Substituting the value of V_1 and h in eqn. (*ii*), we get:

$$0.92 + \frac{V_2^2}{16 \times 2g} - \frac{V_2^2}{2g} = 0$$

or, $$\frac{V_2^2}{2g}\left(1 - \frac{1}{16}\right) = 0.92 \quad \text{or} \quad V_2^2 \times \frac{15}{16} = 0.92$$

or, $$V_2^2 = \frac{0.92 \times 2 \times 9.81 \times 16}{15} = 19.25 \quad \text{or} \quad V_2 = 4.38 \text{ m/s}$$

Rate of flow of oil, $Q = A_2 V_2 = 0.0314 \times 4.38 =$ **0.1375 m³/s (Ans.)**

Orificemeter

- Orificemeter or orifice plate is a device (cheaper than a venturimeter) employed for measuring the discharge of fluid through a pipe. It also works on the same principle of a venturimeter. It consists of a flat circular plate having a circular sharp edged hole (called orifice) concentric
- with the pipe. The diameter of the orifice may vary from 0.4 to 0.8 times the diameter of the pipe but its value is generally chosen as 0.5. A differential manometer is connected at section (1) which is at a distance of 1.5 to 2 times the pipe diameter upstream from the orifice plate, and at section (2) which is at a distance of about half the diameter of the orifice from the orifice plate on the downstream side.
- Let, A1 = Area of pipe at section (1) V1 = Velocity at section (1),
- p1 = Pressure at section (1), and A2 V2 and p2 are corresponding values at section (2).

Applying Bernoulli's equation at sections (1) and (2), we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

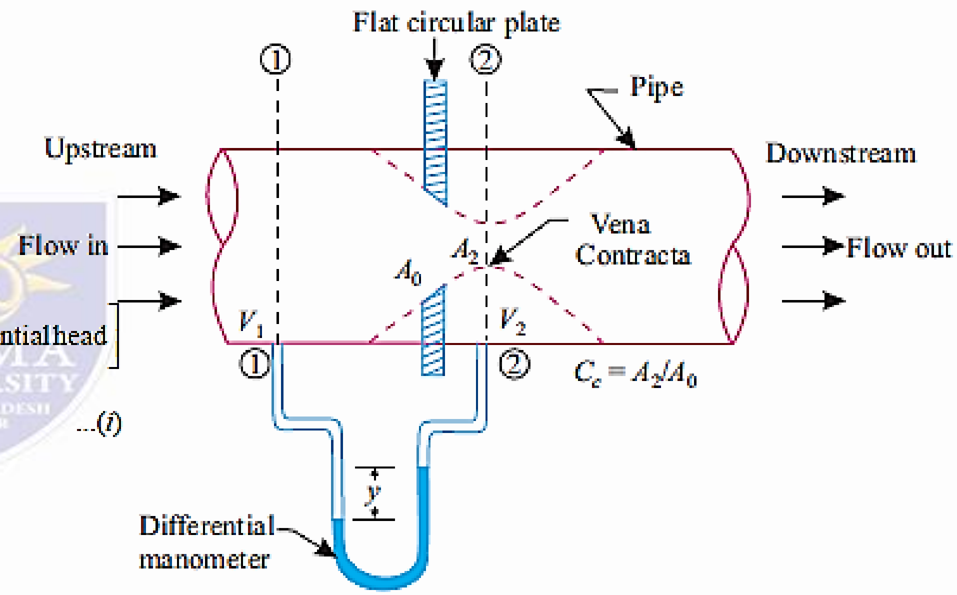
or, $$\left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

or, $$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\left[\because h = \left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = \text{differential head}\right]$$

or, $$\frac{V_2^2}{2g} = h + \frac{V_1^2}{2g} \qquad \text{...(i)}$$

or, $$V_2 = \sqrt{2g\left(h + \frac{V_1^2}{2g}\right)} = \sqrt{2gh + V_1^2}$$



Now, section (2) is at *vena contracta* and A_2 represents the area at vena contracta. If A_0 is the area of orifice, then we have:

$$C_c = \frac{A_2}{A_0}$$

(where, C_c = co-efficient of contraction)

$$\therefore \qquad A_2 = A_0 C_c \qquad \text{...(ii)}$$

Using continuity equation, we get:

$$A_1 V_1 = A_2 V_2 \quad \text{or} \quad V_1 = \frac{A_2 V_2}{A_1}$$

or,

$$V_1 = \frac{A_0 C_c V_2}{A_1}$$

Substituting the value of V_1 in eqn. (i), we get:

$$V_2 = \sqrt{2gh + \frac{A_0^2 . C_c^2 . V_2^2}{A_1^2}}$$

$$V_2^2 = 2gh + \left(\frac{A_0}{A_1}\right)^2 \cdot C_2^2 \cdot V_2^2$$

$$V_2^2 \left[1 - \left(\frac{A_0}{A_1}\right)^2 C_2^2\right] = 2gh$$

$$V_2 = \frac{\sqrt{2gh}}{\sqrt{1 - (A_0 / A_1)^2 C_c^2}}$$

The discharge, $Q = A_2 V_2 = A_0 . C_c . V_2$

[A2 = A0 . Cc... as above {eqn. (ii)}]

$$= A_0 C_c \frac{\sqrt{2gh}}{\sqrt{1 - (A_0 / A_1)^2 C_c^2}}$$

The above expression is simplified by using,

$$C_d = C_c \frac{\sqrt{1 - (A_0 / A_1)^2}}{\sqrt{1 - (A_0 / A_1)^2 C_c^2}}$$

(where, C_d = co-efficient of discharge)

$$C_c = C_d \frac{\sqrt{1 - (A_0 / A_1)^2 C_c^2}}{\sqrt{1 - (A_0 / A_1)^2}}$$

Substituting this value of C_c in eqn. (iv), we get:

$$Q = A_0 \cdot C_d \frac{\sqrt{1 - (A_0 / A_1)^2 C_c^2}}{\sqrt{1 - (A_0 / A_1)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1 - (A_0 / A_1)^2 C_c^2}}$$

$$= \frac{C_d \cdot A_0 \sqrt{2gh}}{\sqrt{1 - (A_0 / A_1)^2}} = \frac{C_d \cdot A_0 \cdot A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

i.e.,

$$Q = C_d \frac{A_0 \cdot A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

It may be noted that C_d (co-efficient of discharge) of an orifice is *much smaller than that of a venturimeter*

Difference between a venturimeter and an orificemeter

- A venturimeter is a device which is inserted into pipeline to measure incompressible fluid low rates. It consists of a convergent section which reduces the diameter to between one-half to one-fourth of the pipe diameters. This is followed by a divergent section. The pressure difference between the position just before the venturi and at the throat of the venturi is measured by a differential manometer. The working of the venturi is based on the Bernoulli's principle, that is, when the velocity head increases in an accelerated flow, there is a corresponding reduction in the piezometric head.
- The orificemeter is opening, usually round, located in the side wall of the tank or reservoir, for measuring the flow of a liquid. The main feature of the orificemeter is that most of the potential energy of the liquid is converted into kinetic energy of the free jet issuing through the orifice.
- The main points of difference between a venturimeter and orificemeter are:
- 1. The venturimeter can be used for measuring the flow rates of all incompressible flows. (gases with low pressure variations, as well as liquids), whereas orificemeters are generally used for measuring the flow rates of liquids.
- 2. Venturimeter is installed in pipeline only, and the accelerated flow through the apparatus, is subsequently decelerated to the original velocity at the outlet of the venturimeter. The flow continues through the pipeline. In the orificemeter the entire potential energy of the fluid is converted to kinetic energy, and the jet discharges freely into the open atmosphere.
- 3. In venturimeter, the flow velocity is measured by noting the pressure difference between the inlet and the throat of the venturimeter, whereas in the orificemeter the discharge velocity is measured by using Pitot tube or by trajectory method.

- The following data relate to an orificemeter: Diameter of the pipe = 240 mm Diameter of the orifice = 120 mm Sp. gravity of oil = 0.88
- Reading of differential manometer = 400 mm of mercury Co-efficient of discharge of the meter = 0.65. Determine the rate of flow of oil

Solution. Diameter of the pipe $D_1 = 240 \text{ mm} = 0.24 \text{ m}$

$\therefore$ Area of the pipe, $A_1 = \frac{\pi}{4} \times 0.24^2 = 0.0452 \text{ m}^2$

Diameter of the orifice, $D_0 = 120 \text{ mm} = 0.12 \text{ m}$

$\therefore$ Area of the orifice, $A_0 = \frac{\pi}{4} \times 0.12^2 = 0.0113 \text{ m}^2$

Co-efficient of discharge, $C_d = 0.65$

Sp. gravity of oil, $S_0 = 0.88$

Reading of differential manometer, $y = 400 \text{ mm of mercury} = 0.4 \text{ m of mercury}$

$\therefore$ Differential head, $h = y\left[\frac{S_{hl}}{S_o} - 1\right]$

[where, S_{hl} = sp. gravity of heavier liquid = 13.6 (for mercury)]

$$= 0.4\left[\frac{13.6}{0.88} - 1\right] = 5.78 \text{ m of oil}$$

Discharge Q:

Using the relation, $Q = C_d \dfrac{A_0 . A_1 . \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$, we have:

$$Q = 0.65 \times \frac{0.0113 \times 0.0452 \times \sqrt{2 \times 9.81 \times 5.78}}{\sqrt{(0.0452)^2 - (0.0113)^2}}$$

$$= \frac{0.000353}{0.0437} = \mathbf{0.08 \text{ m}^3/\text{s (Ans.)}}$$

- Water flows at the rate of 0.015 m3/s through a 100 mm diameter orifice used in a 200 mm pipe. What is the difference of pressure head between the upstream section and the vena contracta section? Take co-efficient of contraction Cc = 0.60 and Cv = 1.0.
- Solution. Given: Q = 0.015 m3/s; D0 = 100 mm = 0.1 m; D1 = 200 mm = 0.2 m; Cc = 0.60; Cv = 1.0
- Difference in pressure head h:

$$A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4} \times 0.2^2 = 0.03142 \text{ m}^2$$

$$A_0 = \frac{\pi}{4}D_0^2 = \frac{\pi}{4} \times 0.1^2 = 0.007854 \text{ m}^2$$

$$C_d = C_c \times C_v = 0.60 \times 1.0 = 0.6$$

Using the relation:

$$Q = C_d \frac{A_0 A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

or,

$$0.015 = 0.6 \times \frac{0.007854 \times 0.03142 \sqrt{2 \times 9.81 \times h}}{\sqrt{(0.03142)^2 - (0.007854)^2}} \quad ...|$$

or,

$$0.015 = 0.6 \times \frac{0.001093\sqrt{h}}{0.03042}$$

or,

$$h = \left(\frac{0.015 \times 0.03042}{0.6 \times 0.001093}\right)^2 = \mathbf{0.484 \text{ m of water (Ans.)}}$$