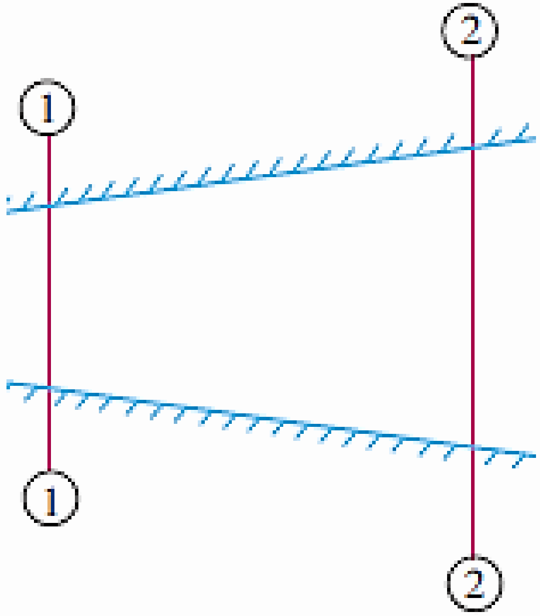
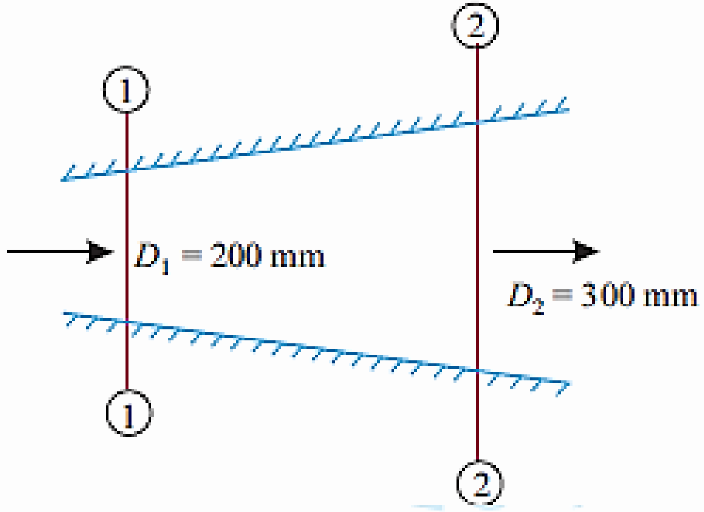
- **RATE OF FLOW OR DISCHARGE**
- Rate of flow (or discharge) is defined as the quantity of a liquid flowing per second through a section of pipe or a channel. It is generally denoted by Q. Let us consider a liquid flowing through a pipe.
- Let, A = Area of cross-section of the pipe, and
- V = Average velocity of the liquid.
- ∴ Discharge, Q = Area × average velocity i.e., Q = A.V
- If area is in m2 and velocity is in m/s, then the discharge,
- Q = m2 × m/s = m3/s = cumecs

- **CONTINUITY EQUATION**
- The continuity equation is based on the principle of conservation of mass. It states as follows: "If no fluid is added or removed from the pipe in any length then the mass passing across different sections shall be same."
- Consider two cross-sections of a pipe as shown in Fig
- Let, A1 = Area of the pipe at section 1–1,
- V1 = Velocity of the fluid at section 1–1,
- ρ1 = Density of the fluid at section 1–1,
- and A2, V2, ρ2 are corresponding values at sections 2–2.
- The total quantity of fluid passing through section 1–1= ρ1 A1 V1
- and, the total quantity of fluid passing through section 2–2 = ρ2A2V2
- From the law of conservation of mass (theorem of continuity), we have:
- ρ1A1V1 = ρ2A2V2 ..
- Eqn. (5.22) is applicable to the compressible as well as
- incompressible fluids and is called Continuity Equation. In case
- of incompressible fluids, ρ1 = ρ2 and the continuity eqn. reduces to:
- A1 V1 = A2V2 ...(5.23)



- The diameters of a pipe at the sections 1-1and 2-2 are 200 mm and 300 mm respectively. If the velocity of water flowing through the pipe at section 1-1 is 4m/s, find:
- (i) Discharge through the pipe, and(ii) Velocity of water at section 2-2



Solution. Diameter of the pipe at *section* 1-1,

$$D_1 = 200 \text{ mm} = 0.2 \text{ m}$$

$$\therefore \quad \text{Area, } A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$$

$$\text{Velocity, } V_1 = 4 \text{ m/s}$$

Diameter of the pipe at *section* 2-2,

$$D_2 = 300 \text{ mm}$$

$$\therefore \quad \text{Area, } A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times 0.3^2 = 0.0707 \text{ m}^2$$

(*i*) Discharge through the pipe, Q:

Using the relation,

$$Q = A_1 V_1, \text{ we have:}$$

$$Q = 0.0314 \times 4 = \mathbf{0.1256 \text{ m}^3/s} \text{ (Ans.)}$$

(*ii*) Velocity of water at section 2-2, V_2:

Using the relation,

$$A_1 V_1 = A_2 V_2, \text{ we have:}$$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.0314 \times 4}{0.0707}$$

$$= \mathbf{1.77 \text{ m/s}} \text{ (Ans.)}$$

Fluid Properties

- A pipe (1) 450 mm in diameter branches into two pipes (2 and 3) of diameters 300 mm and 200 mm respectively as shown in Fig. 5.15. If the average velocity in 450 mm diameter pipe is 3 m/s find:
- (i) Discharge through 450 mm diameter pipe; (ii) Velocity in 200 mm diameter pipe if the average velocity in 300 mm pipe is 2.5 m/s.

Solution. Diameter, $D_1 = 450\text{ mm} = 0.45\text{ m}$

$$\therefore \quad \text{Area, } A_1 = \frac{\pi}{4} \times 0.45^2 = 0.159\text{ m}^2$$

$$\text{Velocity, } V_1 = 3\text{ m/s}$$

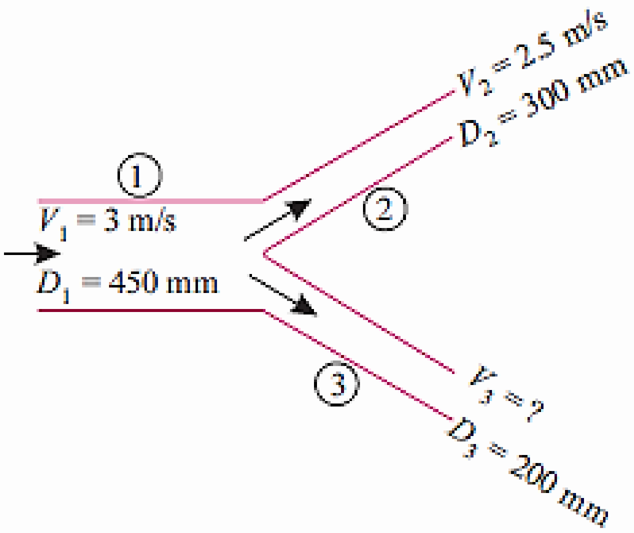
$$\text{Diameter, } D_2 = 300\text{ mm} = 0.3\text{ m}$$

$$\therefore \quad \text{Area, } A_2 = \frac{\pi}{4} \times 0.3^2 = 0.0707\text{ m}^2$$

$$\text{Velocity, } V_2 = 2.5\text{ m/s}$$

$$\text{Diameter, } D_3 = 200\text{ mm} = 0.2\text{ m}$$

$$\text{Area, } A_3 = \frac{\pi}{4} \times 0.2^2 = 0.0314\text{ m}^2$$



(*i*) Discharge through pipe (1) Q_1:

Using the relation, $Q_1 = A_1V_1 = 0.159 \times 3$

$= \mathbf{0.477\ m^3/s\ (Ans.)}$

- (ii) Velocity in pipe of diameter 200 mm i.e. V3: Let Q1, Q2 and Q3 be the discharge in pipes 1, 2 and 3 respectively. Then, according to continuity equation:
- Q1 = Q2 + Q3 ...(i)
- where, Q1 = 0.477 m3/s (calculated earlier)
- and, Q2 = A2V2 = 0.0707 × 2.5 = 0.1767 m3/s
- ∴ 0.477 = 0.1767 + Q2 [from eq. (i)]
- or, Q3 = 0.477 – 0.1767 ⍰ 0.3 m3/s
- But, Q3 = A3V3

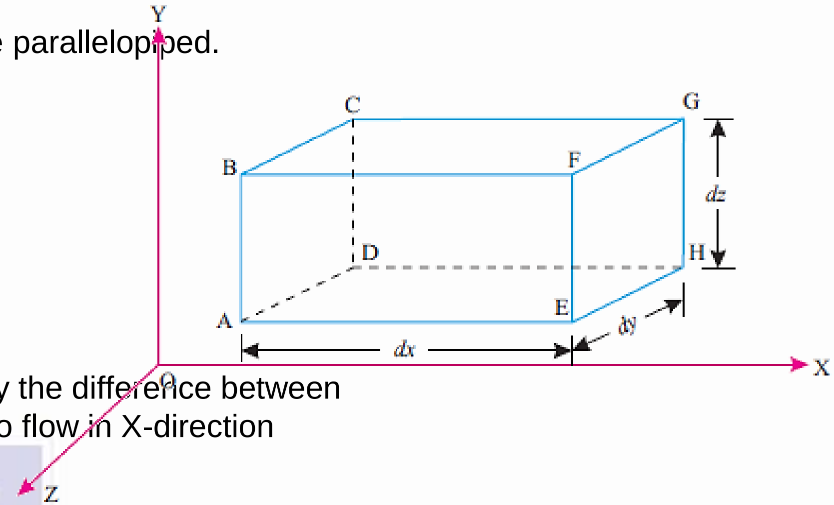
$$V_3 = \frac{Q_3}{A_3} = \frac{0.3}{0.0314} = 9.55\text{ m/s}$$

$$V_3 = \mathbf{9.55\ m/s\ (Ans.)}$$

CONTINUITY EQUATION IN CARTESIAN CO-ORDINATES

Consider a fluid element (control volume) – parallelopiped with sides dx, dy and dz as shown in Fig.
Let, ρ = Mass density of the fluid at a particular instant;
u, v, w = Components of velocity of flow entering the three faces of the parallelopiped.
Rate of mass of fluid entering the face ABCD (i.e. fluid influx).
= ρ × velocity in X-direction × area of ABCD
= ρ udy dz ...(i)
Rate of mass of fluid leaving the face EFGH (i.e. fluid efflux).

$$= \rho u\,dy\,dz + \frac{\partial}{\partial x}(\rho u\,dy\,dz)\,dx$$



The gain in mass per unit time due to flow in the X-direction is given by the difference between the fluid influx and fluid efflux. ∴ Mass accumulated per unit time due to flow in X-direction

$$= \rho u\,dy\,dz - \left[\rho u + \frac{\partial}{\partial x}(\rho u)\,dx\right]dy\,dz$$

$$= -\frac{\partial}{\partial x}(\rho u)\,dx\,dy\,dz$$

Similarly, the gain in fluid mass per unit time in the parallelopiped due to flow in Y and Z-directions

$$= -\frac{\partial}{\partial y}(\rho v)\,dx\,dy\,dz \quad \text{(in Y-direction)} \qquad ...(iv)$$

$$= -\frac{\partial}{\partial z}(\rho w)\,dx\,dy\,dz \quad \text{(in Z-direction)} \qquad ...(v)$$

The total (or net) gain in fluid mass per unit for fluid along three co-ordinate axes

$$= -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right]dx\,dy\,dz \qquad ...(vi)$$

Rate of change of mass of the parallelopiped (control volume)

$$= \frac{\partial}{\partial t}(\rho\,dx\,dy\,dz) \qquad ...(vii)$$

Equations (*vi*) and (*vii*), we get:

$$-\left[\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)\right]dx\,dy\,dz=\frac{\partial}{\partial t}(\rho\,dx\,dy\,dz)$$

Simplification and rearrangement of terms would reduce the above expression to:

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)+\frac{\partial\rho}{\partial t}=0 \qquad \text{...(5.24}$$

This eqn. (5.24) is the general equation of continuity in three-dimensions and is applicable t *any type of flow* and for any fluid whether *compressible or incompressible.*

For *steady flow* $\left(\frac{\partial\rho}{dt}=0\right)$ *incompressible fluids* (ρ = constant) the equation reduces to:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial y}=0 \qquad \text{...(5.25}$$

For *two dimensional flow*, eqn. (5.25) reduces to:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (\because w=0)$$

For *one dimensional flow*, say in *X*-direction, eqn. (5.25) takes the form:

$$\frac{\partial u}{\partial x}=0 \qquad (\because v=0, w=0)$$

Integrating with respect to *x*, we get:

$$u=\text{constant} \qquad \text{...(5.26)}$$

If the area of flow is *a* then the rate of flow is

$$Q=a.u=\text{constant for } \textit{steady flow}$$

which is the same eqn. (5.23) and states that if area of flow *a* is constant the velocity of flow *u* will also be constant.

EQUATION OF CONTINUITY IN POLAR COORDINATES

- Consider a fluid element LMST as shown in Fig. 5.17. The sides of the element has the following dimensions.
- LT = MS = dr; LM = rdθ and ST = (r + dr)dθ
- Let, Vr = Component of the velocity in the radial direction, and Vθ = Component of the velocity in the tangential direction.
- Further, let thickness of the element perpendicular to the plane of paper be unity. As the fluid flows through the element, changes will place in its velocity as well as in the density

Flow in radial direction:

Mass of fluid *entering the face LM* during time *dt* is given by:

$$\text{Fluid } \textit{influx} = \text{Density} \times (\text{velocity} \times \text{area}) \times \text{time}$$

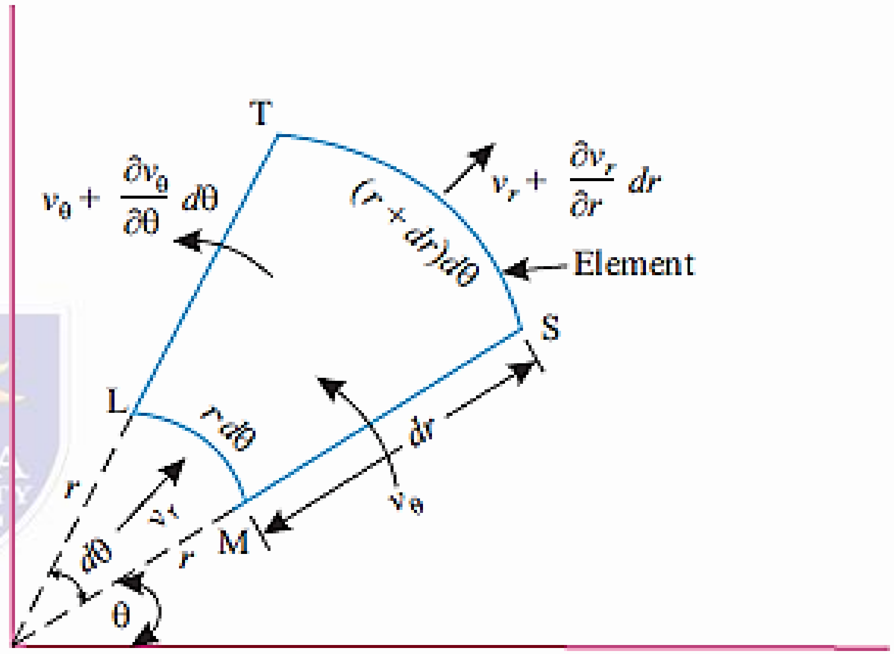
$$= \rho \times (v_r \times r d\theta) \times dt$$

Mass of fluid *leaving the face ST* during the same time *dt* is given by:

$$\text{Fluid } \textit{efflux} = \left[\rho v_r + \frac{\partial}{\partial r}(\rho v_r)\,dr\right](r + dr)\,d\theta . dt$$

Mass accumulated in the element because of flow in radial direction

$$= \text{Fluid influx} - \text{fluid efflux}$$



$$= \rho \times (v_r \times r d\theta) \times dt - \left[\rho v_r + \frac{\partial}{\partial r}(\rho v_r)\,dr\right](r + dr)\,d\theta\,dt$$

$$= -\left[\rho v_r\,dr\,d\theta + \frac{\partial}{\partial r}(\rho v_r)\,dr \cdot d\theta\right]dt$$

[Neglecting terms containing $(dr)^2$] . . . .

Flow in tangential direction:

The mass accumulated due to flow in the tangential direction (by a similar treatment as discussed earlier).

$$= \left[\rho v_\theta\,dr - \left\{\rho v_\theta + \frac{\partial}{\partial \theta}(\rho v_\theta)\,d\theta\right\}dr\right]dt = -\frac{\partial}{\partial \theta}(\rho v_\theta)\,dr\,d\theta\,dt$$

∴ Total gain in fluid mass

$$= -\left[\rho v_r\,dr\,d\theta + \frac{\partial}{\partial r}(\rho v_r)\,dr\;r d\theta + \frac{\partial}{\partial \theta}(\rho v_\theta)\,dr\,d\theta\right]dt$$