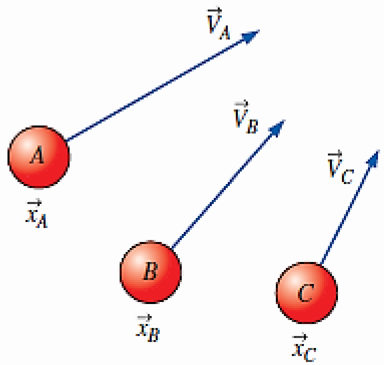
Fluid kinematics

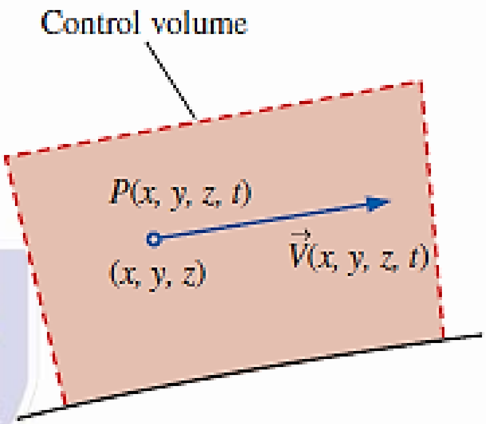
- Fluid kinematics is a branch of ‘Fluid mechanics’ which deals with the study of velocity and acceleration of the particles of fluids in motion and their distribution in space without considering any force or energy involved.
- DESCRIPTION OF FLUID MOTION

Langrangian Method



- The observer concentrates on the movement of a single particle
- The path taken by the particle and the changes in its velocity and acceleration are studied.
- This method entails the following shortcomings
- 1. Cumbersome and complex.
- 2. The equations of motion are very difficult to solve and the motion is hard to understand.

Eulerian Method



The observer concentrates on a point in the fluid system. Velocity, acceleration and other characteristics of the fluid at that particular point are studied.

The velocities at any point (x, y, z) can be written as:

$$\left.\begin{aligned} u &= f_1(x, y, z, t) \\ v &= f_2(x, y, z, t) \\ w &= f_3(x, y, z, t) \end{aligned}\right\}$$

The components of acceleration of the fluid particle can be worked out by partial differentiation as follows:

$$du = \frac{\partial u}{\partial x}.dx + \frac{\partial u}{\partial y}.dy + \frac{\partial u}{\partial z}.dz + \frac{\partial u}{\partial t}.dt$$

$$a_x = \frac{\partial u}{dt} = \left(\frac{\partial u}{\partial x}.\frac{dx}{dt} + \frac{\partial u}{\partial y}.\frac{dy}{dt} + \frac{\partial u}{dz}.\frac{dz}{dt}\right) + \frac{\partial u}{\partial t}.\frac{dt}{dt}$$

$$\frac{dx}{dt} = u, \frac{dy}{dt} = v, \frac{dz}{dt} = w$$

$$\left.\begin{aligned} a_x = \frac{du}{dt} &= \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) + \frac{\partial u}{\partial t} \\ a_y = \frac{dv}{dt} &= \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) + \frac{\partial v}{\partial t} \\ a_z = \frac{dw}{dt} &= \left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) + \frac{\partial w}{\partial t} \end{aligned}\right\}$$

Now, resultant velocity: $V = \sqrt{u^2 + v^2 + w^2}$

Resultant acceleration, $a = \sqrt{a_x^2 + a_y^2 + a_z^2}$

In vector notation:

Velocity vector: $V = ui + vj + wk$

Acceleration vector: $a = \frac{dV}{dt} = \left(u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} + w\frac{\partial V}{\partial z}\right) + \frac{\partial V}{\partial t}$

$$a = a_x i + a_y j + a_z k$$

and, $|V| = \sqrt{u^2 + v^2 + w^2}$

$$|a| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

Vectorially,

$$a = (V.\nabla)V + \frac{\partial V}{\partial t}$$

The velocity, in general, is a function of space(s) and time (t) i.e. V = f (x, y, z,t) or, V = f (s, t)

$$a = \frac{dV}{dt} = \frac{\partial V}{\partial s}.\frac{ds}{dt} + \frac{\partial V}{\partial t}$$

$$a = V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}$$

Fluid Properties

- Thus the acceleration consists of the two parts :
- (i) $V \frac{\partial V}{\partial s}$ This part is due to change in position or movement and is called convective acceleration.

$$\therefore \quad \text{Convective acceleration} = V\frac{\partial V}{\partial s} = \frac{1}{2}\frac{\partial (V)^2}{\partial s}$$

(ii) $\frac{\partial V}{\partial t}$: This part is with respect to time at a given location and is called **local** (or *temporal*) **acceleration**.

$$\text{Local acceleration} = \frac{\partial V}{\partial t} = \frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t}$$

Tangential and normal acceleration:

When the motion is curvilinear eqn. 5.16 gives the *tangential acceleration*. A particle moving in a curved path will always have a normal acceleration $a_n = \frac{V^2}{r}$ towards the centre of the curved path (r being the radius of the path), though its tangential acceleration (a_s) may be zero as in the case of uniform circular motion.

For motion along a curved path, in general,

$$a = a_s + a_n$$

$$= \left(V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} + \frac{V^2}{r} \right) \qquad ...(5.19)$$

TYPES OF FLUID FLOW

- **Steady flow.** The type of flow in which the fluid characteristics like velocity, pressure, density, etc. at a point do not change with time is called steady flow. Mathematically, we have:

$$\left(\frac{\partial u}{\partial t}\right)_{x_0, y_0, z_0} = 0; \left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} = 0; \left(\frac{\partial w}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

$$\left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0; \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0; \text{ and so on}$$

- where (x0, y0, z0) is a fixed point in a fluid field where these variables are being measured w.r.t. time.
- Example. Flow through a prismatic or non-prismatic conduit at a constant flow rate Q m3/s is steady.
- (A prismatic conduit has a constant size shape and has a velocity equation in the form u = ax2 + bx + c, which is independent of time t).
- **Unsteady flow.** It is that type of flow in which the velocity, pressure or density at a point
- change w.r.t. time. Mathematically, we have

$$\left(\frac{\partial u}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \left(\frac{\partial w}{\partial t}\right)_{x_0, y_0, z_0} \neq 0$$

$$\left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \text{ and so on}$$

- Example. The flow in a pipe whose valve is being opened or closed gradually (velocity equation is in the form u = ax2 + bxt).

Fluid Properties

- **Uniform flow.** The type of flow, in which the velocity at any given time does not change with respect to space is called uniform flow. Mathematically, we have:

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} = 0$$

∂V = Change in velocity, and

∂s = Displacement in any direction.

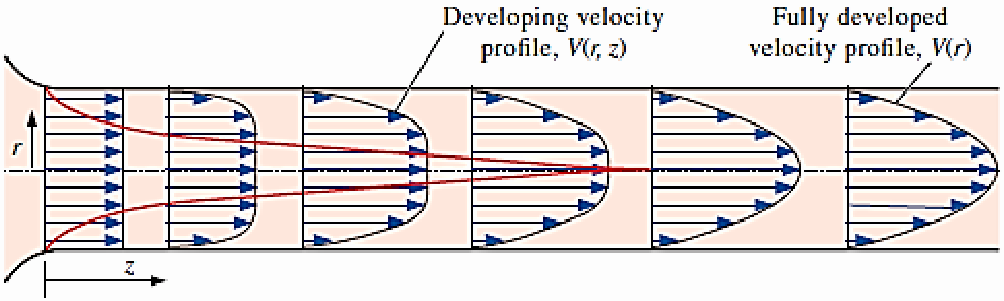
- Example. Flow through a straight prismatic conduit (i.e. flow through a straight pipe of constant diameter).
- **Non-uniform flow.** It is that type of flow in which the velocity at any given time changes with respect to space. Mathematically,

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} \neq 0$$

- Example. (i) Flow through a non-prismatic conduit.
- (ii) Flow around a uniform diameter pipe-bend or a canal bend

Fluid Properties

- **The development of the velocity profile in a circular pipe.**
- **V = V(r, z) and thus the flow is two-dimensional**
- **in the entrance region, and becomes one-dimensional**
- **downstream when the velocity profile fully develops**
- **and remains unchanged in the flow direction, V= V(r).**



- **One dimensional flow.** It is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only. Mathematically:
- u = f (x),v = 0 and w = 0
- where u, v and w are velocity components in x, y and z directions respectively.
- Example. Flow in a pipe where average flow parameters are considered for analysis
- **Two dimensional flow.** The flow in which the velocity is a function of time and two rectangular space coordinates is called two dimensional
- flow. Mathematically:
- u = f1 (x,y)
- v = f2 (x, y)
- w = 0
- Examples. (i) Flow between parallel plates of infinite extent.
- (ii) Flow in the main stream of a wide river.
- **Three dimensional flow**. It is that type of flow in which the velocityis a function of time and three mutually perpendicular directions.
- Mathematically:
- u = f1 (x, y, z)
- v = f2 (x, y, z)
- w = f3 (x, y, z)
- Examples. (i) Flow in a converging or diverging pipe or channel.
- (ii) Flow in a prismatic open channel in which the width and the water depth areof the same order of magnitude.