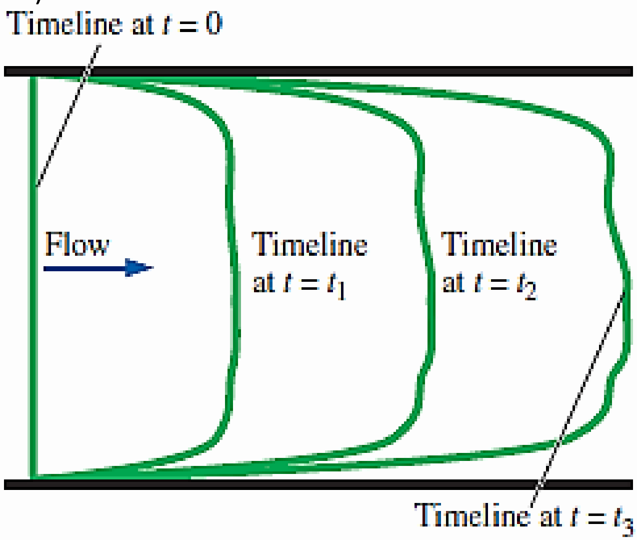
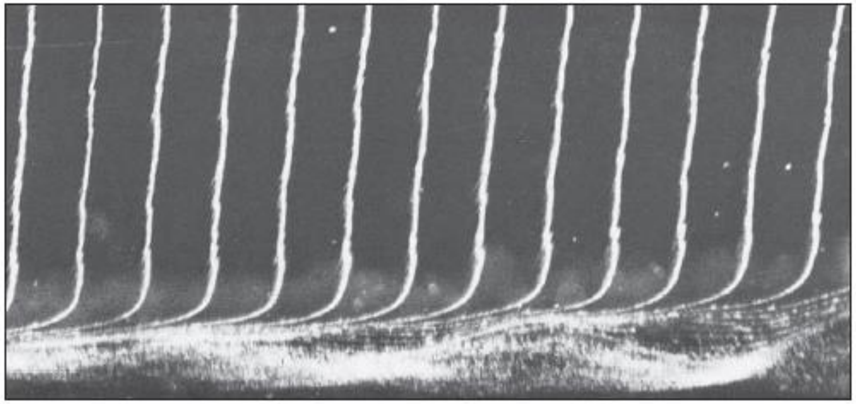
Fluid Properties

- Timelines:
- A timeline is a set of adjacent fluid particles that were marked at the same (earlier) instant in time.
- Timelines are particularly useful in situations where the uniformity of a flow (or lack thereof) is to be examined
- Timelines are formed by marking line of fluid particles, and then that line move (and deform) through the flow field; timelines are shown at t 5 0, t1, t2, and t3



- friction at the walls, the fluid velocity there is zero (the no-slip condition),
- and the top and bottom of the timeline are anchored at their starting locations.
- Timelines can be generated experimentally in a water channel through use of a hydrogen
- Bubble wire. When a short burst of electric current is sent through the cathode wire,
- electrolysis of the water occurs and tiny hydrogen gas bubbles form at the
- wire. Since the bubbles are so small, their buoyancy is nearly negligible,
- and the bubbles follow the water flow nicely



Timelines produced by a hydrogen bubble wire are used to visualize the boundary layer velocity profile shape along a flat plate. Flow is from left to right, and the hydrogen bubble wire is located to the left of the field of view. Bubbles near the wall reveal a flow instability that leads to turbulence.

Numerical on velocities and acceleration

- In a fluid, the velocity field is given by V = (3x + 2y) i + (2z + 3x2) j + (2t – 3z) k Determine:
- (i) The velocity components u, v, w at any point in the flow field;
- (ii) The speed at point (1, 1, 1);
- (iii) The speed at time t = 2s at point (0, 0, 2).
- Also classify the velocity field as steady, or unsteady, uniform or non-uniform and one, two or three dimensional.

Solution. *Given:* Velocity field, $V = (3x + 2y)\,i + (2z + 3x^2)\,j + (2t - 3z)\,k$

(*i*) Velocity components:

The velocity components are:

$$u = 3x + 2y,\ v = (2z + 3x^2),\ w = (2t - 3z) \quad \text{(Ans.)}$$

(*ii*) Speed at point (1, 1, 1), $V_{(1,1,1)}$:

Substituting $x = 1, y = 1, z = 1$ in the expressions for u, v and w, we have:

$$u = (3 + 2) = 5,\ v = (2 + 3) = 5,\ w = (2t - 3)$$

$$\therefore \quad V^2 = u^2 + v^2 + w^2$$

$$= 5^2 + 5^2 + (2t - 3)^2$$

$$= 25 + 25 + 4t^2 - 12t + 9$$

$$= 4t^2 - 12t + 59$$

$$\therefore \quad V_{(1,1,1)} = \sqrt{4t^2 - 12t + 59} \quad \text{(Ans.)}$$

(*iii*) Speed at $t = 2s$ at point (0, 0, 2):

Substituting $t = 2, x = 0, y = 0, z = 2$ in the expressions for u, v and w, we get:

$$u = 0,\ v = (2 \times 2 + 0) = 4,\ w = (2 \times 2 - 3 \times 2) = -2$$

$$\therefore \quad V^2 = u^2 + v^2 + w^2 = 0 + 4^2 + (-2)^2 = 20$$

or, $$V_{(0,0,2)} = \sqrt{20} = 4.472 \text{ units (Ans.)}$$

Velocity field, type:

(*i*) Since V at given (x, y, z) *depends on t* it is **unsteady flow, (Ans.)**

(*ii*) Since at given t velocity changes in the X direction it is **non-uniform flow. (Ans.)**

(*iii*) Since V depends on x, y, z it is three **dimensional flow. (Ans.)**

- Velocity for a two dimensional flow field is given by V = (3 + 2xy + 4t2) i + (xy2 + 3t) j Find the velocity and acceleration at a point (1,2) after 2 sec.

Solution. *Given*: Velocity field: $V = (3 + 2xy + 4t^2)\, i + (xy^2 + 3t)\, j$

Velocity at (1, 2), $V_{(1,2)}$:

Substituting $x = 1$, $y = 2$, $t = 2$ in the expression of velocity field, we get:

$$V = (3 + 2 \times 1 \times 2 + 4 \times 2^2)\, i + (1 \times 2^2 + 3 \times 2)\, j$$
$$= (3 + 4 + 16)\, i + (4 + 6)\, j$$
$$= 23i + 10j$$

$$\therefore \quad V_{(1,2)} = \sqrt{23^2 + 10^2} = \textbf{25.08 units (Ans.)}$$

Acceleration at point (1, 2), $a_{(1,2)}$:

We know that: $$a = \frac{dV}{dt} = \left(u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y}\right) + \frac{\partial V}{\partial t}$$

Also, $$V = (3 + 2xy + 4t^2)i + (xy^2 + 3t)\, j \quad \text{...(Given)}$$

$$\therefore \quad \frac{\partial V}{\partial x} = 2yi + y^2 j,$$

$$\frac{\partial V}{\partial y} = 2xi + 2xyj, \text{ and}$$

$$\frac{\partial V}{\partial t} = 8\, ti + 3\, j$$

$$\therefore \quad a = (3 + 2xy + 4t^2)(2yi + y^2j) + (xy^2 + 3t)(2xi + 2xyj) + (8ti + 3j)$$
$$(\because u = 3 + 2xy + 4t^2 \text{ and } v = xy^2 + 3t)$$

Substituting the values, we get:

$$a = (3 + 2 \times 1 \times 2 + 4 \times 2^2)(2 \times 2i + 2^2 j) + (1 \times 2^2 + 3 \times 2)$$
$$(2 \times 1i + 2 \times 1 \times 2j) + (8 \times 2 \times i + 3j)$$
$$= (3 + 4 + 16)(4i + 4j) + (4 + 6)(2i + 4j) + (16i + 3j)$$
$$= 23(4i + 4j) + 10(2i + 4j) + (16i + 3j)$$
$$= 92i + 92j + 20i + 40j + 16i + 3j$$
$$= 128\, i + 135\, j$$

$$a_{(1,2)} = \sqrt{128^2 + 135^2} = \textbf{186.03 units (Ans.)}$$

- Find the velocity and acceleration at a point (1, 2, 3) after 1 sec. for a threedimensional flow given by u = yz + t, v = xz – t, w = xy m/s.

Solution. *Given*: Three-dimensional flow field is given as:

$$u = yz + t,\ v = xz - t,\ w = xy \text{ m/s}$$

Velocity at a point 1, 2, 3 $V_{(1,2,3)}$:

Velocity at a point (1, 2, 3) after 1s is calculated as follows:

$$u = yz + t = 2 \times 3 + 1 = 7 \text{ m/s},\ v = xz - t = 1 \times 3 - 1 = 2 \text{ m/s and}$$

$$w = xy = 1 \times 2 = 2 \text{ m/s.}$$

$$\therefore \quad V_{(1,2,3)} = 7i + 2j + 2k$$

$$= \sqrt{7^2 + 2^2 + 2^2} = 7.55 \text{ m/s}$$

Hence, $V_{(1,2,3)} = \mathbf{7.55\ m/s}$ **(Ans.)**

Acceleration, $a_{(1,2,3)}$:

Now, $V = (yz + t)i + (xz - t)j + xy\,k$ m/s

Acceleration, $$a = \frac{dV}{dt} = \left(u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} + w\frac{\partial V}{\partial z}\right) + \frac{\partial V}{\partial t}$$

$$a = (yz + t)(zi + yk) + (xz - t)(zi + xk) + xy(yi + xj) + (1i - 1j)$$

$$\therefore \quad a_{(1,2,3)} = 7(3j + 2k) + 2(3i + 1k) + 2(2i + 1j) + (1i - 1j)$$

$$= (21j + 14k) + (6i + 2k) + (4i + 2j) + (1i - 1j)$$

$$= (10i + 23j + 16k) + (1i - 1j)$$

The *convective acceleration components are*: (10, 23, 16) m/s^2

The *local acceleration components are*: (1, –1) m/s^2 along x and y directions.

The total acceleration of fluid particles at the points (1, 2, 3) is given by:

$$a_{(1,2,3)} = \sqrt{(10+1)^2 + [23 + (-1)]^2 + 16^2}$$

$$= \sqrt{11^2 + 22^2 + 16^2} = 29.34 \text{ m/s}^2$$

Hence, $a_{(1,2,3)} = \mathbf{29.34\ m/s^2}$ **(Ans.)**

- For a three-dimensional flow the velocity distribution is given by u = – x, v = 3 – y and w = 3 – z. What is the equation of a streamline passing through (1,2,2)?

Solution. *Given:* $u = -x$, $v = 3 - y$, $w = 3 - z$

Equation of a streamline passing through (1, 2, 2):

The streamlines are defined by:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Substituting for u, v and w, we get:

$$\underset{(i)}{\frac{dx}{-x}} = \underset{(ii)}{\frac{dy}{3-y}} = \underset{(iii)}{\frac{dz}{3-z}}$$

Considering the expressions (*i*) and (*ii*) and integrating, we get:

$$\int \frac{dx}{-x} = \int \frac{dz}{(3-y)}$$

$$= -\log_e x = -\log_e (3-y) + C_1$$

(where, C_1 = constant of integration).

Since the streamline passes through $x = 1, y = 2$ $\quad \therefore C_1 = 0$

$\therefore \quad (x)^{-1} = (3-y)^{-1} \quad$ or $\quad x = (3-y) \quad$ ...(1)

Considering the expressions (*i*) and (*iii*), and integrating, we get:

$$\int \frac{dx}{-x} = \int \frac{dy}{3-z}$$

or, $\quad -\log_e x = -\log (3-z) + C_2$

(where C_2 = constant of integration)

Since the streamline passes through $x = 1, z = 2$ $\quad \therefore C_2 = 0$

$\therefore \quad x^{-1} = (3-z)^{-1}$

or, $\quad x = (3-z) \quad$ ...(2)

From (1) and (2), the equation of the streamline passing through (1, 2, 2) is given as:

$x = (3-y) = (3-z)$ **(Ans.)**

- Obtain the equation to the streamlines for the velocity field given as: V = 2x3i – 6x2yj

Solution. *Given*: Velocity field, $V = 2x^3 i - 6x^2 yj$

Here, $u = 2x^3, v = 6x^2 y$

The streamlines in two dimensions are defined by:

$$\frac{dx}{u} = \frac{dy}{v}$$

or, $$\frac{dy}{dx} = \frac{v}{u} = \frac{-6x^2 y}{2x^3} = \frac{-3y}{x}$$

Separating the variables, we have:

$$\frac{dy}{y} = \frac{-3dx}{x}$$

Integrating, we get:

$$ln\,(y) = -3ln\,(x) + C_1$$

or, $$ln\,(y) + 3\,ln\,(x) = C_1$$

or, $$yx^3 = C \text{ (Ans.)}$$