$$= \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g}$$

Eqn. (6.5) gives the discharge under ideal conditions and is called *theoretical discharge*. Actual discharger (Q_{act}) which is less than the theoretical discharge (Q_{th}) is given by:

$$Q_{act} = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh} \quad ...(6.6)$$

where, C_d = *Co-efficient of venturimeter* (or co-efficient of discharge) and its value is *less than unity* (varies between 0.96 and 0.98)

- *Due to variation of C_d venturimeters are not suitable for very low velocities.*

Value of 'h' by differential U-tube manometer:

Case. I. *Differential manometer containing a liquid heavier than the liquid flowing through the pipe.*

Let, S_{hl} = Sp. gravity of heavier liquid,
S_p = Sp. gravity of liquid flowing through pipe, and
y = Difference of the heavier liquid column in U-tube.

$$h = y\left[\frac{S_{hl}}{S_p} - 1\right]$$

Case. II. Differential manometer containing a liquid lighter than the liquid flowing through the pipe.

Let, Sll = Sp. gravity of lighter liquid,
Sp = Sp. gravity of liquid flowing through pipe, and
y = Difference of lighter liquid column in U-tube.

Then, $$h = y\left[1 - \frac{S_{ll}}{S_p}\right]$$

A horizontal venturimeter with inlet diameter 200 mm and throat diameter 100 mm is used to measure the flow of water. The pressure at inlet is 0.18 N/mm2 and the vacuum pressure at the throat is 280 mm of mercury. Find the rate of flow. The value of Cd may be taken as 0.98.

Solution. Inlet diameter of venturimeter, $D_1 = 200 \text{ mm} = 0.2 \text{ m}$

$\therefore$ Area of inlet, $A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$

Throat diameter, $D_2 = 100 \text{ mm} = 0.1 \text{ m}$

$\therefore$ Area of throat, $A_2 = \frac{\pi}{4} \times 0.1^2 = 0.00785 \text{ m}^2$

Pressure at inlet, $p_1 = 0.18 \text{ N/mm}^2 = 180 \text{ kN/m}^2$

$\therefore$ $\frac{p_1}{w} = \frac{180}{9.81} = 18.3 \text{ m}$

Vacuum pressure at the throat,

$$\frac{p_2}{w} = -280 \text{ mm of mercury}$$

$$= -0.28 \text{ m of mercury} = -0.28 \times 13.6 = -3.8 \text{ m of water}$$

Co-efficient of discharge, $C_d = 0.98$

$\therefore$ Differential head, $h = \frac{p_1}{w} - \frac{p_2}{w} = 18.3 - (-3.8) = 22.1 \text{ m}$

Rate of flow, Q:

Using the relation,

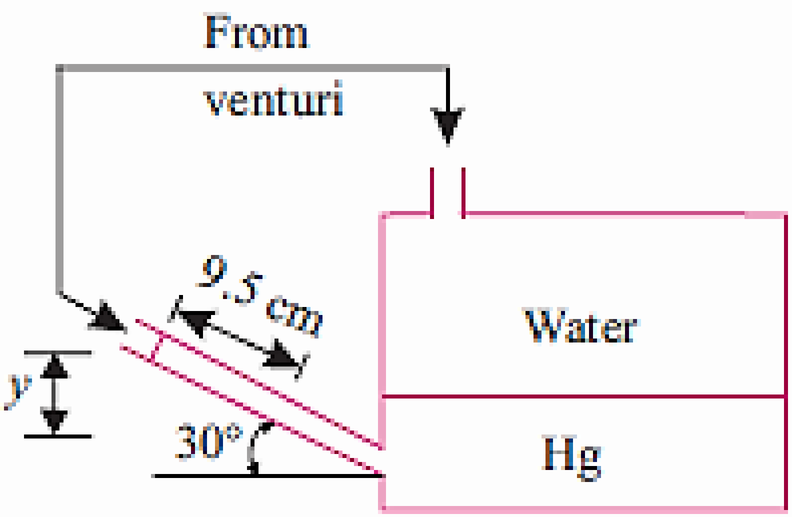
$$Q = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}, \text{ we have:}$$

$$= 0.98 \times \frac{0.0314 \times 0.00785}{\sqrt{(0.0314)^2 - (0.00785)^2}} \times \sqrt{2 \times 9.81 \times 22.1}$$

$$= \frac{0.000241}{0.0304} \times 20.82$$

or $Q = \mathbf{0.165 \text{ m}^3/\text{s}}$ **(Ans.)**

- A venturimeter (throat diameter = 10.5 cm) is fitted to a water pipeline (internal diameter = 21.0 cm) in order to monitor flow rate. To improve accuracy of measurement, pressure difference across the venturimeter is measured with the help of an inclined tube manometer, the angle of inclination being 30° (Fig. 6.30). For a manometer reading of 9.5 cm of mercury, find the flow rate. Discharge co-efficient of venturimeter is 0.984. (GATE



- Solution ; Internal dia., D1 = 21.0 cm = 0.21 m;

$$\text{Area of inlet, } A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.21)^2 = 0.0346 \text{ m}^2$$

$$\text{Throat dia, } D_2 = 10.5 \text{ cm} = 0.105 \text{ m}$$

$$\therefore \quad \text{Area at throat, } A_2 = \frac{\pi}{4} \times D_2^2 = \frac{\pi}{4} \times (0.105)^2 = 0.00866 \text{ m}^2$$

Discharge co-efficient of venturimeter, $C_d = 0.984$

$$\text{Pressure head, } h = y\left[\frac{S_{Hg}}{S_{water}} - 1\right] = (9.5 \sin 30°)\left[\frac{13.6}{1} - 1\right]$$

$$= 59.85 \text{ cm} = 0.5985 \text{ m}$$

Discharge (Q) through a venturimeter is given by:

$$Q = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$

$$= 0.984 \times \frac{0.0346 \times 0.00866}{\sqrt{(0.0346)^2 - (0.00866)^2}} \times \sqrt{2 \times 9.81 \times 0.5985}$$

$$= 0.984 \times 0.008945 \times 3.427 = 0.0302 \text{ m}^3\text{/s (Ans.)}$$

- Vertical and inclined venturimeters
- Vertical or inclined venturimeters are employed for measuring discharge on pipelines which arenot horizontal. The same formula for discharge as used for horizontal venturimeter holds good in these cases as well

Here, $$h = \left(\frac{p_1}{w} - \frac{p_2}{w}\right) + (z_1 - z_2)$$

[In horizontal venturimeters $z_1 - z_2 = 0$ as $z_1 = z_2$]

- A 200 mm × 100 mm venturimeter is provided in a vertical pipe carrying water, flowing in the upward direction. A differential mercury manometer connected to the inlet and throat gives a reading of 220 mm. Find the rate of flow. Assume Cd = 0.98.
- Solution. Diameter at the inlet, D1 = 200 mm = 0.2 m

$$\text{Area of inlet, } A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$$

$$\text{Diameter at the throat, } D_2 = 100 \text{ mm} = 0.1 \text{ m}$$

$$\text{Area at the throat, } A_2 = \frac{\pi}{4} \times 0.1^2 = 0.00785 \text{ m}^2$$

- Sp. gravity of heavy liquid (in the manometer), Shl = 13.6
- Sp. gravity of liquid flowing through pipe, Sp = 1.0
- Co-efficient of discharge, Cd = 0.98
- Reading of the differential manometer, y = 220 mm = 0.22 m

Rate of flow, Q:

Differential head,

$$h = \left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = y\left|\frac{S_{hl}}{S_p} - 1\right|$$

$$= 0.22\left(\frac{13.6}{1.0} - 1.0\right) = 2.77 \text{ m}$$

Using the relation, $$Q = C_d \cdot \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}\text{, we have}$$

$$Q = 0.98 \times \frac{0.0314 \times 0.00785}{\sqrt{0.0314^2 - 0.00785^2}} \times \sqrt{2 \times 9.81 \times 2.77}$$

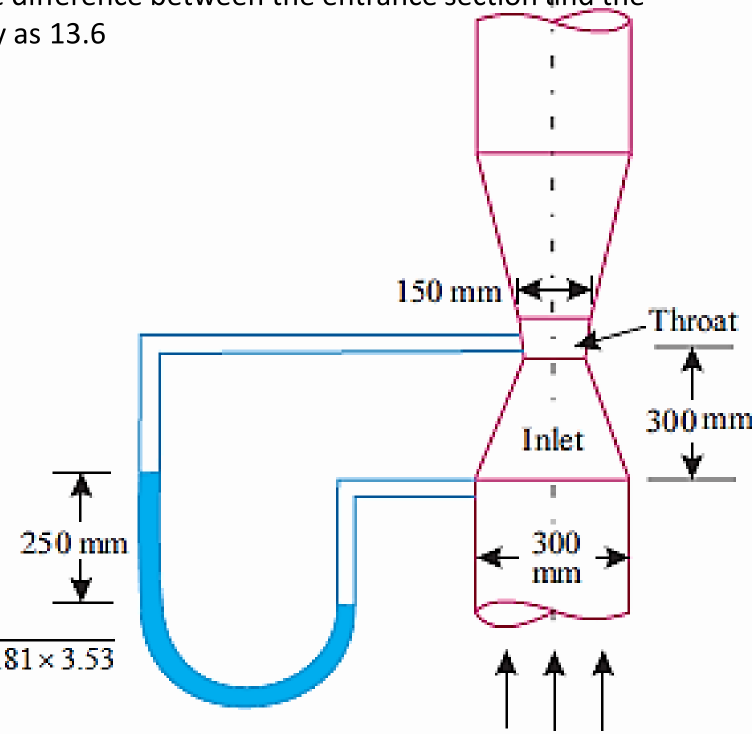
$$= \frac{0.000241}{0.0304} \times 7.34 = \mathbf{0.0584 \text{ m}^3/\text{s}} \text{ (Ans.)}$$

- A 300 mm × 150 mm venturimeter is provided in a vertical pipeline carryingoil of specific gravity 0.9, flow being upward. The difference in elevation of the throat section and entrance section of the venturimeter is 300 mm. The differential U-tube mercury manometer shows gauge deflection of 250 mm. Calculate: (i) The discharge of oil, and (ii) The pressure difference between the entrance section and the throat section. Take the co-efficient of meter as 0.98 and specific gravity of mercury as 13.6
- Solution. Diameter at inlet, D1 = 300 m m = 0.3 m

$$\text{Area of inlet, } A_1 = \frac{\pi}{4} \times 0.3^2 = 0.07\,\text{m}^2$$

$$\text{Diameter at throat, } D_2 = 150\text{ mm} = 0.15\text{ m}$$

$$\text{Area at throat, } A_2 = \frac{\pi}{4} \times 0.15^2 = 0.01767\text{ m}^2$$



- Specific gravity of heavy liquid (mercury)
- in U-tube manometer, Shl =13.6 Specific gravity of liquid (oil) flowing
- through pipe, Sp = 0.9 Reading of differential manometer,
- y = 250 mm = 0.25 m The differential 'h' is given by:

$$h = \left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right)$$

$$= y\left[\frac{S_{hl}}{S_p} - 1\right] = 0.25\left[\frac{13.6}{0.9} - 1\right]$$

$$= 3.53 \text{ m of oil}$$

(i) **Discharge of oil, Q:**

Using the relation,

(i) Discharge of oil, Q: Using the relation,

$$Q = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}\text{, we have:}$$

$$Q = 0.98 \times \frac{0.07 \times 0.01767}{\sqrt{0.07^2 - 0.01767^2}} \times \sqrt{2 \times 9.81 \times 3.53}$$

$$= \frac{0.001212}{0.0677} \times 8.32 = \mathbf{0.1489\ m^3/s.\ (Ans.)}$$

(ii) **Pressure difference between entrance and throat sections, $p_1 - p_2$:**

We know that, $$h = \left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = 3.53$$

or, $$\left(\frac{p_1}{w} - \frac{p_2}{w}\right) + (z_1 - z_2) = 3.53$$

But, $$z_2 - z_1 = 300 \text{ mm or } 0.3 \text{ m}$$

$$\therefore \quad \left(\frac{p_1}{w} - \frac{p_2}{w}\right) - 0.3 = 3.53 \quad \text{or} \quad \frac{p_1 - p_2}{w} = 3.83$$

or, $$p_1 - p_2 = (9.81 \times 0.9) \times 3.83 = \mathbf{33.8\ kN/m^2\ (Ans.)}$$

Fluid Properties

- The following data relate to an inclined venturimeter: Diameter of the pipeline = 400 mm Inclination of the pipeline with the horizontal = 30° Throat diameter = 200 mm The distance between the mouth and throat of the meter = 600 mm Sp. gravity of oil flowing through the pipeline = 0.7 Sp. gravity of heavy liquid (U-tube) = 13.6 Reading of the differential manometer = 50 mm The co-efficient of the meter = 0.98 Determine the rate of flow in the pipeline.
- Solution. Diameter at inlet, D1 = 400 mm = 0.4 m

$\therefore$ Area of inlet, $A_1 = \frac{\pi}{4} \times 0.4^2 = 0.1257 \text{ m}^2$

Throat diameter, $D_2 = 200 \text{ mm} = 0.2 \text{ m}$

$\therefore$ Area at throat, $A_2 = \frac{\pi}{4} \times 0.02^2 = 0.0314 \text{ m}^2$

Reading of the differential manometer (U-tube), $y = 50 \text{ mm} = 0.05 \text{ m}$

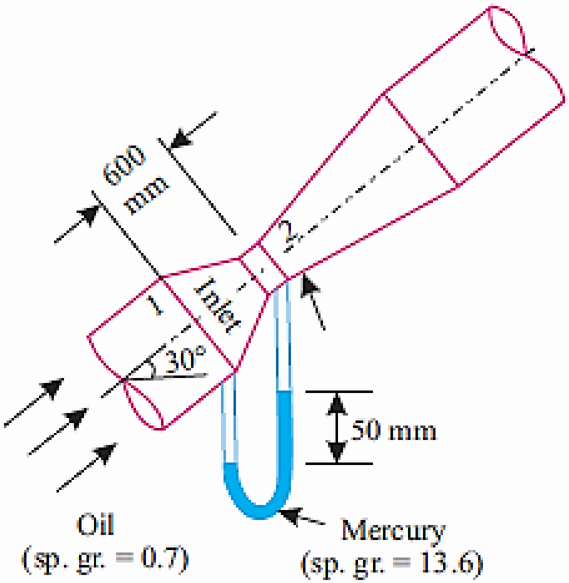
Difference of pressure head h is given by:

$$h = y\left[\frac{S_{hl}}{S_p} - 1\right]$$

where, S_{hl} = Sp. gravity of heavy liquid (*i.e.*, mercury) in U-tube = 13.6, and

S_p = Sp. gravity of liquid (*i.e.*, oil) flowing through the pipe = 0.7

$$h = 0.05\left(\frac{13.6}{0.7} - 1\right) = 0.92 \text{ m of oil}$$



It may be noted that differential gauge reading will include in itself the difference of pressure head and the difference of datum head. Thus, eqn. (i) reduces to:

Now, applying Bernoulli's equation at sections '1' and '2', we get:

$$\frac{p_1}{w} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{w} + z_2 + \frac{V_2^2}{2g}$$

or, $$\left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = 0$$

But, $$\left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = h$$

or, $$\left(\frac{p_1}{w} - \frac{p_2}{w}\right) + (z_1 - z_2) = h$$

$$h + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = 0$$

Applying continuity equation at sections '1' and '2' we get:

$$A_1V_1 = A_2V_2$$

or, $$V_1 = \frac{A_2V_2}{A_1} = \frac{(\pi/4) \times 0.2^2}{(\pi/4) \times 0.4^2} \times V_2 = \frac{V_2}{4}$$