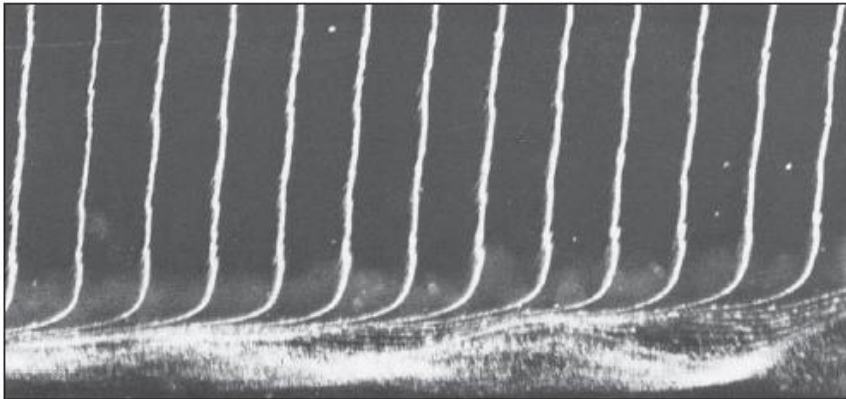
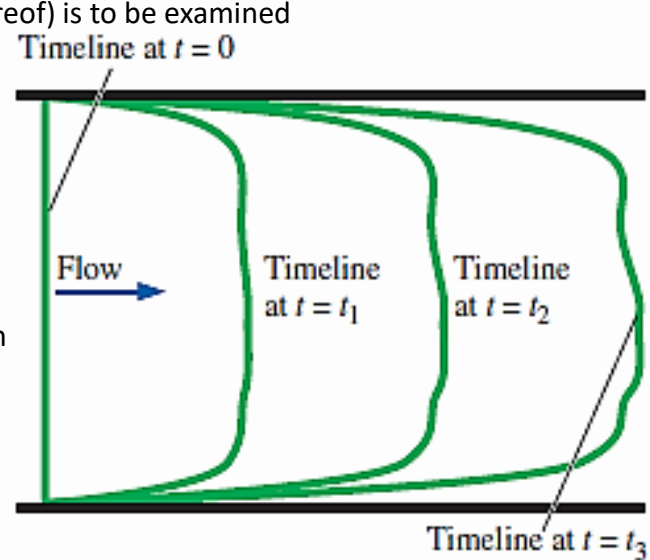


Fluid Properties

- Timelines:
- A timeline is a set of adjacent fluid particles that were marked at the same (earlier) instant in time.
- Timelines are particularly useful in situations where the uniformity of a flow (or lack thereof) is to be examined
- Timelines are formed by marking line of fluid particles, and then that line move (and deform) through the flow field; timelines are shown at $t = 0$, t_1 , t_2 , and t_3

- friction at the walls, the fluid velocity there is zero (the no-slip condition),
- and the top and bottom of the timeline are anchored at their starting locations.
- Timelines can be generated experimentally in a water channel through use of a hydrogen
- Bubble wire. When a short burst of electric current is sent through the cathode wire,
- electrolysis of the water occurs and tiny hydrogen gas bubbles form at the
- wire. Since the bubbles are so small, their buoyancy is nearly negligible,
- and the bubbles follow the water flow nicely



Timelines produced by a hydrogen bubble wire are used to visualize the boundary layer velocity profile shape along a flat plate. Flow is from left to right, and the hydrogen bubble wire is located to the left of the field of view. Bubbles near the wall reveal a flow instability that leads to turbulence.

Numerical on velocities and acceleration

- In a fluid, the velocity field is given by $V = (3x + 2y) i + (2z + 3x^2) j + (2t - 3z) k$ Determine:
- (i) The velocity components u, v, w at any point in the flow field;
- (ii) The speed at point $(1, 1, 1)$;
- (iii) The speed at time $t = 2s$ at point $(0, 0, 2)$.
- Also classify the velocity field as steady, or unsteady, uniform or non-uniform and one, two or three dimensional.

Solution. Given: Velocity field, $V = (3x + 2y) i + (2z + 3x^2) j + (2t - 3z) k$

(i) Velocity components:

The velocity components are:

$$u = 3x + 2y, v = (2z + 3x^2), w = (2t - 3z) \text{ (Ans.)}$$

(ii) Speed at point $(1, 1, 1)$, $V_{(1,1,1)}$:

Substituting $x = 1, y = 1, z = 1$ in the expressions for u, v and w , we have:

$$u = (3 + 2) = 5, v = (2 + 3) = 5, w = (2t - 3)$$

$$\begin{aligned} \therefore V^2 &= u^2 + v^2 + w^2 \\ &= 5^2 + 5^2 + (2t - 3)^2 \\ &= 25 + 25 + 4t^2 - 12t + 9 \\ &= 4t^2 - 12t + 59 \end{aligned}$$

$$\therefore V_{(1,1,1)} = \sqrt{4t^2 - 12t + 59} \text{ (Ans.)}$$

(iii) Speed at $t = 2s$ at point $(0, 0, 2)$:

Substituting $t = 2, x = 0, y = 0, z = 2$ in the expressions for u, v and w , we get:

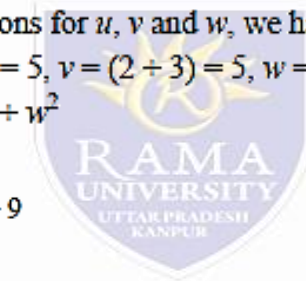
$$u = 0, v = (2 \times 2 + 0) = 4, w = (2 \times 2 - 3 \times 2) = -2$$

$$\therefore V^2 = u^2 + v^2 + w^2 = 0 + 4^2 + (-2)^2 = 20$$

$$\text{or, } V_{(0,0,2)} = \sqrt{20} = 4.472 \text{ units (Ans.)}$$

Velocity field, type:

- (i) Since V at given (x, y, z) depends on t it is **unsteady flow**, (Ans.)
- (ii) Since at given t velocity changes in the X direction it is **non-uniform flow**, (Ans.)
- (iii) Since V depends on x, y, z it is **three dimensional flow**, (Ans.)



Fluid Properties

- Velocity for a two dimensional flow field is given by $V = (3 + 2xy + 4t^2) i + (xy^2 + 3t) j$ Find the velocity and acceleration at a point (1,2) after 2 sec.

Solution. Given: Velocity field: $V = (3 + 2xy + 4t^2) i + (xy^2 + 3t) j$

Velocity at (1, 2), $V_{(1,2)}$:

Substituting $x = 1, y = 2, t = 2$ in the expression of velocity field, we get:

$$\begin{aligned} V &= (3 + 2 \times 1 \times 2 + 4 \times 2^2) i + (1 \times 2^2 + 3 \times 2) j \\ &= (3 + 4 + 16) i + (4 + 6) j \\ &= 23i + 10j \end{aligned}$$

$$\therefore V_{(1,2)} = \sqrt{23^2 + 10^2} = 25.08 \text{ units (Ans.)}$$

Acceleration at point (1, 2), $a_{(1,2)}$:

We know that:

$$a = \frac{dV}{dt} = \left(u \frac{\partial V}{\partial x} + v \frac{\partial V}{\partial y} \right) + \frac{\partial V}{\partial t}$$

Also,

$$V = (3 + 2xy + 4t^2) i + (xy^2 + 3t) j \quad \dots(\text{Given})$$

$$\therefore \frac{\partial V}{\partial x} = 2yi + y^2 j,$$

$$\frac{\partial V}{\partial y} = 2xi + 2xyj, \text{ and}$$

$$\frac{\partial V}{\partial t} = 8ti + 3j$$

$$\begin{aligned} \therefore a &= (3 + 2xy + 4t^2) (2yi + y^2 j) + (xy^2 + 3t) (2xi + 2xyj) + (8ti + 3j) \\ (\because u &= 3 + 2xy + 4t^2 \text{ and } v = xy^2 + 3t) \end{aligned}$$

Substituting the values, we get:

$$\begin{aligned} a &= (3 + 2 \times 1 \times 2 + 4 \times 2^2) (2 \times 2i + 2^2 j) + (1 \times 2^2 + 3 \times 2) \\ &\quad (2 \times 1i + 2 \times 1 \times 2j) + (8 \times 2 \times i + 3j) \\ &= (3 + 4 + 16) (4i + 4j) + (4 + 6) (2i + 4j) + (16i + 3j) \end{aligned}$$

$$\begin{aligned} &= 23(4i + 4j) + 10(2i + 4j) + (16i + 3j) \\ &= 92i + 92j + 20i + 40j + 16i + 3j \\ &= 128i + 135j \end{aligned}$$

$$a_{(1,2)} = \sqrt{128^2 + 135^2} = 186.03 \text{ units (Ans.)}$$

Fluid Properties

Find the velocity and acceleration at a point (1, 2, 3) after 1 sec. for a three-dimensional flow given by $u = yz + t$, $v = xz - t$, $w = xy$ m/s.

Solution. Given: Three-dimensional flow field is given as:

$$u = yz + t, v = xz - t, w = xy \text{ m/s}$$

Velocity at a point 1, 2, 3 $V_{(1,2,3)}$:

Velocity at a point (1, 2, 3) after 1s is calculated as follows:

$$u = yz + t = 2 \times 3 + 1 = 7 \text{ m/s}, v = xz - t = 1 \times 3 - 1 = 2 \text{ m/s and}$$

$$w = xy = 1 \times 2 = 2 \text{ m/s.}$$

$$\begin{aligned} \therefore V_{(1,2,3)} &= 7i + 2j + 2k \\ &= \sqrt{7^2 + 2^2 + 2^2} = 7.55 \text{ m/s} \end{aligned}$$

Hence, $V_{(1,2,3)} = 7.55 \text{ m/s (Ans.)}$

Acceleration, $a_{(1,2,3)}$:

Now,

$$V = (yz + t)i + (xz - t)j + xy k \text{ m/s}$$

Acceleration,

$$a = \frac{dV}{dt} = \left(u \frac{\partial V}{\partial x} + v \frac{\partial V}{\partial y} + w \frac{\partial V}{\partial z} \right) + \frac{\partial V}{\partial t}$$

$$a = (yz + t)(zi + yk) + (xz - t)(zi + xk) + xy(yi + xj) + (1i - 1j)$$

$$\begin{aligned} \therefore a_{(1,2,3)} &= 7(3j + 2k) + 2(3i + 1k) + 2(2i + 1j) + (1i - 1j) \\ &= (21j + 14k) + (6i + 2k) + (4i + 2j) + (1i - 1j) \\ &= (10i + 23j + 16k) + (1i - 1j) \end{aligned}$$

The convective acceleration components are: (10, 23, 16) m/s^2

The local acceleration components are: (1, -1) m/s^2 along x and y directions.

The total acceleration of fluid particles at the points (1, 2, 3) is given by:

$$\begin{aligned} a_{(1,2,3)} &= \sqrt{(10 + 1)^2 + [23 + (-1)]^2 + 16^2} \\ &= \sqrt{11^2 + 22^2 + 16^2} = 29.34 \text{ m/s}^2 \end{aligned}$$

Hence, $a_{(1,2,3)} = 29.34 \text{ m/s}^2 \text{ (Ans.)}$

Fluid Properties

For a three-dimensional flow the velocity distribution is given by $u = -x$, $v = 3 - y$ and $w = 3 - z$. What is the equation of a streamline passing through (1,2,2)?

Solution. Given: $u = -x$, $v = 3 - y$, $w = 3 - z$

Equation of a streamline passing through (1, 2, 2):

The streamlines are defined by:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Substituting for u, v and w , we get:

$$\begin{aligned} \frac{dx}{-x} &= \frac{dy}{3-y} = \frac{dz}{3-z} \\ (i) \quad (ii) \quad (iii) \end{aligned}$$

Considering the expressions (i) and (ii) and integrating, we get:

$$\begin{aligned} \int \frac{dx}{-x} &= \int \frac{dy}{(3-y)} \\ &= -\log_e x = -\log_e (3-y) + C_1 \end{aligned}$$

(where, C_1 = constant of integration).

Since the streamline passes through $x = 1, y = 2 \quad \therefore C_1 = 0$

$$\therefore (x)^{-1} = (3-y)^{-1} \quad \text{or} \quad x = (3-y) \quad \dots(1)$$

Considering the expressions (i) and (iii), and integrating, we get:

$$\begin{aligned} \int \frac{dx}{-x} &= \int \frac{dz}{3-z} \\ \text{or,} \quad -\log_e x &= -\log (3-z) + C_2 \end{aligned}$$

(where C_2 = constant of integration)

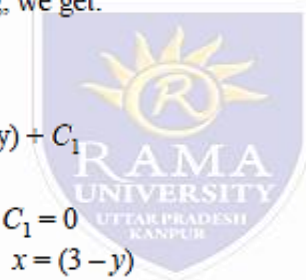
Since the streamline passes through $x = 1, z = 2 \quad \therefore C_2 = 0$

$$\therefore x^{-1} = (3-z)^{-1}$$

or, $x = (3-z) \quad \dots(2)$

From (1) and (2), the equation of the streamline passing through (1, 2, 2) is given as:

$$x = (3-y) = (3-z) \quad \text{(Ans.)}$$



Fluid Properties

- Obtain the equation to the streamlines for the velocity field given as: $V = 2x^3i - 6x^2yj$

Solution. Given: Velocity field, $V = 2x^3i - 6x^2yj$

Here, $u = 2x^3, v = -6x^2y$

The streamlines in two dimensions are defined by:

$$\frac{dx}{u} = \frac{dy}{v}$$

or,
$$\frac{dy}{dx} = \frac{v}{u} = \frac{-6x^2y}{2x^3} = \frac{-3y}{x}$$

Separating the variables, we have:

$$\frac{dy}{y} = \frac{-3dx}{x}$$

Integrating, we get:

$$\ln(y) = -3\ln(x) + C_1$$

or, $\ln(y) + 3\ln(x) = C_1$

or, $yx^3 = C \text{ (Ans.)}$

