Momentum correction factor (β):

- 'Momentum correction factor' is defined as the ratio of momentum of the flow per second based on actual velocity to the momentum of the flow per second based on average velocity across a section. It is denoted by β.

Mathematically,
$$\beta = \frac{\text{Momentum per second based on actual velocity}}{\text{Momentum per second based on average velocity}}$$

Refer to Fig. 6.59.

The momentum of fluid mass m is

$$= m\bar{u} = (\rho A\bar{u})\bar{u} = \rho A\bar{u}^2$$

The true momentum at the section LL is given as:

$$\int_{LL} dm.u = \int_{LL} (\rho dA.u)u = \int_{A} \rho u^2 dA$$

$$\beta = \frac{\int_A \rho u^2 dA}{\rho A\bar{u}^2} = \frac{1}{A}\int_A \left(\frac{u}{\bar{u}}\right)^2 dA$$

$\beta = 1$ for *uniform flow,*

$\beta = 1.01$ to 1.07 for *turbulent flow in pipes,* and

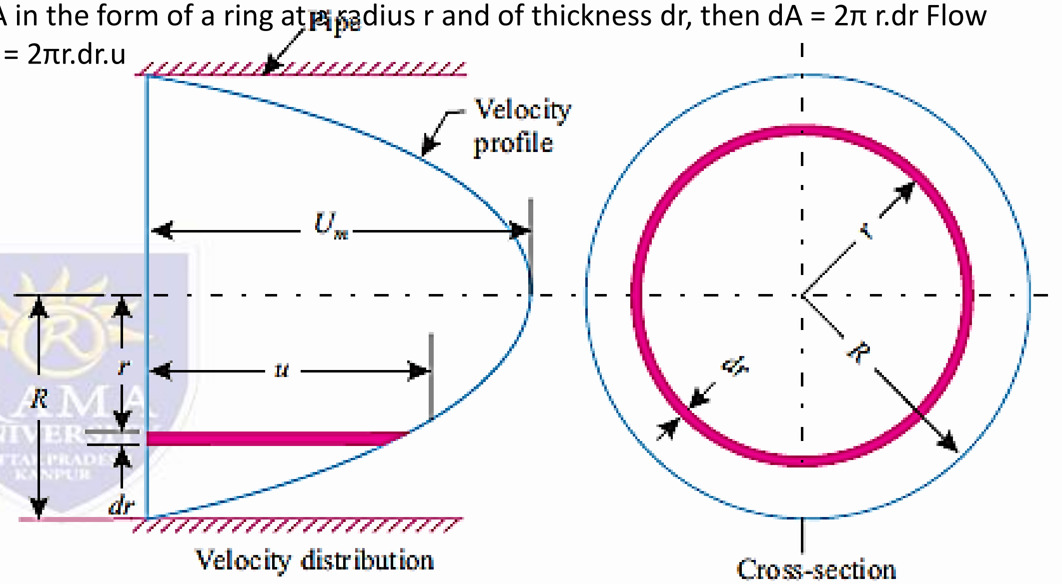
$\beta = \frac{4}{3} = 1.33$ for *laminar flow in pipes.*

- The value of β may be greater for open channel flow. In most cases, β is taken as 1.

- In a circular pipe the velocity profile is given as where u is the velocity at any radius r, Um is the velocity at the pipe axis, and R is the radius of the pipe. Find:
- (i) Average velocity,

$$u = U_m\left[1-\left(\frac{r}{R}\right)^2\right]$$

- (ii) Energy correction factor, and
- (iii) Momentum correction factor.
- Solution. Refer to Fig. 6.61. Consider an elementary area dA in the form of a ring at a radius r and of thickness dr, then dA = 2π r.dr Flow rate through the ring = dQ = Elemental area × local velocity = 2πr.dr.u



$$\text{Total flow } Q = \int_0^R 2\pi r.u.dr$$

$$= \int_0^R 2\pi\, U_m\left(1-\frac{r^2}{R^2}\right) r\,dr$$

$$= 2\pi U_m \int_0^R \left(r-\frac{r^3}{R^2}\right) dr = 2\pi U_m\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]_0^R$$

$$= 2\pi U_m\left(\frac{R^2}{2}-\frac{R^2}{4}\right) = 2\pi U_m\left(\frac{R^2}{4}\right)$$

$$Q = 2\pi U_m\left(\frac{R^2}{4}\right)$$

(ii) Average velocity, $\bar{u}$:

If $\bar{u}$ is the average flow velocity, then:

$$Q = A\bar{u} = \pi R^2 \bar{u}$$

From (i) and (ii), we get:

$$\pi R^2 \bar{u} = 2\pi U_m\left(\frac{R^2}{4}\right)$$

$$\therefore \quad \bar{u} = \frac{2\pi U_m\left(\frac{R^2}{4}\right)}{\pi R^2} = \frac{U_m}{2} \quad \text{(Ans.)}$$

- (iii) Kinetic energy correction factor, α

$$\alpha = \frac{1}{A\bar{u}^3}\int_0^R u^3\,dA$$

$$= \frac{1}{A\bar{u}^3}\int_0^R U_m^3\left[1-\left(\frac{r}{R}\right)^2\right]^3 2\pi\, r dr$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3}\int_0^R \left(1-\frac{3r^2}{R^2}+\frac{3r^4}{R^4}-\frac{r^6}{R^6}\right) r\,dr$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3}\int_0^R \left(r-\frac{3r^3}{R^2}+\frac{3r^5}{R^4}-\frac{r^7}{R^6}\right) dr$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3}\left[\frac{r^2}{2}-\frac{3r^4}{4R^2}+\frac{3r^6}{6R^4}-\frac{r^8}{8R^6}\right]_0^R$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3}\left[\frac{R^2}{2}-\frac{3}{4}R^2+\frac{R^2}{2}-\frac{R^2}{8}\right]$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3}\left(\frac{R^2}{8}\right)$$

$A = \pi R^2$ and $\bar{u} = \dfrac{U_m}{2}$, we get:

$$\alpha = \frac{2\pi U_m^3}{\pi R^2 \times \left(\frac{U_m}{2}\right)^3} \times \left(\frac{R^2}{8}\right) = \mathbf{2\ (Ans.)}$$

(iv) Momentum correction factor, β:

$$\beta = \frac{1}{A\bar{u}^2}\int u^2\,dA$$

$$= \frac{1}{A\bar{u}^2}\int_0^R U_m^2\left[1-\left(\frac{r}{R}\right)^2\right]^2 2\pi r\,dr$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2}\int_0^R \left(1-2\times\frac{r^2}{R^2}+\frac{r^4}{R^4}\right) r\,dr$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2}\int_0^R \left(r-2\times\frac{r^3}{R^2}+\frac{r^5}{R^4}\right) dr$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2}\left[\frac{r^2}{2}-2\times\frac{r^4}{4R^2}+\frac{1}{6}\times\frac{r^6}{R^4}\right]_0^R$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2}\left[\frac{R^2}{2}-\frac{R^2}{2}+\frac{R^2}{6}\right]$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2}\left(\frac{R^2}{6}\right)$$

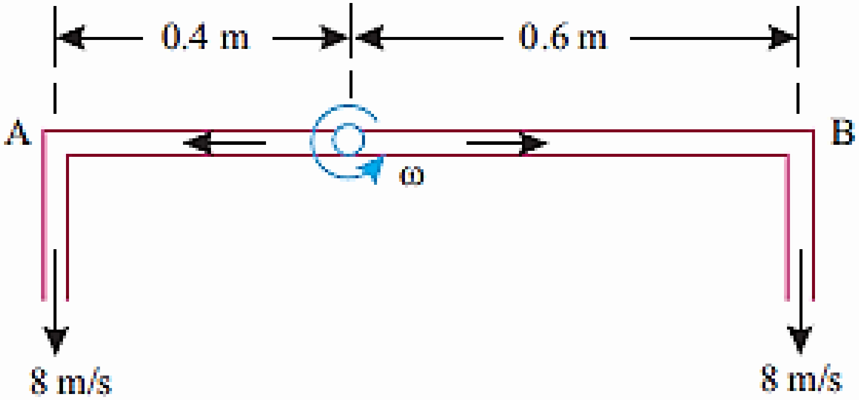
Substituting the values $A = \pi R^2$ and $\bar{u} = \dfrac{U_m}{2}$, we get:

$$\beta = \frac{2\pi U_m^2}{\pi R^2 \times \left(\frac{U_m}{2}\right)^2}\left(\frac{R^2}{6}\right) = \mathbf{1.33\ (Ans.)}$$

MOMENT OF MOMENTUM EQUATION

- Moment of momentum equation is derived from moment of momentum principle which states as follows:
- "The resulting torque acting on a rotating fluid is equal to the rate of change of moment of momentum". When the moment of momentum of flow leaving a control volume is different from that entering it, the result is a torque acting over the control volume.
- Let, Q = Steady rate of flow of fluid,
- ρ = Density of fluid,
- $V1$ = Velocity of fluid at section 1,
- $r1$ = Radius of curvature at section 1, and
- $V2$ and $r2$ = Velocity and radius of curvature at section 2.
- Momentum of fluid at section 1= Mass × velocity = $\rho Q \times V1$
- ∴ Moment of momentum per second of fluid at section 1 = $\rho Q \times V1 \times r1$
- Similarly, moment of momentum per second of fluid at section 2 = $\rho Q \times V2 \times r2$
- ∴ Rate of change of moment of momentum = $\rho Q\ V2\ r2 - \rho Q V1 r1 = \rho Q(V2\ r2 - V1 r1)$
- According to the moment of momentum principle,
- Resultant torque = Rate of change of moment of momentum
- $T = rQ\ (V2\ r2 - V1r1)$ ...
- Above Eqn. is known as moment of momentum equation. This equation is used:
- (i) To find torque exerted by water on sprinkler, and
- (ii) To analyse flow problems in turbines and centrifugal pumps

- An unsymmetrical sprinkler. It has a frictionless shaft and equal flow through each nozzle with a velocity of 8 m/s relative to the nozzle. Find the speed of rotation in r.p.m.



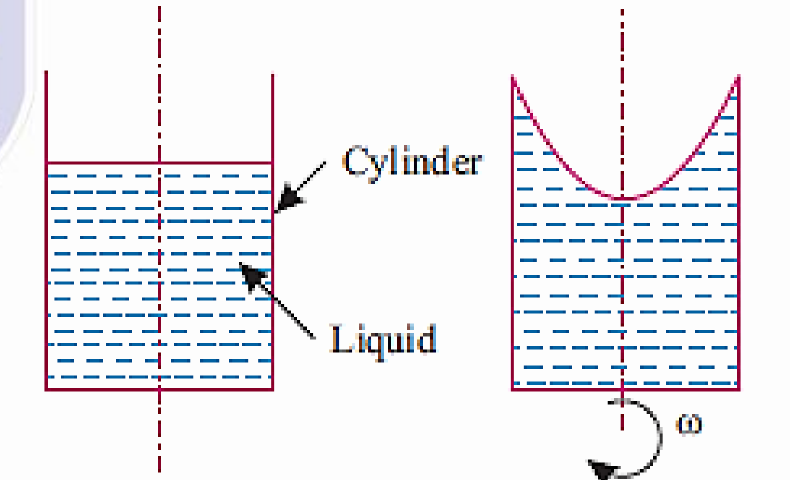
- Solution. Refer to Fig. 6.62.
- $r_A = 0.4$ m, $r_B = 0.6$ m
- Velocity relative to the nozzle
- V_A (= V_B) = 8 m/s
- Let, ω = Angular velocity of the sprinkler.
- Absolute velocity, $V_1 = V_A + \omega r_A = 8 + \omega \times 0.4 = 8 + 0.4\omega$
- Absolute velocity, $V_2 = V_B - \omega r_B = 8 - \omega \times 0.6 = 8 - 0.6\omega$
- Speed of rotaiton, N (r.p.m.):
- The moment of momentum of the fluid entering sprinkler is given zero and also there is no
- external torque applied on the sprinkler. Hence resultant torque is zero, i.e.
- $T = 0$
- $\therefore \rho Q (V_2 r_2 - V_1 r_1) = 0$
- or, $V_2 r_2 - V_1 r_1 = 0$ (□ ρQ 0)
- or, $(8 - 0.6\omega) \times 0.6 = (8 + 0.4\omega) \times 0.4$
- or, $4.8 - 0.36\omega = 3.2 + 0.16\omega$
- or, $0.52\omega = 1.6$
- or, $\omega = 3.077$ rad/s
- But, $\omega = 2\pi N / 60$

$$N = \frac{60 \times \omega}{2\pi} = \frac{60 \times 3.077}{2\pi} = 29.4 \text{ r.p.m. (Ans.)}$$

VORTEX MOTION

- Vortex motion is defined as a motion in which the whole fluid mass rotates about an axis. A mass of fluid in rotation about a fixed axis is called vortex. A vortex motion is characterised by a flow pattern wherein the stream lines are curved. When fluid flows between curved stream lines, centrifugal forces are set up and these are counter-balanced by the pressure force acting in the radial direction.
- The vortex flow is of the following types:
- 1. Forced vortex flow, and
- 2. Free vortex flow.

- **Forced Vortex Flow**
- Forced vortex flow is one in which the fluid mass is made to rotate by means of some external agency. The external agency is generally the mechanical power which imparts a constant torque on the fluid mass. Then, in such a flow there is always xpenditure of energy. The forced vortex motion is also called flywheel vortex or rotational vortex.
- In this type of flow, the fluid mass rotates at a
- constant angular velocity ω. The tangential velocity
- of any fluid particle is given by:
- $v = \omega r$ ...(6.32)
- (where, r = radius of the fluid particle from the axis
- of rotation)
- ∴ Angular velocity. w = vr
- = constant . ..[6.32 (a)]
- Example:
- 1. Rotation of water through the runner of a turbine.
- 2. Rotation of liquid inside the impeller of a centrifugal pump.
- 3. Rotation of liquid in a vertical cylinder (Fig. 6.64).



(a) Stationary cylinder (b) Rotating cylinder