DESIGN OF CONCRETE STRUCTURES -1

Topics to be covered:

- Design of Rectangular Singly Reinforced Sections by Working Stress Method.
- Equivalent or Transformed Section
- Strain -Stress Diagram
- Neutral Axis
- Stresses in Concrete and Steel
- Dimensions of the Beam and Area of Steel
- Percentage of Steel
- Lever arm
- Moment of resistance

PREPARED BY:

SHASHIKANT SRIVASTAVA

ASSISTANT PROFESSOR

DEPARTMENT OF CIVIL ENGINEERING

FACULTY OF ENGINEERING & TECHNOLOGY

RAMA UNIVERSITY UTTAR PRADESH, KANPUR (INDIA)

ANALYSIS OF SINGLY REINFORCED BEAM WORKING STRESS METHOD :

ANALYSIS OF SINGLY REINFORCED BEAM WORKING STRESS METHOD :

Analysis of singly reinforced beam Working stress method : A singly reinforced beam section is shown in Figure below. To analyse this section, it is necessary to convert it into a transformed or equivalent section of concrete.

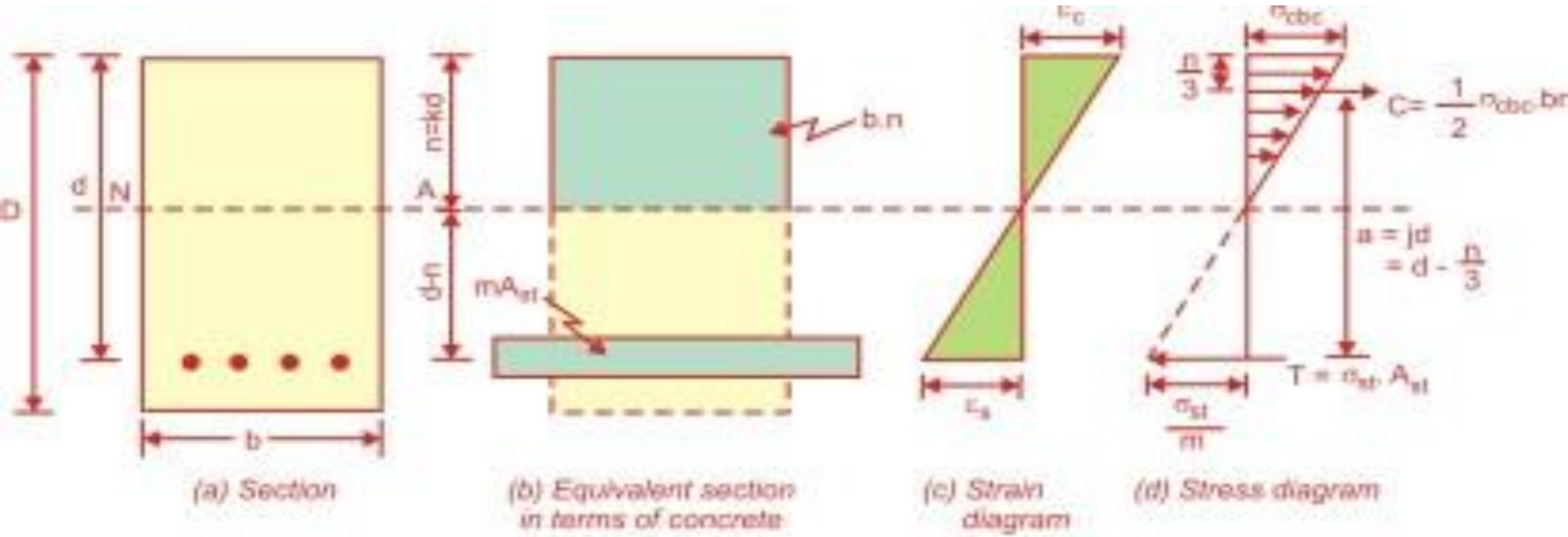


Fig. 2.3. Singly reinforced beam.

ANALYSIS OF SINGLY REINFORCED BEAM WORKING STRESS METHOD :

ASSUMPTIONS IN WORKING STRESS METHOD :

Working Stress Method designing is a method used for Reinforced Concrete Designing, where concrete is assumed elastic and both steel and concrete act together elastically when the relationship between loads and stresses is linear. The analysis of structures using Working Stress Method is completely based on few assumptions as:

(1) The plane section before bending remains plane after bending.

(2) Bond between steel and concrete is perfect within the elastic limit of steel.

(3) Both the steel and concrete behave as linearly elastic materials.

(4) All tensile stress is taken by steel and none by concrete.

(5) The stresses in steel and concrete are related using a factor commonly known as 'Modular ratio' (m).

(6) The stress strain relationship of and concrete is a straight line under working loads.

(7) The factor of safety for concrete is 3 and for steel is 1.8 with respect to yield stress.

MODULAR RATIO : It is defined as the ratio of moduli of steel to the moduli of concrete. It is denoted by the letter "m".

m=Es/Ec

The modular ratio is not constant for all grades of concrete. It varies with the grade of concrete. Es/Ec is generally not used to calculate modular ratio for reinforced concrete designs. **AS PER IS: 456-1978;** m is calculated by the following formula:

$m = 280/3\sigma_{cbc}$, where, σ_{cbc} = permissible compressive stress in concrete in bending.

Calculation of Modular ratio values for different grades of concrete

Grade of Concrete	Modular Ratio
M15	m = 280/3×5 = 18.66
M20	m = 280/3×7 = 13.33
M25	m = 280/3×8.5 = 10.98
M30	m = 280/3×10 = 9.33

ANALYSIS OF SINGLY REINFORCED BEAM WORKING STRESS METHOD :

PERMISSIBLE STRESSES IN CONCRETE :

Reinforced concrete designs make use of M15 grade concrete. The permissible stresses for different grades of concrete is different. **They are given below:**

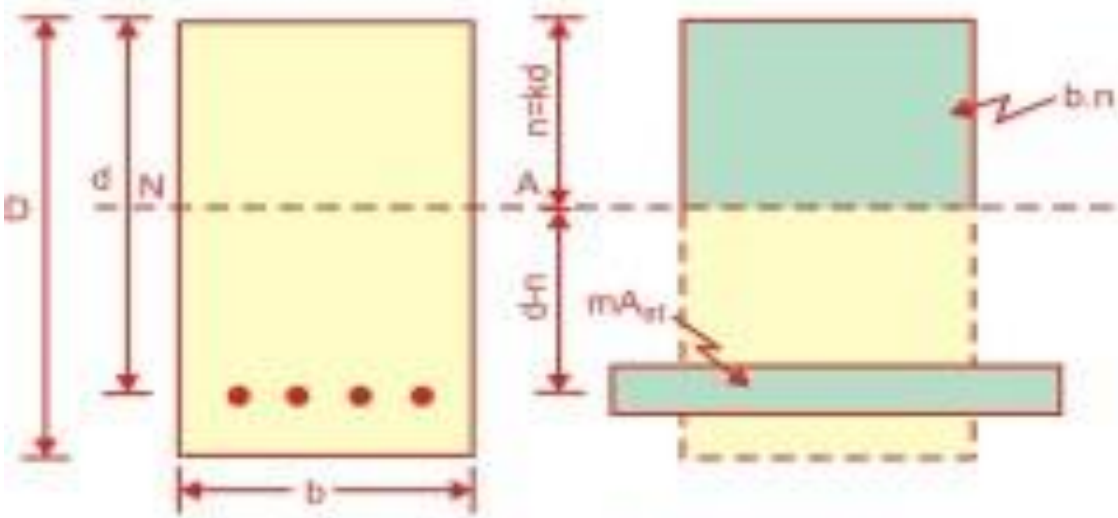
Sr. No.	Concrete Grade	M15	M20	M25	M30
1.	Stress in compression Pure Bending	5	7	8.5	10
	Stress in compression Direct Bending	4	5	6	8
2.	Stress in bond (average) for plain bars	0.6	0.8	0.9	1.0
3.	Characteristics compressive strength	15	20	25	30

PERMISSIBLE STRESSES IN STEEL :

S. No.	Type of Stress in Steel Reinforcement	Permissible Stresses in N/mm^2		High yield strength deformed bar (HYSD) conforming to IS 1786 (Grade Fe 415)
		Mild steel bars conforming to Grade I of IS 432 (Part I)	Medium tensile steel conforming to IS 432 (Part I)	
1.	Tension (σ_{st} or σ_{sv}) (i) Upto and including 20mm (ii) Over 20mm	140 130	Half the guaranted yield stress subject to maximum of 190 190	230 230
2.	Compression in column bars (σ_{sc})	130	130	190
3.	Compression in bars in beam or slab when compressive resistance of concrete is taken into account	The calculated compressive stresses in the surrounding concrete multiplied by 1.5 times the modular ratio or s_{sc} whichever is lower=1.5m$_c$ or s_{sc}			

EQUIVALENT OR TRANSFORMED SECTION :

As per the assumption, all the tensile stresses are taken by steel and none by concrete i.e., concrete in the tensile zone is cracked. So, the concrete area below the neutral axis is neglected and the effective area or the equivalent area of the section in terms of concrete is shown in Figure below. The equivalent area is equal to the area of concrete in the compression zone and an additional concrete area mA_{st} of concrete corresponding to steel area, A_{st}



(a) Section

(b) Equivalent section in terms of concrete

STRAIN -STRESS DIAGRAM :

STRAIN DIAGRAM :

As per the assumption of elastic theory, the strain distribution is linear, with value zero at the neutral axis to maximum at the top and bottom fibre. The strain diagram for the given R.C.C. section is shown in Figure below.

STRESS DIAGRAM :

As per the assumption (4) of the elastic theory the stress-strain relationship is linear for concrete. So, the stress diagram is also a straight line with value zero at neutral axis and varying linearly with the distance as shown in Fig.ure below.

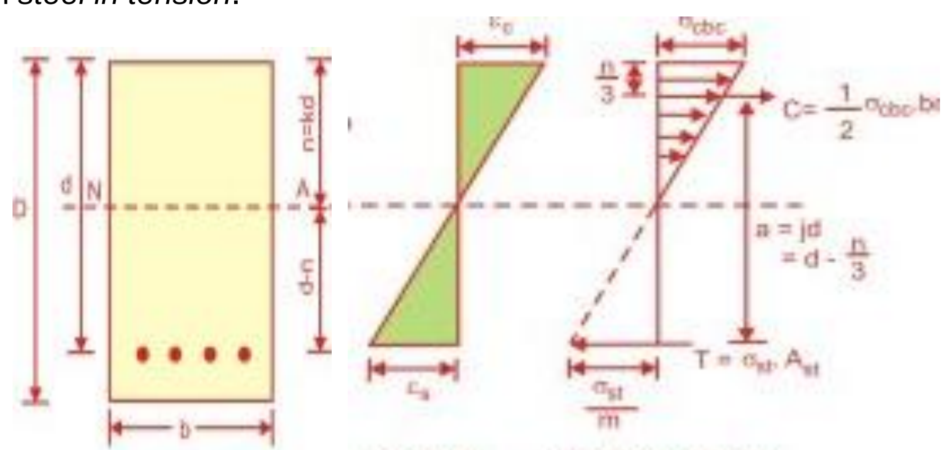
Maximum permissible stress at the top most fibre in concrete = σ_{cbc}

Maximum permissible stress in steel = σ_{st}

Maximum stress in equivalent concrete area at the level of steel = σ_{stm}

NOTE:

1. The suffix cbc in σ_{cbc} stands for permissible stress in *concrete in bending compression*.
2. The suffix *st* in σ_{st} stands for permissible stress in *steel in tension*.



(a) Section (c) Strain diagram (d) Stress diagram

NEUTRAL AXIS :

NEUTRAL AXIS (N) :

Neutral axis lies at the centre of gravity of the section. *It is defined as that axis at which the stresses are zero.* It divides the section into tension and compression zone. The position of the neutral axis depends upon the shape (dimensions) of the section and the amount of steel provided. The position of neutral axis of any rectangular section can be found by the following two methods :

STRESSES IN CONCRETE AND STEEL ARE KNOWN :

Let us consider the R.C.C. section shown in Fig. 2.4(a) the stress σ_c in concrete's top most fibre and σ_s in steel reinforcement are known. Then from stress diagram: $\frac{\sigma_c}{n} = \frac{\sigma_s/m}{d-n}$

From Similar Triangles : $\frac{m.\sigma_c}{\sigma_s} = \frac{n}{d-n}$

If the stresses in concrete and steel are permissible then equation for n is written as: $\frac{m.\sigma_{cbc}}{\sigma_{st}} = \frac{n}{d-n}$

This neutral axis, corresponding to permissible values of stresses of concrete and steel is called as critical neutral axis n_c .

n_c =kd where k is the neutral axis depth factor.

Now we have, $\frac{m.\sigma_{cbc}}{\sigma_{st}} = \frac{kd}{d-kd}$

On rearranging, we get : $k = \frac{m.\sigma_{cbc}}{m.\sigma_{cbc} + \sigma_{st}}$

Putting m=280/ 3σ_{cbc} in the above equation for k, we can see that k does not depend upon grade of concrete. It depends upon grade of steel only.

$$k = \frac{280/3}{280/3 + \sigma_{st}}$$

NEUTRAL AXIS :

Dimensions of the Beam and Area of Steel are Known

The moment of the tensile and compressive area should be equal at the neutral axis.

The neutral axis obtained by this method is called as *actual neutral axis*.

Moment of compressive area = Area in compression × Distance between c.g. of compressive area and neutral axis

$$= b.n.\frac{n}{2} = \frac{bn^2}{2}$$

Moment of tensile area = Equivalent tensile area × Distance of centroid of steel reinforcement from neutral axis

$$= m.A_{st} \times (d - n)$$

Moment of compressive area = Moment of tensile area

$$\frac{bn^2}{2} = m.A_{st}(d - n)(iii)$$

It is a quadratic equation which will give two values of n. Out of these two values only one value (+ve) of n is possible.

PERCENTAGE OF STEEL (P_t) :

The percentage of steel in R.C.C. sections means the area of steel (Ast) expressed as percentage of total area of concrete.

$$Byequation(iii), \frac{b.n^2}{2} = m.A_{st}(d-n)$$

On rearranging, we get

$$A_{st} = \frac{b.n^2}{2m(d-n)}$$

$$P_1 = \frac{50n^2}{md(d-n)}$$

Putting n = kd

$$P_1 = \frac{50k^2}{m(1-k)}$$

LEVER ARM :

Lever arm is the distance between the resultant compressive force and the resultant tensile force. It is denoted as a in the stress diagram. As the compressive area is triangular, the resultant compressive force (C) will act at

$$\frac{n}{3}$$

from the top compressive fibre. The resultant tensile force (T) will act the centroid of the steel reinforcement.

Lever arm =

$$a = d - \frac{n}{3}$$

, it is also expressed as a = jd where j is the lever arem depth factor.

$$jd = d - \frac{kd}{3}$$

$$j = 1 - \frac{k}{3}$$

MOMENT OF RESISTANCE (M_R) :

MOMENT OF RESISTANCE (M_R) :

Moment of resistance is the resistance offered by the beam against external loads. As there is no resultant force acting on the beam and the section is in equilibrium, the total compressive force is equal to the total tensile force. These two forces (equal and opposite separated by a distance) will form a couple and the moment of this couple is equal to the resisting moment or moment of resistance of the section.

Total compression = C = $\left(\frac{1}{2}\sigma_{cbc} \times n\right) b = \frac{1}{2}\sigma_{cbc} bn$ acting n / 3 from top

Total tension = T= **σst . Ast**

acting at centroid of steel reinforcement.

Moment of resistance = C . a or T . a

$$M_r = \frac{1}{2}\sigma_{cbc} b.\, n\left(d - \frac{n}{3}\right)$$

Putting n = kd in the equation

$$M_r = \frac{1}{2}\sigma_{cbc} b.\, kd\left(d - \frac{kd}{3}\right)$$

$$= \frac{1}{2}\sigma_{cbc} k.\left(1 - \frac{k}{3}\right) b.\, d^2$$

$$M_r = \frac{1}{2}\sigma_{cbc} k.\, j.\, b.\, d^2$$

$$M_r = Rbd^2$$

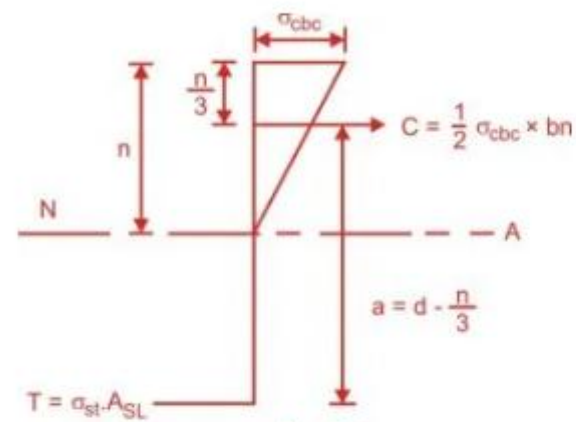


Fig. 2.5.

Moment of Resistance

where R is called as resisting moment factor.

$$R = \frac{1}{2}\sigma_{cbc} kj$$

The factor *k, j* and *R* are constant for a given type of steel and concrete and do not depend upon the beam dimension. These are called as *design constants.*

DESIGN METHODS FOR SINGLY REINFORCED SECTIONS:

Design Methods for Singly reinforced Sections:

Let,

b = breadth of a rectangular beam

d = effective depth of a beam

x = depth of neutral axis below the compression edge

Ast = cross-sectional area of steel in tension

σ_{cbc} = permissible compressive stress in concrete in bending

σ_{st} = permissible stress in steel

m = modular ratio

Neutral axis : Neutral axis is denoted as NA.

We will follow a simple two step procedure.

Step One:

Given that:

Dimensions of the section (b and d)

Permissible stresses in concrete and steel (σ_{cbc} and σ_{st})

Modular ratio (m)

From the above diagram, the formula is as follows:

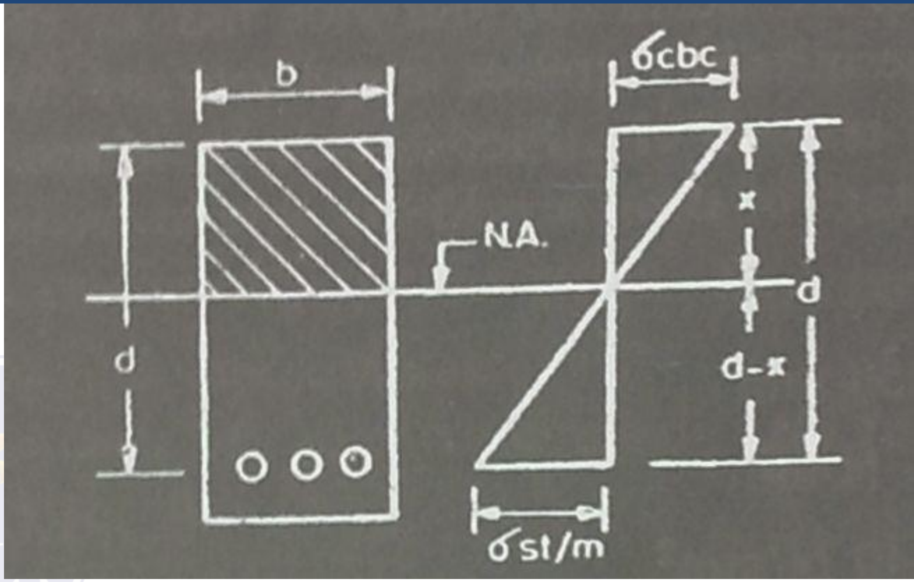
σcbc/(σst/m) = x/(d – x) ——————————— equation 1

From the above equation 1, the value of x can be determined.

Step two:

To find area of steel

Equating total compressive force (C) to total tensile force (T)



C = T

C = area x average compressive stress

= (b.x) X (σcbc + 0)/2

= bx (σcbc/2)

T = area x tensile stress

= Ast x σst

Therefore, bx (σcbc/2) = Ast x σst —————-equation 2

Calculation of NA can be done from eq. 1 and the area of steel from equation 2.

The area of tensile steel is expressed as a percentage (Pt) of the effective section.

Pt = Ast x 100/bd

"Thank you"

Have Any Query ?

Ask us @ shashikant.fet@ramauniversity.ac.in or shashikantchitransh3@gmail.com