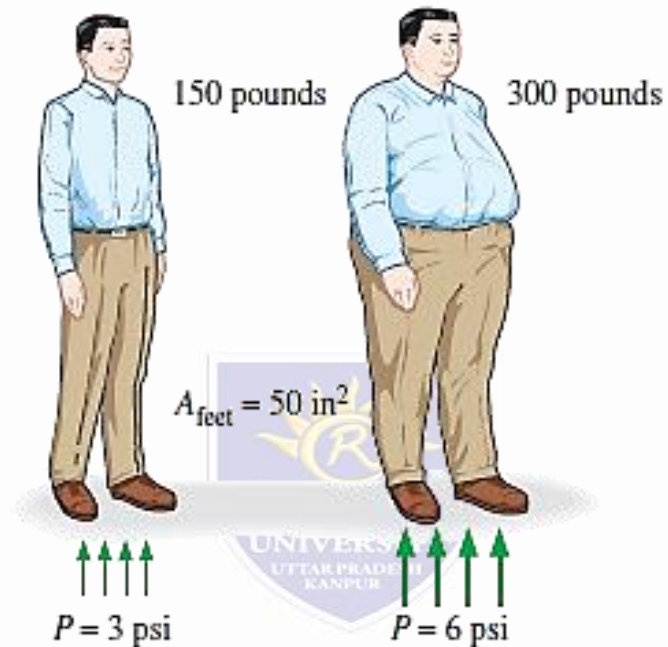


# Fluid Static - Concept of Pressure



$$P = \sigma_n = \frac{W}{A_{\text{feet}}} = \frac{150 \text{ lbf}}{50 \text{ in}^2} = 3 \text{ psi}$$

The normal stress (or “pressure”) on the feet of a chubby person is much greater than on the feet of a slim person.

# Fluid Properties

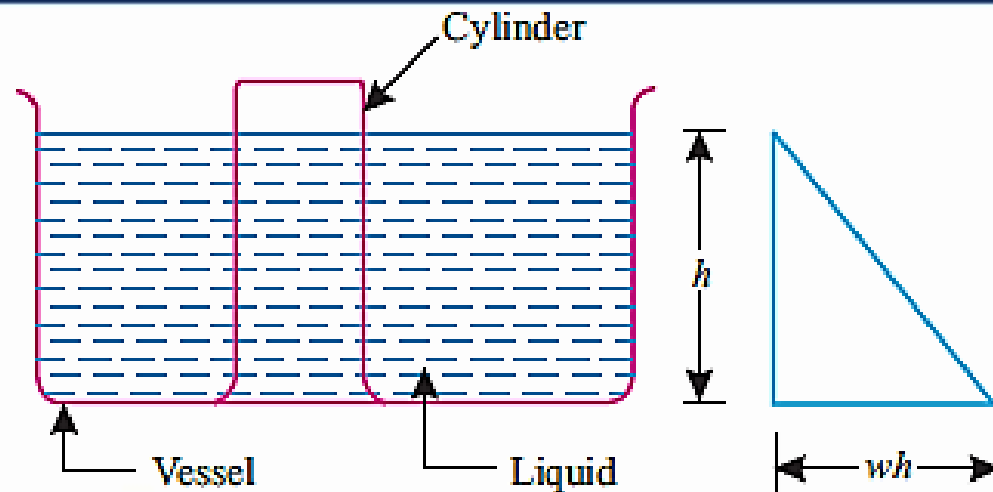
- Pressure is defined as a normal force exerted by a fluid per unit area. We speak of pressure only when we deal with a gas or a liquid.
- The counterpart of pressure in solids is normal stress. Since pressure is defined as force per unit area, it has the unit of newtons per square meter (N/m<sup>2</sup>), which is called a pascal (Pa).
- 1 Pa = 1 N/m<sup>2</sup>
- 1 bar = 10<sup>5</sup> Pa = 0.1 MPa = 100 kPa
- 1 atm = 101,325 Pa = 101.325 kPa = 1.01325 bars
- 1 kgf/cm<sup>2</sup> = 9.807 N/cm<sup>2</sup> = 9.807 × 10<sup>4</sup> N/m<sup>2</sup> = 9.807 × 10<sup>4</sup> Pa
- = 0.9807 bar
- = 0.9679 atm
- Pressure is also used on solid surfaces as synonymous to normal stress, which is the force acting perpendicular to the surface per unit area. For example, a 150-pound person with a total foot imprint area of 50 in<sup>2</sup> exerts a pressure of 150 lbf/50 in<sup>2</sup> = 3.0 psi on the floor
- This also explains how a person can walk on fresh snow without sinking by wearing large snowshoes, and how a person cuts with little effort when using a sharp knife.
- **PRESSURE OF A LIQUID**
- When a fluid is contained in a vessel, it exerts force
- at all points on the sides and bottom and top of the
- container. The force per unit area is called pressure.
- If,  $P$  = The force, and
- $A$  = Area on which the force acts; then intensity of pressure,  $p = P/A$ ,
- The pressure of a fluid on a surface will always act normal to the surface

# Fluid Properties

- PRESSURE HEAD OF A LIQUID
- Let,  $h$  = Height of liquid in the cylinder,
- $A$  = Area of the cylinder base,
- $w$  = Specific weight of the liquid,
- and,  $p$  = Intensity of pressure.
- Now, Total pressure on the base of the cylinder  
= Weight of liquid in the cylinder
- i.e.,  $p \cdot A = wAh$

$$p = \frac{wAh}{A} = wh$$

$$\text{i.e., } p = wh$$



- As  $p = wh$ , the intensity of pressure in a liquid due to its depth will vary directly with depth. As the pressure at any point in a liquid depends on height of the free surface above that point, it is sometimes convenient to express a liquid pressure by the height of the free surface which would cause the pressure,

$$h = \frac{p}{w}$$

- The height of the free surface above any point is known as the static head at that point. In this case, static head is  $h$ .
- Hence, the intensity of pressure of a liquid may be expressed in the following two ways:
- 1. As a force per unit area (i.e.,  $N/mm^2$ ,  $N/m^2$ ), and
- 2. As an equivalent static head (i.e., metres, mm or cm of liquid).

# Fluid Properties

- Pressure variation in fluid at rest:
- to determine the pressure at any point in a fluid at rest “hydrostatic law” is used; the
- law states as follows:
- “The rate of increase of pressure in a
- vertically downward direction must be equal to
- the specific weight of the fluid at that point.”
- The proof of the law is as follows. Refer to Fig.
- Let,  $p$  = Intensity of pressure on face LM,
- $\Delta A$  = Cross-sectional area of the element,
- $Z$  = Distance of the fluid element from
- free surface, and  $\Delta Z$  = Height of the fluid element.
- The forces acting on the element are:
- (i) Pressure force on the face LM =  $p \times \Delta A$  ... (acting downward)

(ii) Pressure force on the face ST =  $\left( p + \frac{\partial p}{\partial Z} \times \Delta Z \right) \times \Delta A$  ...

(iii) Weight of the fluid element = Weight density  $\times$  volume  
 $= w \times (\Delta A \times \Delta Z)$

(iv) Pressure forces on surfaces MT and LS ..... are equal and opposite.

For equilibrium of the fluid element, we have:

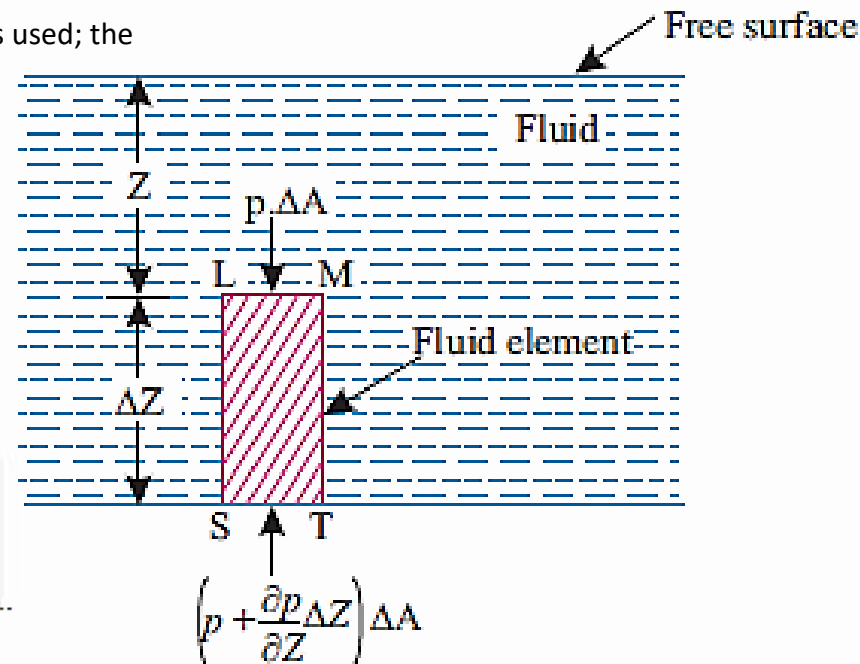
$$p \times \Delta A - \left[ p + \frac{\partial p}{\partial Z} \times \Delta Z \right] \times \Delta A + w \times (\Delta A \times \Delta Z) = 0$$

$$\text{or, } p \times \Delta A - p \times \Delta A - \frac{\partial p}{\partial Z} \times \Delta Z \times \Delta A + w \times \Delta A \times \Delta Z = 0$$

$$\text{or, } \frac{\partial p}{\partial Z} \Delta Z \times \Delta A + w \times \Delta A \times \Delta Z = 0$$

$$\text{or, } \frac{\partial p}{\partial Z} = w \text{ (cancelling } \Delta Z \times \Delta A \text{ from both the sides)}$$

$$\text{or, } \frac{\partial p}{\partial Z} = \rho \times g \quad (\because w = \rho \times g)$$



states that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point. This is “hydrostatic law”.

$$\int dp = \int \rho g . dZ$$

$$\text{or, } p = \rho g . Z (= wZ)$$

where,  $p$  is the pressure above atmospheric pressure.

From eqn. (2.4), we have:

$$Z = \frac{p}{\rho g} \left( = \frac{p}{w} \right)$$

Here  $Z$  is known as pressure head.

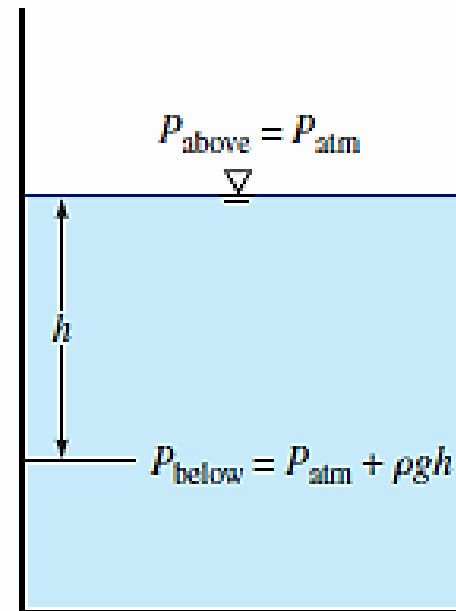
# Fluid Properties

- Find the pressure at a depth of 15 m below the free surface of water in a reservoir.
- Solution. Depth of water,  $h = 15$  m
- Specific weight of water,  $w = 9.81$  kN/m<sup>3</sup>
- Pressure  $p$ :
- We know that,  $p = wh = 9.81 \times 15 = 147.15$  kN/m<sup>2</sup>
- i.e.,  $p = 147.15$  kN/m<sup>2</sup> = 147.15 kPa (Ans.)

- Find the height of water column corresponding to a pressure of 54 kN/m<sup>2</sup>.

- Solution. Intensity of pressure,  $p = 54$  kN/m<sup>2</sup>
- Specific weight of water,  $w = 9.81$  kN/m<sup>3</sup>
- Height of water column,  $h$ :
- Using the relation:  $p = wh$ ;

$$h = \frac{p}{w} = \frac{54}{9.81} = 5.5 \text{ m (Ans.)}$$



# Fluid Static –Pascal Law

- PASCAL'S LAW
- The Pascal's law states as follows :
- "The intensity of pressure at any point in a liquid at rest, is the same in all directions".
- Proof. Let us consider a very small wedge shaped element LMN of a liquid, as shown in Fig. 2.3.
- Let,  $p_x$  = Intensity of horizontal pressure on the element of liquid,
- $p_y$  = Intensity of vertical pressure on the element of liquid,
- $p_z$  = Intensity of pressure on the diagonal of the right angled triangular element,
- $\alpha$  = Angle of the element of the liquid,
- $P_x$  = Total pressure on the vertical side LN of the liquid,
- $P_y$  = Total pressure on the horizontal side MN of the liquid, and
- $P_z$  = Total pressure on the diagonal LM of the liquid.
- Now,  $P_x = p_x \times LN$  ...(i)
- and,  $P_y = p_y \times MN$  ...(ii)
- and,  $P_z = p_z \times LM$  ...(iii)
- As the element of the liquid is at rest, therefore the sum of horizontal and vertical components of the liquid pressures must be equal to zero.
- Resolving the forces horizontally:
- $P_z \sin \alpha = P_x$

