- The force P1 acts at a depth of h1 bar from free liquid surface, which is given by:

$$\bar{h}_1 = \frac{I_G}{A\bar{x}_1} + \bar{x}_1$$

$$I_G = \frac{bd^3}{12} = \frac{2 \times 1.2^3}{12} = 0.288 \text{ m}^4$$

$$A = 2.4 \text{ m}^2, \bar{x} = 1.5 + \frac{1.2}{2} = 2.1 \text{ m}$$

$$\bar{h}_1 = \frac{0.288}{2.4 \times 2.1} + 2.1 = 2.157 \text{ m}$$

$\therefore$ Distance of P_1 from the hinge $= (1.5 + 1.2) - \bar{h}_1 = 2.7 - 2.157 = 0.543$ m

Similarly the force P_2 acting at a depth of $\bar{h}_2$ from the liquid surface is given by:

$$\bar{h}_2 = \frac{I_G}{A\bar{x}_2} + \bar{x}_2$$

where, $I_G = 0.288 \text{ m}^4$ (as above); $\bar{x}_2 = \frac{1.2}{2} = 0.6$ m; $A = 2.4 \text{ m}^2$

$\therefore$ $\bar{h}_2 = \frac{0.288}{2.4 \times 0.6} + 0.6 = 0.8$ m

$\therefore$ Distance of P_2 from the hinge $= 1.2 - 0.8 = 0.4$ m

Now the resultant force will act at a distance given by:

$$\frac{71.67 \times 0.543 - 14.13 \times 0.4}{57.54} = \textbf{0.578 m above the hinge (Ans.)}$$

(ii) Force required to open the gate, F:

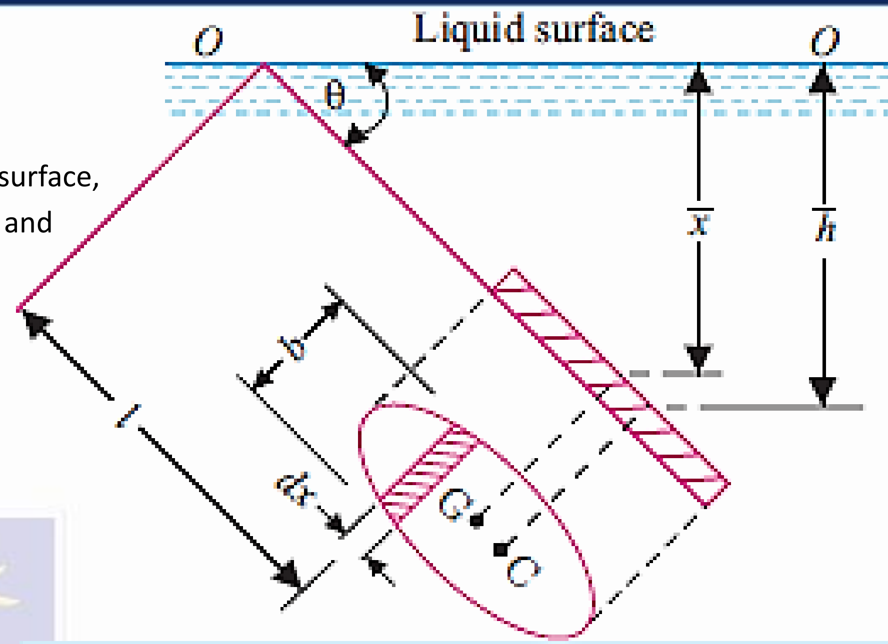
Taking moments of P_1, P_2 and F about the hinge, we get:

$$F \times 1.2 + P_2 \times 0.4 = P_1 \times 0.543$$

or, $F \times 1.2 + 14.13 \times 0.4 = 71.67 \times 0.543$

or, $$F = \frac{71.67 \times 0.543 - 14.13 \times 0.4}{1.2} = \textbf{27.72 kN (Ans.)}$$

INCLINED IMMERSED SURFACE



Refer to Fig. Consider a plane inclined surface, immersed in a liquid.

Let, A = Area of the surface,

Xbar = Depth of centre of gravity of immersed surface from the free liquid surface,

θ = Angle at which the immersed surface is inclined with the liquid surface, and

w = Specific weight of the liquid.

(a) Total pressure (P):

Consider a strip of thickness dx, width b at a distance l from O (A point, on the liquid surface, where the immersed surface will meet, if produced).

The intensity of pressure on the strip

= wl sinθ

Area of the strip = b.dx

Pressure on the strip

= Intensity of pressure × area

= wl sin θ . b. dx

Now total pressure on the surface,

$$P = \int wl \sin\theta \,.\, b \,.\, dx = w \sin\theta \int l \,.\, b \,.\, dx$$

$$\int l \,.\, b \,.\, dx = \text{moment of surface area about } 00$$

$$= \frac{A\bar{x}}{\sin\theta},$$

$$P = w \sin\theta \,.\, \frac{A\bar{x}}{\sin\theta} = wA\bar{x} \text{ (same as in Arts. 3.3 and 3.4)}$$

- (b) Centre of pressure (hbar):
- Referring to Fig 3.27, let C be the centre of pressure of the inclined surface.
- Let, h–bar = Depth of centre of pressure below free liquid surface,
- IG = Moment of inertia of the immersed surface about OO,
- X-bar = Depth of centre of gravity of the surface from the liquid surface,
- θ = Angle at which the immersed surface is inclined with the liquid surface, and

A = Area of the surface.

- Consider a strip of thickness of dx, width b and at distance l from OO.
- The intensity of pressure on the strip = wlsin θ
- Area of strip = b . dx
- ∴ Pressure on the strip =
- Intensity of pressure × area = wl sinθ b . dx
- Moment of the pressure about OO
- = (wl sin θ . b.dx) l = wl2 sinθ . b . dx
- Now sum of moments of all such pressures about O,

$$M = \int wl^2 \sin\theta \, . \, b \, dx = w \sin\theta \int l^2 \, . \, b \, . \, dx$$

But, $\int l^2 \, . \, b \, . \, dx = I_0$ = moment of inertia of the surface about the point 0 (or the second moment of area)

$$M = w \sin\theta \, . \, I_0 \quad ...(i)$$

The sum of moments of all such pressures about O is also equal to $\frac{P\bar{h}}{\sin\theta}$ ...(ii)

where, P is the total pressure on the surface.

Equating eqns. (i) and (ii), we get:

$$\frac{P\bar{h}}{\sin\theta} = w \sin\theta \, . \, I_0$$

$$\frac{wA\bar{x}\bar{h}}{\sin\theta} = w \sin\theta \, . \, I_0 \quad (\because P = wA\bar{x})$$

or,

$$\bar{h} = \frac{I_0 \sin^2\theta}{A\bar{x}} \quad ...(iii)$$

Also, $I_0 = I_G + Ah^2$ ...Theorem of parallel axes.

where, I_G = Moment of inertia of figure about horizontal axis through its centre of gravity, and

h = Distance between 0 and the centre of gravity of the figure $= l \left(= \frac{x}{\sin\theta} \right)$ in this case.

Rearranging equation (iii), we have:

$$\bar{h} = \frac{\sin^2\theta}{A\bar{x}} (I_G + Al^2)$$

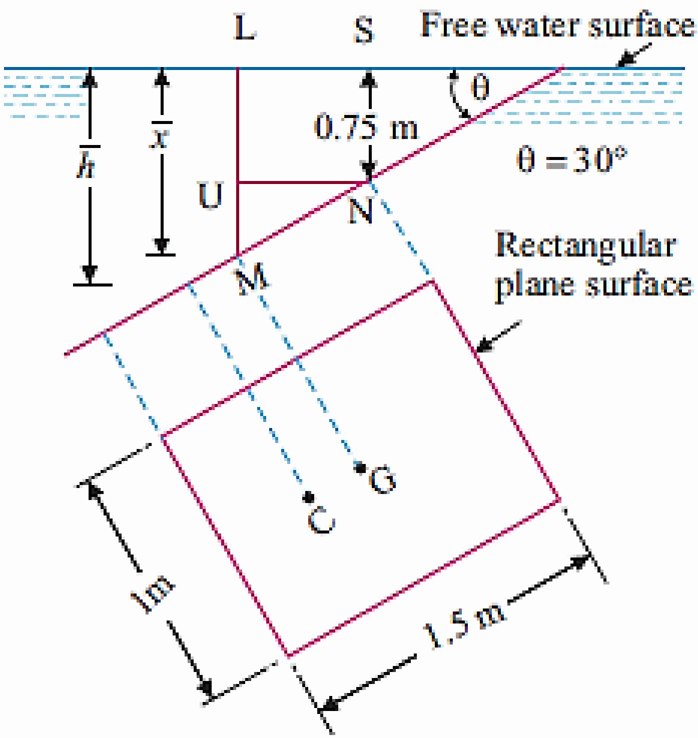
$$= \frac{\sin^2\theta}{A\bar{x}} \left[I_G + A\left(\frac{\bar{x}}{\sin\theta}\right)^2 \right] = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$$

$$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$$

- It will be noticed that if θ = 90° eqn (3.3) becomes the same as equation

Numerical

- A 1m wide and 1.5 m deep rectangular plane surface lies in water in such a way that its plane makes an angle of 30° with the free water surface. Determine the total pressure and position of center of pressure when the upper edge is 0.75 m below the free water surface.



- Solution. Width of the plane surface = 1m
- Depth of the plane surface = 1.5 m
- Inclination, θ = 30°
- Distance of upper edge from free water surface
- = 0.75 m
- (i) Total pressure, P:
- Using the relation, P = wA –xbar
- where, w = 9.81 kN/m3,
- Area, A = 1.5 × 1 = 1.5 m2,
- Xbar = LU + UM = 0.75 + MN sin 30°
- = 0.75 + 1.5/2
- × 0.5 = 1.125 m
- P = 9.81 × 1.5 × 1.125 m = 16.55 kN (Ans.)

(ii) Centre of pressure, $\bar{h}$:

Using the relation, $\bar{h} = \dfrac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$

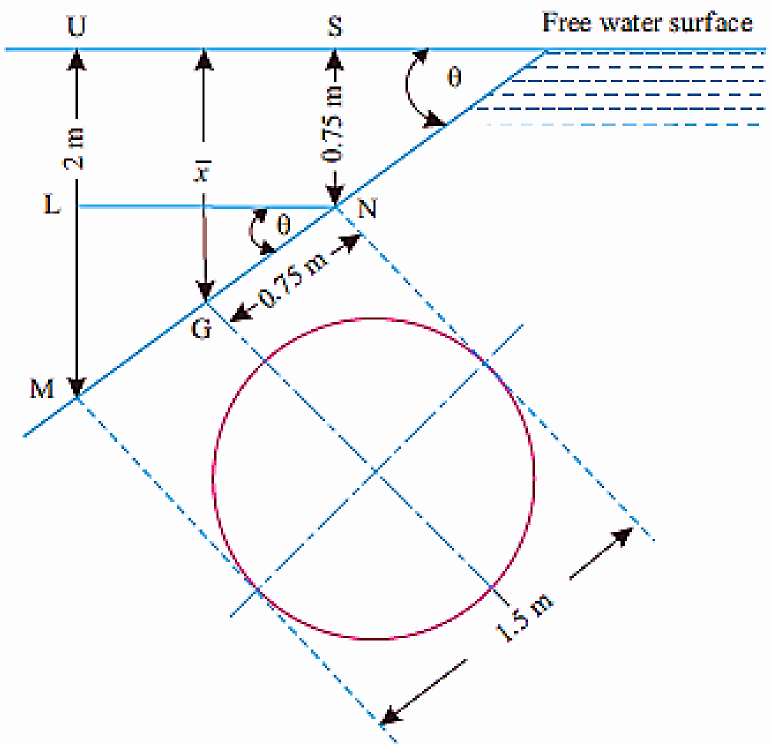
where, $I_G = \dfrac{1 \times 1.5^3}{12} = 0.281\ \text{m}^4$

i.e., $\bar{h} = \dfrac{0.281 \times (0.5)^2}{1.5 \times 1.125} + 1.125 = \mathbf{1.166\ m\ (Ans.)}$

Fluid Properties

- A circular plate 1.5 m diameter is submerged in water, with its greatest and least depths below the surface being 2 m and 0.75 m respectively. Determine: (i) The total pressure on one face of the plate, and (ii) The position of the centre of pressure.
- Solution. Diameter of the plate, = 1.5 m
- Area of the plate,
- Distance, SN = 0.75 m, UM = 2m
- Distance of c.g. from free surface,

$$A = \frac{\pi}{4} . d^2 = \frac{\pi}{4} \times 1.52 = 1.767 \text{ m}^2$$



$$\bar{x} = SN + GN \sin\theta$$
$$= 0.75 + 0.75 \sin\theta$$

But, $$\sin\theta = \frac{LM}{MN} = \frac{UM - UL}{MN}$$
$$= \frac{2 - 0.75}{1.5} = 0.8333$$

$\therefore$ $$\bar{x} = 0.75 + 0.75 \times 0.8333 = 1.375 \text{ m}$$

(i) **Total pressure, *P*:**

We know that,

$$P = wA\bar{x} = 9.81 \times 1.767 \times 1.375$$
$$= \mathbf{23.83 \text{ kN (Ans.)}}$$

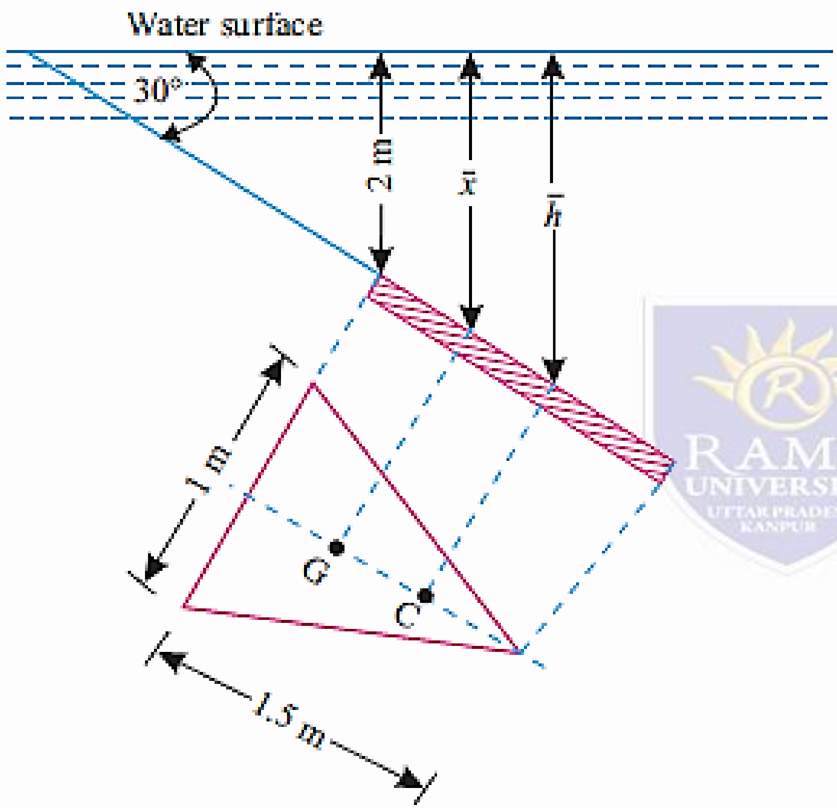
(ii) **Centre of pressure, $\bar{h}$:**

Using the relation,

$$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$$
$$= \frac{\pi/64 \times 1.5^4 \times (0.8333)^2}{1.767 \times 1.375} + 1.375 = 1.446$$

i.e., $$\bar{h} = \mathbf{1.446 \text{ m (Ans.)}}$$

- A triangular plate of 1 metre base and 1.5 metre altitude is immersed in water. The plane of the plate is inclined at 30° with free water surface and the base is parallel to and at a depth of 2 metres from water surface. Find the total pressure on the plate and the position of centre of pressure.



Solution. Refer to Fig. 3.31.

Area of the plate,

$$A = \frac{1}{2} \times 1 \times 1.5 = 0.75 \text{ m}^2$$

Inclination of the plate, $\theta = 30°$

Total pressure on the plate, P:

The depth of *c.g.* of the plate from water surface,

$$\bar{x} = 2 + \frac{1.5}{3} \sin 30°$$

$$= 2 + 0.5 \times 0.5 = 2.25 \text{ m}$$

Using the relation,

$$P = wA\bar{x} = 9.81 \times 0.75 \times 2.25$$

$$= \mathbf{16.55 \text{ kN (Ans.)}}$$

Depth of centre of pressure, $\bar{h}$:

Moment of inertia of a triangular section about its *c.g.*,

$$I_G = \frac{1 \times 1.5^3}{36} = 0.09375 \text{ m}^4$$

$$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x} = \frac{0.09375 \sin^2 30°}{0.75 \times 2.25} + 2.25$$

$$= \mathbf{2.264 \text{ m (Ans.)}}$$