.I. \text{ of gate} = \frac{bd^3}{12} = \frac{5\times 1.2^3}{12} = 0.72 \text{ m}^4, \theta = 45°$$

i.e., $$\bar{h} = \frac{0.72\times\sin^2 45°}{6\times 4.576} + 4.576 = 4.589 \text{ m}$$

From Fig. 3.33, we have $\frac{\bar{h}}{OC} = \sin 45°$

$$\text{Distance, } OC = \frac{\bar{h}}{\sin 45°} = \frac{4.589}{0.707} = 6.49 \text{ m;}$$

$$\text{Distance, } OB = \frac{5}{\sin 45°} = 7.072 \text{ m}$$

$\therefore$ $$\text{Distance, } BC = OB - OC = 7.072 - 6.49 = 0.582 \text{ m}$$

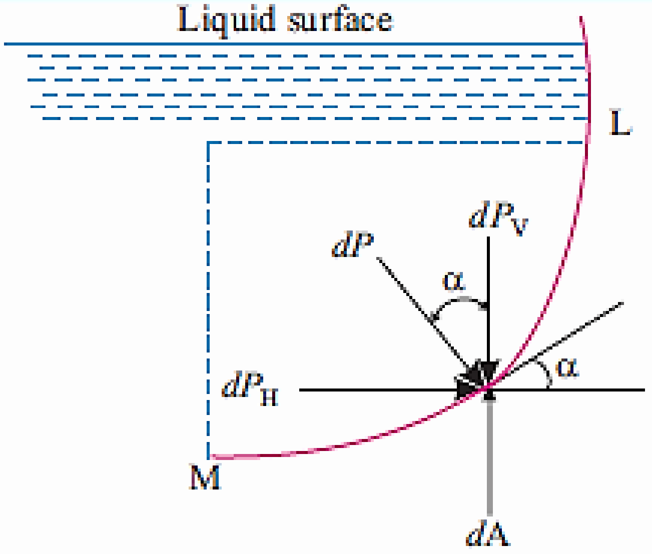
$$\text{Distance, } AC = AB - BC = 1.2 - 0.582 = 0.618 \text{ m}$$

Taking moments about the hinge A, we get:

$$F\times AB = P\times AC$$

or, $$F = \frac{P\times AC}{AB} = \frac{269.3\times 0.618}{1.2} = \textbf{138.69 kN (Ans.)}$$

CURVED IMMERSED SURFACE



Consider a curved surface LM submerged in a static fluid as shown in Fig. At any point on the curved surface, the pressure acts normal to the surface. Thus if dA is the area of a small element of the curved surface lying at a vertical depth of h from surface of the liquid, then the total pressureon the elemental area is,
dp = p × dA = (wh) × dA

This force *dP* acts normal to the surface. Further integration of eqn. (3.4) would provide the total pressure on the curved surface and hence,

$$P = \int wh\,dA$$

But, is case of curved surface the direction of the total pressures on the elementary areas are not in the same direction, but varies from point to point. Thus the ntegration of eqn. for curved surface is impossible. The problem, however, can be solved by resolving the force *P* into horizontal and vertical components *P*H and *P*V. Then total force on the curved surface is,

$$P = \sqrt{P_H^2 + P_V^2}$$

The direction of the resultant force P with the horizontal is given by: $\tan\theta = \dfrac{P_V}{P_H}$

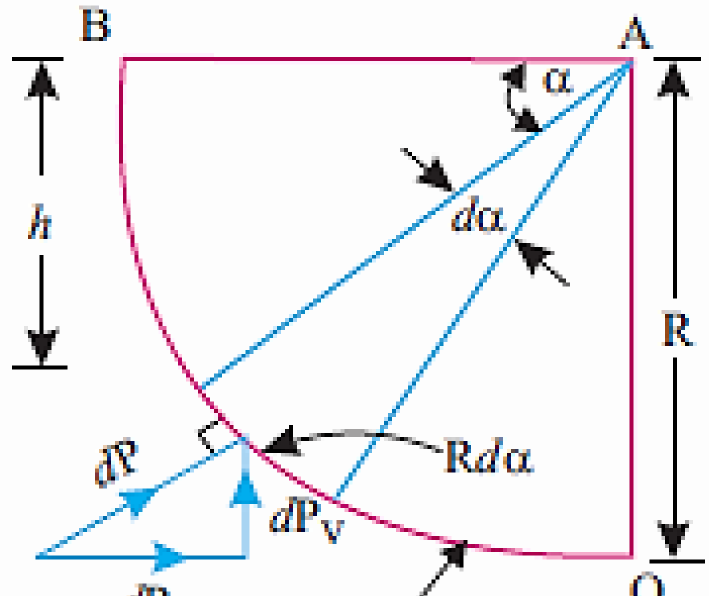
or, $$\theta = \tan^{-1}\left(\frac{P_V}{P_H}\right) \qquad ...(3.7)$$

Here, P_H = Total pressure force on the projected area of the curved surface on vertical plane, and

P_V = Weight of the liquid supported by the curved surface upto free surface of liquid.

Fluid Properties

- The profile of a vessel is quadrant of a circle of radius R. Determine the horizontal and vertical components of the total pressure force, from the first principles.
- Solution. Consider an elementary strip of radius R at depth h
- and subtending an angle as shown in Fig.
- Let the vessel has a unit depth perpendicular to the plane of
- paper. Then Area of the element, dA = Rdα × unit depth = Rdα
- Depth, h = R sinα
- Intensity of pressure, p = wh = w R sinα
- Pressure force, dp = p × dA = wRsinα × Rdα
- = wR2 sin α dα
- Vertical component of dP,dPV = wR2 sin α dα × sin α = w R2 sin2α dα
- Horizontal component of dP,
- dPH = wR2 sin α dα × cos α = wR2 sinα cosα dα
- ∴ Total vertical pressure force,



Profile of a vessel

$$P_V = \int_0^{\pi/2} wR^2 \sin^2\alpha \, d\alpha$$

$$= \frac{wR^2}{2}\left[\int_0^{\pi/2}\left(\frac{1-\cos 2\alpha}{2}\right) d\alpha\right]$$

$$= \frac{wR^2}{2}\left[\left|\alpha\right|_0^{\pi/2} - \left|\frac{\sin 2\alpha}{2}\right|_0^{\pi/2}\right] = \frac{wR^2\pi}{4}$$

$$= w\left(\frac{\pi R^2}{4} \times \text{unit length}\right)$$

Thus the vertical component of pressure force on a curved surface equals the weight of the volume liquid extending vertically from the curved surface to the free surface of liquid.
(Ans.)
Total horizontal pressure force,

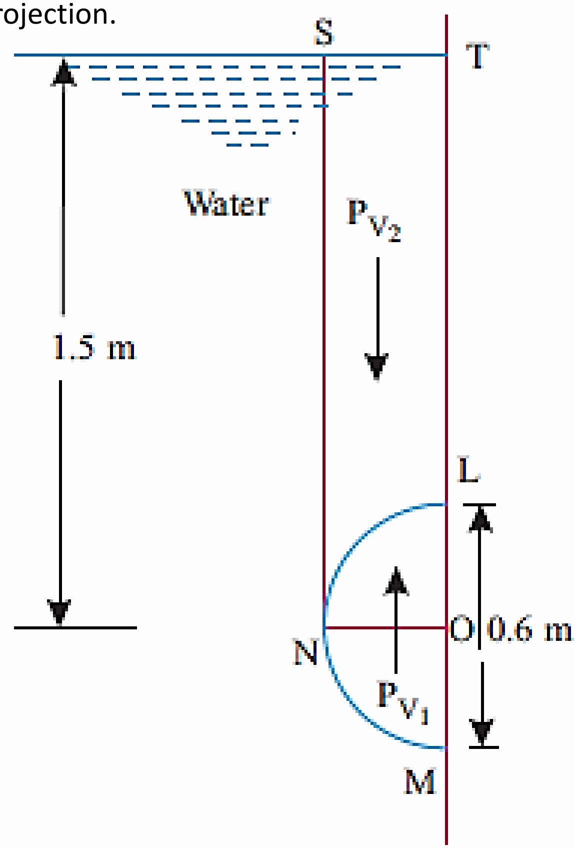
$$P_H = \int_0^{\pi/2} wR^2 \sin\alpha \cos\alpha \, d\alpha$$

$$= \frac{wR^2}{2}\int_0^{\pi/2} 2\sin\alpha \cos\alpha \, d\alpha$$

$$= \frac{wR^2}{2}\int_0^{\pi/2} \sin 2\alpha \, d\alpha = \frac{wR^2}{2}\left[-\frac{\cos 2\alpha}{2}\right]_0^{\pi/2} = \frac{wR^2}{2}$$

$$= w\,(R \times \text{unit length}) \times \frac{R}{2} \equiv w A \bar{x}$$

- A hemisphere projection of diameter 0.6 m exists on one of the vertical sides of a tank. If the tank contains water to an elevation of 1.5 m above the centre of the hemisphere, calculate the vertical and horizontal forces acting on the projection.



- Solution. Refer to Fig. Vertical force, PV = PV1
- – PV2 = Weight volume of water MNST –
- weight of volume of water LNST
- = Weight of water contained by the
- hemisphere LNM

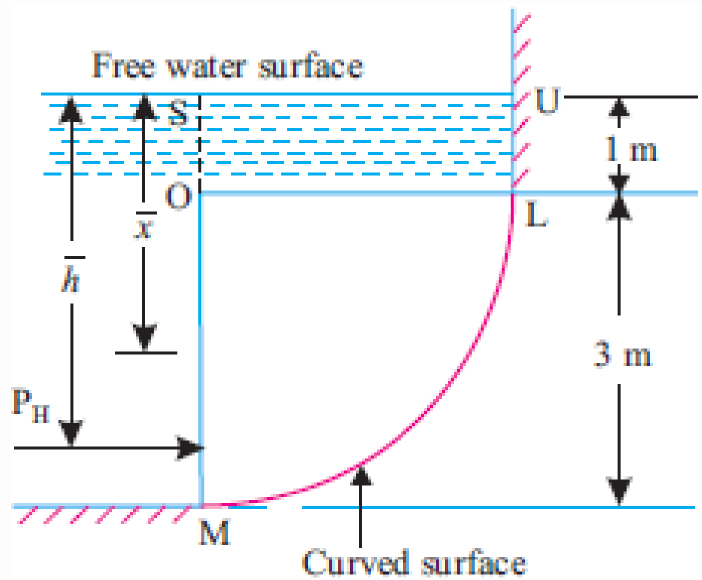
$$= w \times \frac{1}{2}\left(\frac{4}{3}\pi R^3\right)$$

$$= 9.81 \times \frac{1}{2} \times \frac{4}{3} \times \pi \times (0.3)^3$$

$$= \mathbf{0.555\ kN\ (Ans.)}$$

- Horizontal force, PH = wAx-bar
- = 9.81× π × (0.3)2 × 1.5 = 4.16 kN (Ans.)

- Fig. . shows a curved surface LM, which is in the form of a quadrant of a circle of radius 3 m, immersed in the water. If the width of the gate is unity, calculate the horizontal and vertical components of the total force acting on the curved surface.
- Solution. Radius of the gate = 3 m
- Width of the gate = 1m
- Refer to Fig. Distance LO = OM = 3 m
- Horizontal component of total force, PH:
- Horizontal force (PH) exerted by water on gate is given by,
- PH = Total pressure force on the projected area
- of curved surface LM on vertical plane
- = Total pressure force on OM (projected area of curved surface on vertical plane



$$= OM \times 1) = wA\bar{x}$$

But, $A = OM \times 1 = 3 \times 1 = 3\text{m}^2$ and $\bar{x} = 1 + \frac{3}{2} = 2.5 \text{ m}$

$$P_H = 9.81 \times (3 \times 1) \times 2.5 = \mathbf{73.57 \text{ kN (Ans.)}}$$

The point of application of P_H is given by:

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

where, $I_G = M.O.I.$ of OM about its c.g. $= \frac{bd^3}{12} = \frac{1 \times 3^3}{12} = 2.25 \text{ m}^4$

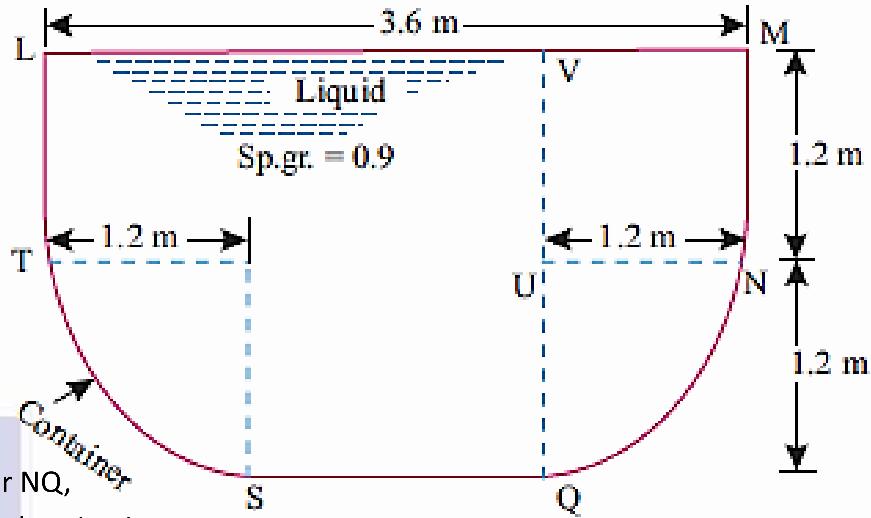
$$\therefore \quad \bar{h} = \frac{2.25}{(3 \times 1) \times 2.5} + 2.5 = \mathbf{2.8 \text{ m from water surface (Ans.)}}$$

Vertical component of total force, P_V:

Vertical force (P_V) exerted by water is given by:

$$\begin{aligned} P_V &= \text{Weight of water supported by } LM \text{ upto free surface} \\ &= \text{weight of portion } ULMOS \\ &= \text{weight of } ULOS + \text{weight of water in } LOM \\ &= w\,(\text{volume of } ULOS + \text{volume of } LOM) \\ &= 9.81\left[UL \times LO + \frac{\pi \times (LO)^2}{4} \times 1\right] = 9.81\left[1 \times 3 + \frac{\pi \times 3^2}{4} \times 1\right] \\ &= 9.81\,(3 + 7.068) \text{ kN} = \mathbf{98.77 \text{ kN (Ans.)}} \end{aligned}$$

- A liquid of specific gravity 0.9 is filled in a container, shown in Fig., upto a depth of 2.4 m. Determine the magnitude and direction of hydrostatic pressure force per unit length of container exerted on its vertical face MN and curved corner NQ.



Vertical face MN:

$$P = wA\bar{x} = w \times (MN \times \text{unit length}) \times \frac{MN}{2}$$

$$= (9.81 \times 0.9) \times (1.2 \times 1) \times \frac{1.2}{2} = \mathbf{6.357\ kN\ (Ans.)}$$

This force acts horizontally towards right and its point of application is given by:

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x} = \frac{1 \times \frac{1.2^3}{12}}{(1.2 \times 1) \times \frac{1.2}{2}} + \frac{1.2}{2} = \mathbf{0.8\ m\ (Ans.)}$$

- Curved surface NQ:
- Horizontal component of hydrostatic pressure force on the curved corner NQ,
- PH = Specific weight × vertical projected area × dep th of centre of vertical projection
- Vertical component of hydrostatic pressure force on the curved corner,
- PV = Weight of liquid contained in portion MNQUV
- = Specific weight [Volume of liquid in portion MNUV + volume
- of liquid in portion NQU]

$$= w \times (QU \times \text{unit length}) \times \left(VU + \frac{UQ}{2}\right)$$

$$= (9.81 \times 0.9) \times (1.2 \times 1) \times \left(1.2 + \frac{1.2}{2}\right) = 19.07\ \text{kN}$$

$$= w\,[MN \times NU \times \text{unit length} + \frac{1}{4}\pi \times (NU)^2 \times \text{unit length}]$$

$$= (9.81 \times 0.9)\,[1.2 \times 1.2 \times 1 + \frac{1}{4} \times \pi \times (1.2)^2 \times 1] = 22.7\ \text{kN}$$

$$\text{Resultant pressure force, } P = \sqrt{P_H^2 + P_V^2} = \sqrt{(19.07)^2 + (22.7)^2} = \mathbf{29.65\ kN\ (Ans.)}$$

The angle made by the resultant with the horizontal,

$$\theta = \tan^{-1}\left(\frac{P_V}{P_H}\right) = \tan^{-1}\left(\frac{22.7}{19.07}\right) = \mathbf{49.97^\circ\ (Ans.)}$$