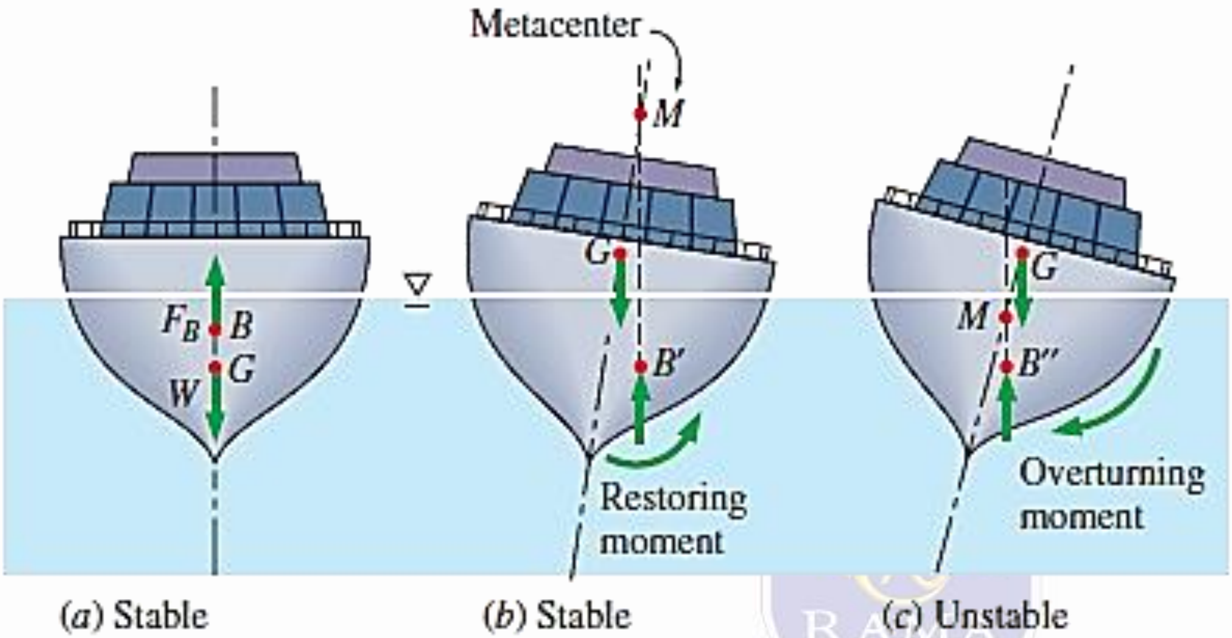
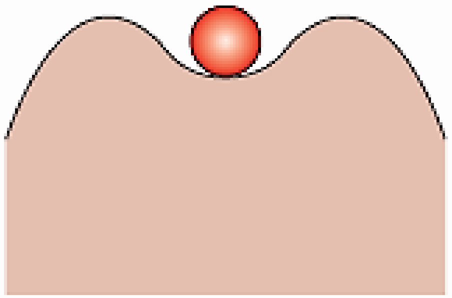
Stabilities of Bodies : Floating



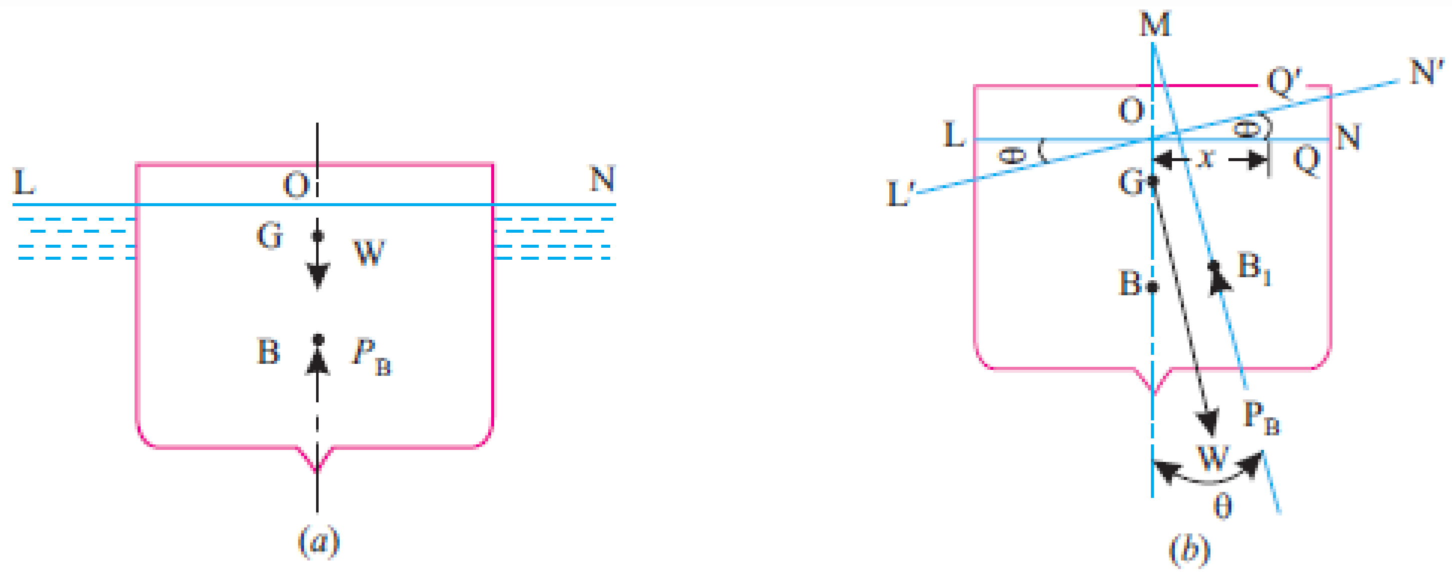


A floating body is *stable* if the body is (*a*) bottom-heavy and thus the center of gravity *G* is below the centroid *B* of the body, or (*b*) if the metacenter *M* is above point *G*. However, the body is (*c*) *unstable* if point *M* is below point *G*.

A ball in a trough between two hills is stable for small disturbances, but unstable for large disturbances

Fluid Properties

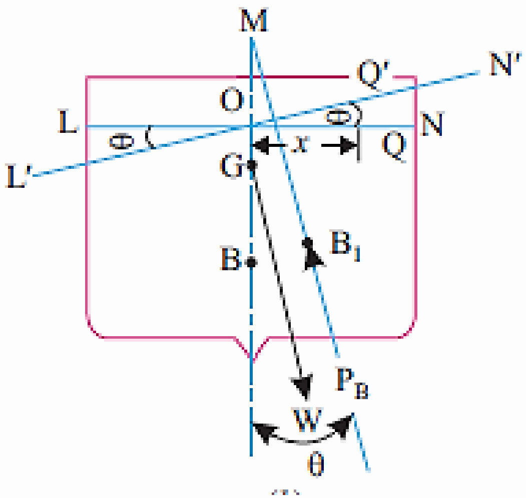
- Metacentre :
- Fig. shows body floating in a liquid in a state of equilibrium. When it is given a small angular displacement [see Fig. it starts oscillating about some point (M). This point, about which the body starts oscillating, is called metacentre.



- The metacentre may also be defined as a point of intersection of the axis of body passing through c.g.(G) and, original centre of buoyancy (B) and a vertical line passing through the centre of buoyancy (B1) of the tilted position of the body. The position of metacentre, M remains practically constant for the small angle of tilt θ.
- Metacentric height:
- The distance between the centre of gravity of a floating body and the metacentre (i.e. distance GM as shown in Fig. is called metacentric height. For stable equilibrium, the position of metacentre M remains higher than c.g. of the body, G. For unstable equilibrium, the position of metacentre M remains lower than G. For neutral equilibrium, the position of metacentre M coincides with G.

DETERMINATION OF METACENTRIC HEIGHT

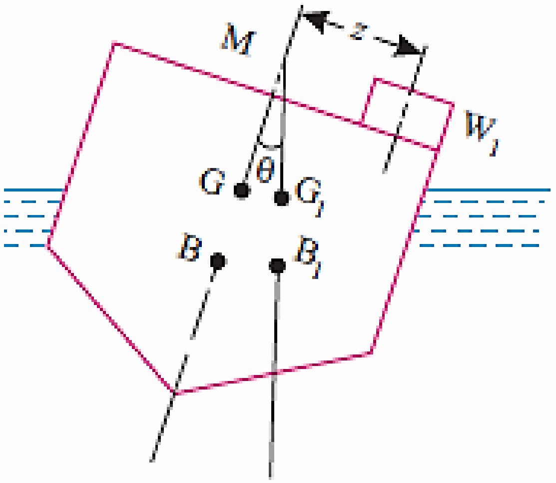
- Analytical Method



$$BM = \frac{1}{V}$$

- Now metacentric height, GM = BM ± BG
- + ve sign : when G is lower than B
- – ve sign : when G is higher than B

- Experimental Method



$\therefore \quad W_1.z = W.Gm.\tan\theta \quad \text{or} \quad GM = \dfrac{W_1.z}{W.\tan\theta}$

If, $\quad l$ = Length of plumb bob, and

d = Displacement of the plumb bob,

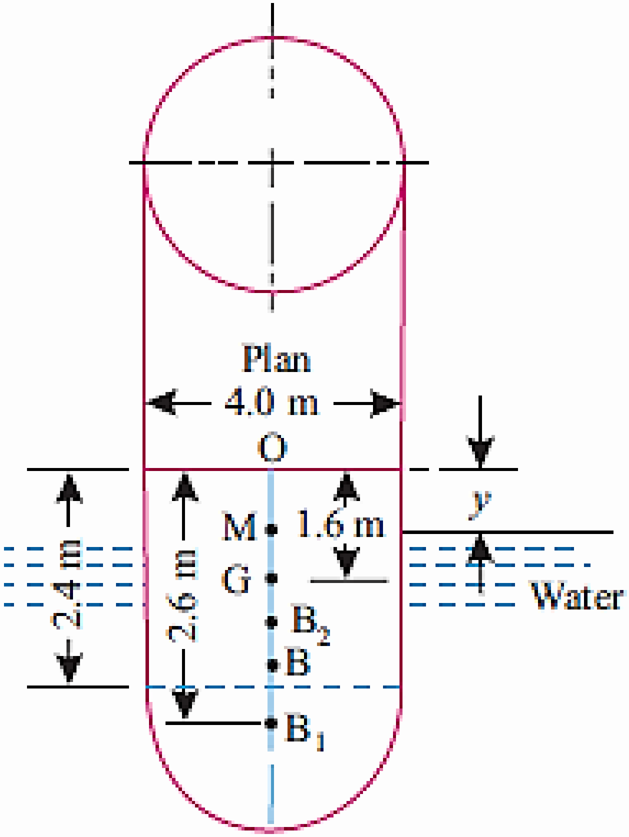
Then, $\quad \tan\theta = \dfrac{d}{l}$

and, metacentric height is given by:

$$GM = \frac{W_1.z.l}{W.d} \quad ...(4.3)$$

- A body has the cylindrical upper portion of 4m diameter and 2.4 m deep. The lower portion, which is curved, displaces a volume of 800 litres of water and its centre of buoyancy is situated 2.6 m below the top of the cylinder. The centre of gravity of the whole body is 1.6 m below the top of the cylinder and the total displacement of water is 52 kN. Find the metacentric height of the body.

- Solution. Given: Diameter of body, d = 4 m
- Depth of cylindrical portion = 2.4 m
- Volume of curved portion = 800 litres = 0.8 m3
- Distance between centre of buoyancy of curved portion and top of body,
- OB1 = 2.6 m
- Distance between centre of gravity of the whole body andtop of the cylinder, OG = 1.6 m



Total volume of water displaced, $V = \frac{52}{9.81} = 5.3 \text{ m}^3$

Volume of water displaced by the cylindrical portion

$$= 5.3 - 0.8 = 4.5 \text{ m}^3$$

If y is the distance between the water surface and the top of the body, then:

$$4.5 = \frac{\pi}{4} \times 4^2 \times (2.4 - y)$$

$$y = 2.4 - \frac{4.5 \times 4}{\pi \times 4^2} = 2.04 \text{ m}$$

Distance of the centre of buoyancy of the cylindrical portion from the top of the body,

$$OB_2 = y + \left(\frac{2.4 - y}{2}\right) = 2.04 + \frac{2.4 - 2.04}{2} = 2.22 \text{ m}$$

If B be the centre of buoyancy of the whole body, then:

$$OB = \frac{(0.8 \times 2.6) + (4.5 \times 2.22)}{0.8 + 4.5} = 2.227 \text{ m}$$

Now, $$BG = OB - OG = 2.277 - 1.6 = 0.677 \text{ m}$$

Now, $$BM = \frac{I}{V}$$

where, I = Moment of inertia of the cylindrical portion (top portion) about its $c.g.$

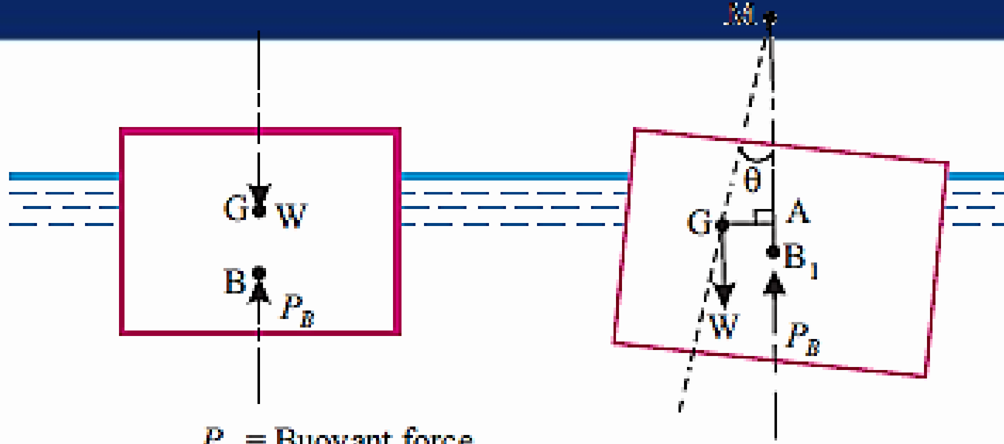
$$= \frac{\pi}{64} \times 4^4 \text{ m}^4 = 12.566 \text{ m}^4$$

and, $$V = 5.3 \text{ m}^3 \text{ (already calculated earlier)}$$

$$BM = \frac{12.566}{5.3} = 2.37$$

Metacentric height, $$GM = BM - BG = 2.37 - 0.677 = 1.693 \text{ m (Ans.)}$$

W = Weight of floating body,
θ = Angle (in radians) through which the body is depressed,
α = Angular acceleration of the body in rad/s2,
T = Time of rolling (i.e. one complete oscillation) in seconds,
k = Radius of gyration about G, and
I = Moment of inertia of the body about its centre of gravity G
= W k 2
g



P_B = Buoyant force

Angular acceleration of the body, $\alpha = -\dfrac{d^2\theta}{dt^2}$

– ve sign indicates that the force is acting in such a way that it tends to *decrease* the angle θ.

Also, Inertia torque = Moment of inertia × Angular acceleration

$$= I.\alpha = -\frac{W}{g}k^2 \times \frac{d^2\theta}{dt^2} \quad ...($$

Equating (*i*) and (*ii*), we get:

$$W.\ GM.\theta = -\frac{W}{g}k^2 \times \frac{d^2\theta}{dt^2} \quad \text{or} \quad \frac{W}{g}k^2\frac{d^2\theta}{dt^2} + W.GM.\theta = 0$$

Dividing both sides by *W*, we get:

$$\frac{k^2}{g} \times \frac{d^2\theta}{dt^2} + GM.\theta = 0$$

Again, dividing both sides by $\dfrac{k^2}{g}$, we get:

$$\frac{d^2\theta}{dt^2} + \frac{GM.g\theta}{k^2} = 0$$

The above equation is a differential equation of second degree, whose solution is:

$$Q = C_1 \sin\left[\sqrt{\frac{GM.g}{k^2}} \times t\right] + C_2 \cos\left[\sqrt{\frac{GM.g}{k^2}} \times t\right] \quad ...(i$$

where, C_1 and C_2 are constants of integration.

The values of C_1 and C_2 are obtained from the following boundary conditions:

1. At $t = 0, \theta = 0$

$C_2 = 0$ [By substitution of $t = 0, \theta = 0$ in (*iii*)]

GM = Metacentric height of the body. When the force which has caused angular displacement is removed the only force acting on the body is due to the restoring couple due to the weight W of the body and the force of buoyancy PB. ∴ Restoring couple = W × GA
= W × GM tan θ ...(i) = W . GM . Θ [assuming θ to be small (tan θ = θ)]

2. At $t = \dfrac{T}{2}, \theta = 0$

$$\therefore 0 = C_1 \sin\left[\sqrt{\frac{GM.g}{k^2}} \times \frac{T}{2}\right]$$

Since C_1 cannot be equal to zero, therefore:

$$\sin\left[\sqrt{\frac{GM.g}{k^2}} \times \frac{T}{2}\right] = 0 \quad \text{or} \quad \sqrt{\frac{GM.g}{k^2}} \times \frac{T}{2} = \pi$$

or, $$T = 2\pi\sqrt{\frac{k^2}{GM.g}}$$