- For the following stream functions calculate velocity at a point (1, 2): (i) ψ = 3 xy (ii) y = 3x2y – y

Solution. *(i)* $\psi = 3xy$: ...(Given)

$$u = \frac{\partial \psi}{\partial y} = 3x$$

$$v = -\frac{\partial \psi}{\partial x} = -3y$$

At (1,2): $u = 3 \times 1 = 3$

$$v = -3 \times 2 = -6$$

$$\therefore \quad V = \sqrt{u^2 + v^2} = \sqrt{(3)^2 + (-6)^2} = \sqrt{45} \text{ units. (Ans.)}$$

(ii) $\psi = 3x^2y - y^3$: ...(Given)

$$u = \frac{\partial \psi}{\partial y} = 3x^2 - 3y^2$$

$$v = -\frac{\partial \psi}{\partial x} = -6xy$$

At (1,2): $u = 3 \times (1)^2 - 3 \times (2)^2 = -9$

$$v = -6 \times 1 \times 2 = -12$$

$$V = \sqrt{u^2 + v^2} = \sqrt{(-9)^2 + (-12)^2} = 15 \text{ (Ans.)}$$

- A two-dimensional flow field i(i) The stream function.
- (ii) The velocity at L(2, 6) and M (6,6) and the pressure difference between the points L and M.
- (iii) The discharge between the stream lines passing through the points L and M.s given by $\phi = 3xy$, determine:

Solution. *Given.* $\phi = 3xy$ ...Flow field

***(i)* The stream function ψ:**

We know that: $$u = -\frac{\partial \phi}{\partial x} = -\frac{\partial}{\partial x}(3xy) = -3y$$

$$v = -\frac{\partial \phi}{\partial y} = -\frac{\partial}{\partial y}(3xy) = -3x$$

Also, $$u = \frac{\partial \psi}{\partial y} = -3y, \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} = -3x$$

Again, $$d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy \quad \text{or} \quad \partial\psi = 3x dx + (-3y)\, dy$$

Integrating both sides, we get:

$$\psi = \int 3x\, dx + \int (-3y)\, dy$$

$$= 3 \times \frac{x^2}{2} - 3 \times \frac{y^2}{2} + C = \frac{3}{2}(x^2 - y^2) + C$$

(where, C = constant of integration.)

For $\psi = 0$ at the origin, the constant $C = 0$

$$\therefore \quad \psi = \frac{3}{2}(x^2 - y^2) \quad \text{(Ans.)}$$

***(ii)* Velocities at L and M:**

At $L(2, 6)$: $u = -3 \times 6 = -18$, $v = -3 \times 2 = -6$

$$\therefore \quad V_L = \sqrt{u^2 + v^2} = \sqrt{(-18)^2 + (-6)^2} = 18.97 \text{ units (Ans.)}$$

At M(6, 6): $u = -3 \times 6 = -18$, $v = -3 \times 6 = -18$

$$\therefore \quad V_M = \sqrt{u^2 + v^2} = \sqrt{(-18)^2 + (-18)^2} = 25.45 \text{ units (Ans.)}$$

Pressure difference between *L* and *M*:

For two-dimensional plane flow:

$$\frac{p_L}{w} + \frac{V_L^2}{2g} = \frac{p_M}{w} + \frac{V_M^2}{2g}$$

$$\therefore \quad \frac{p_L - p_M}{w} = \frac{1}{2g}(V_M^2 - V_L^2) = \frac{648 - 360}{2 \times 9.81} = 14.68 \text{ units (Ans.)}$$

***(iii)* The discharge between the streamlines, *q*:**

$$\psi = \frac{3}{2}(x^2 - y^2)$$

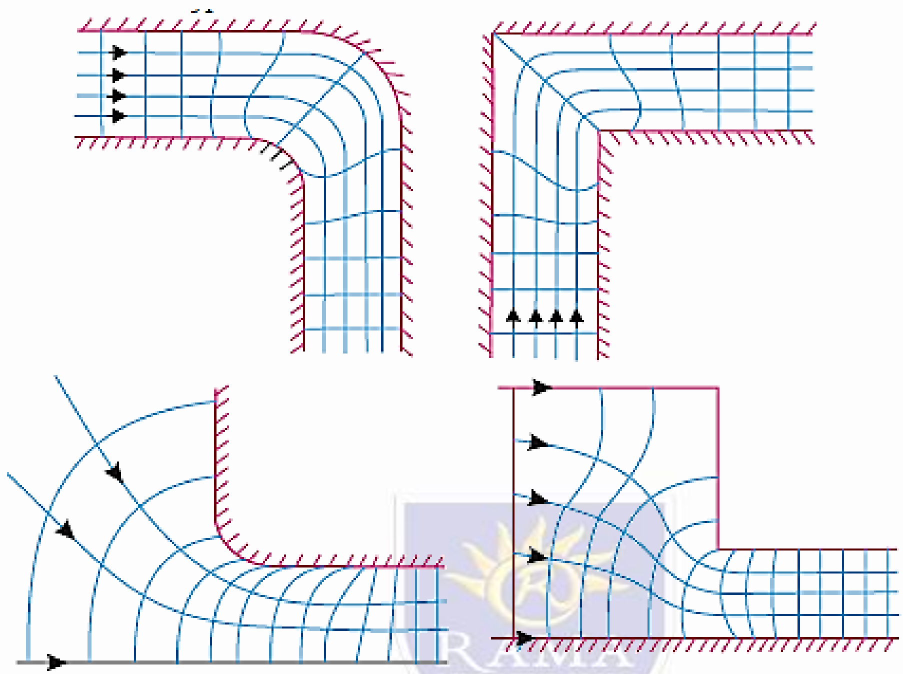
$$\psi_{L(2,6)} = \frac{3}{2}(2^2 - 6^2) = -48 \text{ units.}$$

$$\psi_{M(2,6)} = \frac{3}{2}(6^2 - 6^2) = 0$$

$$\therefore \quad q = \psi_M - \psi_L = 0 - (-48) = 48 \text{ units (Ans.)}$$

FLOW NETS

- FLOW NETS; A grid obtained by drawing a series of stream lines and equipotential lines is known as a flow net. The flow net provides a simple graphical technique for studying two-dimensional irrotational flows especially in the cases where mathematical relations for stream function and velocity function are either not available or are rather difficult and cumbersome to solve.
- Methods of Drawing Flow Nets
- The following methods are used for drawing flow nets:
- 1. Analytical method (or Mathematical analysis):
- — Here, the equations corresponding to the curves ϕ and ψ are first obtained and the same are plotted to give the flow net pattern for the flow of fluid between the given boundary shape.
- — This method can be applied to problems with simple and ideal boundary conditions.
- 2. Graphical method:
- — A graphical method consists of drawing stream lines and equipotential lines such that they cut orthogonally and form curvilinear squares.
- — This method consumes lot of time and requires lot of erasing to get the proper shape of a flow net.
- 3. Electrical analogy method:
- — This method is a practical method of drawing a flow net for a particular set of boundaries.
- — It is based on the fact that the flow of fluids and flow of electricity through a conductor are analogous. These two systems are similar in the respect that electric potential is analogous to the velocity potential, the electric current is analogous to the velocity of flow, and the homogeneous conductor is analogous to the homogeneous fluid.
- 4. Hydraulic models:
- — Stream lines can be traced by injecting a dye in a seepage model or Heles haw apparatus.
- — Then, by drawing equipotential lines the flow net is completed.



Use of flow nets:
The following are the uses of flow-net analysis:
1. To determine the stream lines and equipotential lines.
2. To determine quantity of seepage and upward lift pressure below hydraulic structure
3. To determine the velocity and pressure distribution, for given boundaries of flow (provided the velocity distribution and pressure at any reference section are known).
4. To determine the design of the outlets for their streamlining.
Limitations of flow nets:
The following are the limitations of flow net:
1. The flow net analysis cannot be applied in the region close to the boundary where the effects of viscosity are predominant.
2. In case of a flow of a fluid past a solid body, while the flow net gives a fairly accurate picture of the flow pattern for the upstream part of the solid body, it can give little information concerning the flow conditions at the rear because of separation and eddies.

Fluid Properties

1. The motion of fluid particles may be described by which of the following methods?
(a) Langrangian method
(b) Eulerain method
(c) Both (a) and (b)
(d) None of the above.
2. In which of the following methods, the observer concentrates on a point in the fluid system?
(a) Langrangian method
(b) Eulerian method
(c) Any of the above
(d) None of the above.
3. Normal acceleration in fluid-flow situation exists only when
(a) the flow is unsteady
(b) the flow is two-dimensional
(c) the streamlines are straight and parallel
(d) the streamlines are curved.
4. In a steady flow the velocity
(a) does not change from place to place
(b) at a given point does not change with time
(c) may change its direction but the magnitude remains unchanged
(d) none of the above.
5. The flow in a pipe whose valve is being opened or closed gradually is an example of
(a) steady flow (b) unsteady flow
(c) rotational flow
(d) compressible flow.
6. The type of flow in which the velocity at any given time does not change with respect to space is called
(a) steady flow
(b) compressible flow
(c) uniform flow (d) rotational flow.
7. Flow in a pipe where average flow parameters are considered for analysis is an example of
(a) incompressible flow
(b) one-dimensional flow
(c) two-dimensional flow
(d) three-dimensional flow.
8. The flow in a river during the period of heavy rainfall is
(a) steady, non-uniform and three-dimensional
(b) steady, uniform, two-dimensional
(c) unsteady, uniform, three-dimensional
(d) unsteady, non-uniform and three-dimensional.
9. Flow between parallel plates of infinite extent is an example of
(a) one-dimensional flow
(b) two-dimensional flow
(c) three-dimensional flow
(d) compressible flow.
10. If the flow is irrotational as well as steady it is known as
(a) non-uniform flow
(b) one-dimensional flow
(c) potential flow
(d) none of the above.

1. (c) 2. (d) 3. (b) 4. (b) 5. (b) 6. (c) 7. (b) 8. (d) 9. (b) 10. (c)

BERNOULLI’S EQUATION

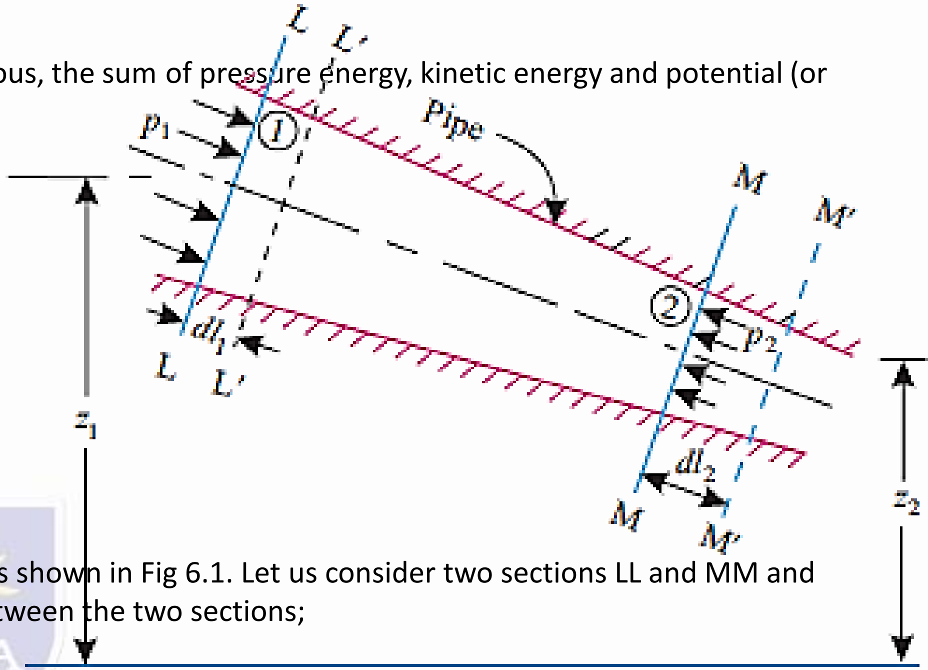
- Bernoulli’s equation states as follows:
- “In an ideal incompressible fluid when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential (or datum) energy is constant along a stream line.”
- Mathematically,

$$\frac{p}{w} + \frac{V^2}{2g} + z = \text{constant}$$

$$\frac{p}{w} = \text{Pressure energy,}$$

$$\frac{V^2}{2g} = \text{Kinetic energy, and}$$

$$z = \text{Datum (or elevation) energy.}$$



- Proof:
- Consider an ideal incompressible liquid through a non-uniform pipe as shown in Fig 6.1. Let us consider two sections LL and MM and assume that the pipe is running full and there is continuity of flow between the two sections;
- Let, p1 = Pressure at LL,
- V1 = Velocity of liquid at LL,
- z1 = Height of LL above the datum,
- A1 = Area of pipe at LL, and
- p2, V2, z2, A2 = Corresponding values at MM.
- Let the liquid between the two sections LL and MM move to L′ L′ and M′M′ through very small lengths dl1 and dl2 as shown in Fig. 6.1. This movement of liquid between LL and MM is equivalent to the movement of the liquid between LL and L’L’ and MM and M’ M’, the remaining liquid between L’ L’ and MM being unaffected. Let, W = Weight of liquid between LL and L’ L’ .
- As the flow is continuous,
- ∴ W = wA1 . dl1 = wA2 . dl2 or, A1 . dl1 = W/w ...(i)
- Similarly, A2 . dl2 = W/w...(ii)
- ∴ A1 . dl1 = A2.dl2
- Work done by pressure at LL, in moving the liquid to L’ L’ = Force × distance = p1 . A1 . dl1