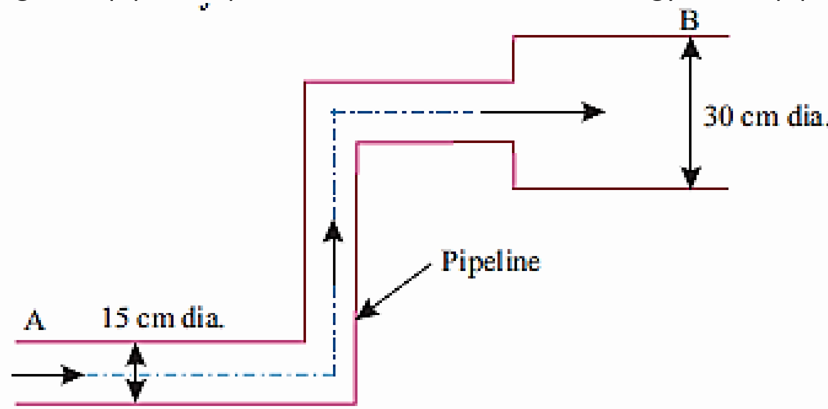
Numericals

- A pipeline (Fig. 6.5) is 15 cm in diameter and it is at an elevation of 100 m at section A. At section B it is at an elevation of 107 m and has diameter of 30 cm. When a discharge of 50 litre/sec of water is passed through this pipeline, pressure at A is 35 kPa. The energy loss in pipe is 2m of water. Calculate pressure at B if flow is from A to B.
- Solution
- DA = 15 cm = 0.15 m; DB = 30 cm = 0.3 m;
- pA = 35 kPa; Q = 50 litre/sec = 0.05 m3/s;
- hf = 2 m of water; Direction of flow: from A to B



Pressure at B, p_B:

$$V_A = \frac{Q}{\frac{\pi}{4}\times D_A^2} = \frac{0.05}{\frac{\pi}{4}\times 0.15^2} = 2.829 \text{ m/s}$$

$$V_B = \frac{Q}{\frac{\pi}{4}\times D_B^2} = \frac{0.05}{\frac{\pi}{4}\times (0.3)^2} = 0.707 \text{ m/s}$$

Applying Bernoulli's equation between sections A and B, we get:

$$\frac{P_A}{w} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{w} + \frac{V_B^2}{2g} + z_B + h_f$$

or, $$\frac{p_B}{w} = \frac{p_A}{w} + \left(\frac{V_A^2 - V_B^2}{2g}\right) + (z_A - z_B) - h_f$$

or, $$p_B = p_A + w\left[\left(\frac{V_A^2 - V_B^2}{2g}\right) + (z_A - z_B) - h_f\right]$$

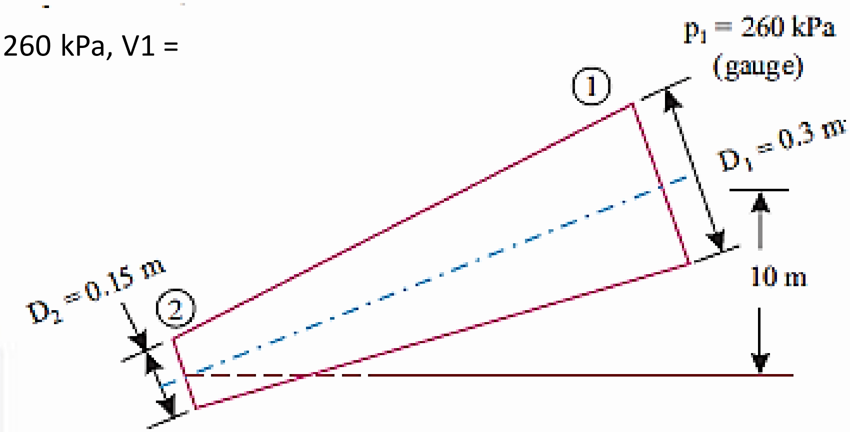
$$= 35 + \frac{(1000\times 9.81)}{1000}\left[\left(\frac{2.829^2 - 0.707^2}{2\times 9.81}\right) + (100 - 107) - 2\right]$$

$$35 + 9.81\,(0.3824 - 7 - 2) = -49.54 \text{ kPa.}$$

i.e., p_B **= – 49.54 kPa.** This shows that the given pressure at A, 35 kPa is *gauge pressure* and hence there is *vacuum at point B*. (Ans.)

- Water flows in a circular pipe. At one section the diameter is 0.3 m, the static pressure is 260 kPa gauge, the velocity is 3 m/s and the elevation is 10 m above ground level. The elevation at a section downstream is 0 m, and the pipe diameter is 0.15 m. Find out the gauge pressure at the downstream section. Frictional effects may be neglected. Assume density of water to be 999 kg/m3.

- Solution. Refer to Fig. 6.7. D1 = 0.3 m; D2 = 0.15 m; z1 = 0; z2 = 10 m; p1 = 260 kPa, V1 =
- 3 m/s; ρ = 999 kg/m3. From continuity equation, A1 V1 = A2V2,



$$V_2 = \frac{A_1}{A_2} \times V_1 = \left(\frac{\frac{\pi}{4} D_1^2}{\frac{\pi}{4} D_2^2} \right) \times V_1$$

$$= \left(\frac{D_1}{D_2} \right)^2 \times V_1 = \left(\frac{0.3}{0.15} \right)^2 \times 3 = 12 \text{ m/s}$$

Weight density of water, $w = \rho g = 999 \times 9.81 = 9800.19 \text{ N/m}^3$

From Bernoulli's equation between sections 1 and 2 (neglecting friction effects as given), we have:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

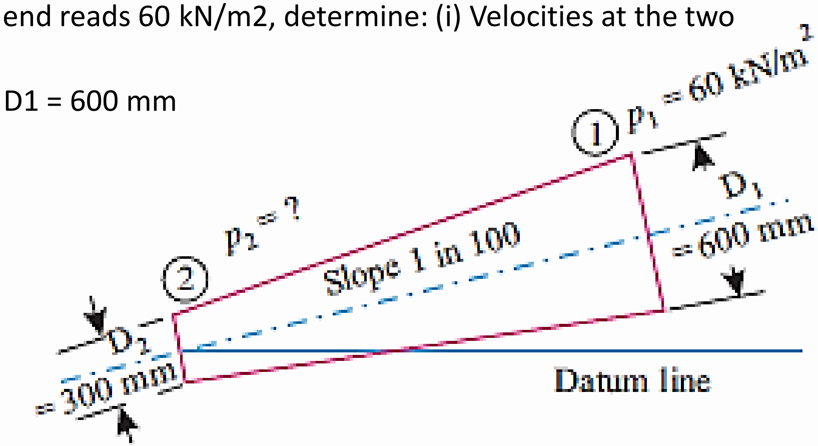
$$\frac{260 \times 1000}{9800.19} + \frac{(3)^2}{2 \times 9.81} + 10$$

$$= \frac{p_2}{9800.19} + \frac{(12)^2}{2 \times 9.81} + 0$$

$$26.53 + 0.459 + 10 = \frac{p_2}{9800.19} + 7.34$$

$$p_2 = 290566 \text{ N/m}^2 = \mathbf{290.56 \text{ kPa (Ans.)}}$$

- A pipe 200 m long slopes down at 1 in 100 and tapers from 600 mm diameter at the higher end to 300 mm diameter at the lower end, and carries 100 litres/sec of oil (sp. Gravity 0.8). If the pressure gauge at the higher end reads 60 kN/m2, determine: (i) Velocities at the two ends; (ii) Pressure at the lower end. Neglect all losses.
- Solution. Length of the pipe, l = 200 m; diameter of the pipe at the higher end, D1 = 600 mm
- = 0.6 m



$$\therefore \text{Area},\ A_1 = \frac{\pi}{4} \times 0.6^2 = 0.283\ \text{m}^2$$

Diameter of the pipe at the lower end,

$$D_2 = 300\ \text{mm} = 0.3\ \text{m}$$

$$\therefore \text{Area},\ A_2 = \frac{\pi}{4} \times 0.3^2 = 0.0707\ \text{m}^2$$

Height of the higher end, above datum,

$$z_1 = \frac{1}{100} \times 200 = 2\ \text{m}$$

Height of the lower end, above datum $z_2 = 0$

- Rate of oil flow, Q = 100 litres/sec = 0.1 m3/s Pressure at the higher end, p1 = 60 kN/m2 (i) Velocities, V1, V2: Now, Q = A1 V1 = A2 V2
- where, V1 and V2 are the velocities at the higher and lower ends respectively.

$$V_1 = \frac{Q}{A_1} = \frac{0.1}{0.283} = 0.353\ \text{m/s (Ans.)}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.1}{0.0707} = 1.414\ \text{m/s (Ans.)}$$

- (ii) Pressure at the lower end p2:
- Using Bernoulli's equation for both ends of pipe, we have:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$\frac{60}{0.8 \times 9.81} + \frac{0.353^2}{2 \times 9.81} + 2 = \frac{p_2}{0.8 \times 9.81} + \frac{1.414^2}{2 \times 9.81} + 0$$

$$7.64 + 0.00635 + 2 = \frac{p_2}{0.8 \times 9.81} + 0.102$$

$$\frac{p_2}{0.8 \times 9.81} = 9.54\ \text{m}$$

$$p_2 = 74.8\ \text{kN/m}^2\ \text{(Ans.)}$$

- Gasoline (sp. gr. 0.8) is flowing upwards a vertical pipeline which tapers from 300 mm to 150 mm diameter. A gasoline mercury differential manometer is connected between 300 mm and 150 mm pipe section to measure the rate of flow. The distance between the manometer tappings is 1 metre and gauge reading is 500 mm of mercury. Find: (i) Differential gauge reading in terms of gasoline head; (ii) Rate of flow. Neglect friction and other losses between tappings.
- Solution. Sp. gravity of gasoline = 0.8
- At Inlet: Diameter, D1 = 300 mm = 0.3 m

$$\therefore \quad \text{Area, } A_1 = \frac{\pi}{4} \times 0.3^2 = 0.0707 \text{ m}^2$$

At Outlet:

$$\text{Diameter, } D_2 = 150 \text{ mm} = 0.15 \text{ m}$$

$$\therefore \quad \text{Area, } A_2 = \frac{\pi}{4} \times 0.15^2 = 0.01767 \text{ m}^2$$

$$\text{Length of the pipe} = 1\text{m}$$

$$\text{Let datum of the pipe at inlet, } z_1 = 0$$

$$\therefore \text{ Datum of the pipe at outlet, } z_2 = 0 + 1 = 1 \text{ m}$$

Gauge reading, $h = 500$ mm of mercury $= 0.5$ m of mercury.

***(i)* Differential gauge reading in terms of gasoline head:**

$$\begin{aligned}\text{The gauge reading} &= 0.5 \text{ m of mercury}\\ &= \frac{13.6 - 0.8}{0.8} \times 0.5 \text{ of gasoline}\\ &= \mathbf{8 \text{ m of gasoline (Ans.)}}\end{aligned}$$

***(ii)* Rate of flow, Q:**

(ii) Rate of flow, Q:

Let, V1 = Velocity of gasoline at the inlet, and

V2 = Velocity of gasoline at the outlet.

We know that, as per equation of continuity:

A1V1 = A2V2

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.0707 \times V_1}{0.01767} = 4V_1$$

Now, using Bernoulli's equation for the inlet and outlet of the pipe, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$\left\{\frac{p_1}{w} - \frac{p_2}{w}\right\} + \left\{\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right\} + (z_1 - z_2) = 0$$

$$8 + \left[\frac{V_1^2}{2g} - \frac{(4V_1)^2}{2g}\right] + [0 - 1] = 0$$

or,

$$8 - \frac{15V_1^2}{2g} - 1 = 0$$

or,

$$\frac{15V_1^2}{2g} = 7$$

$$\therefore \quad V_1 = \left[\frac{7 \times 2 \times 9.81}{15}\right]^{1/2} = 3.026 \text{ m/s.}$$

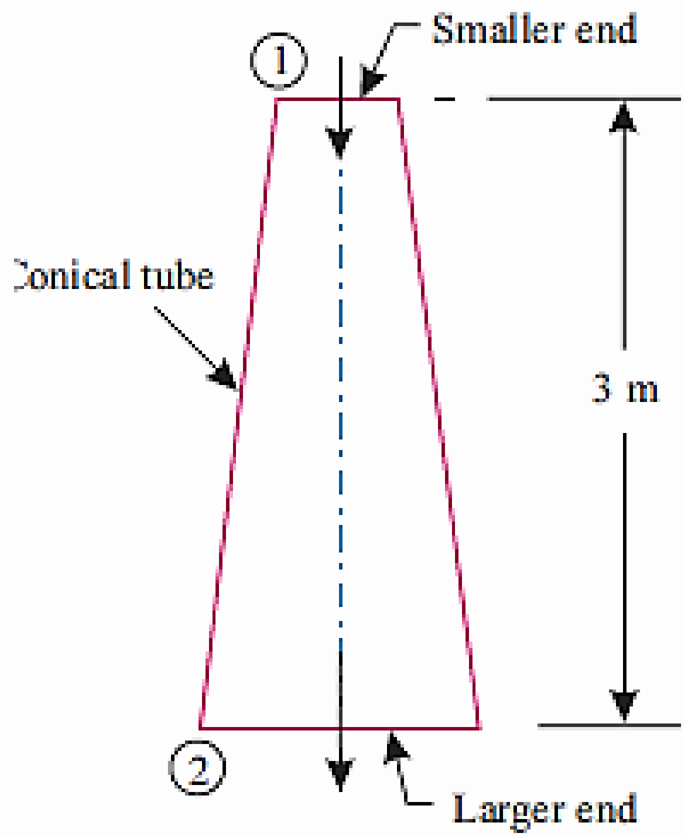
Rate of flow, $Q = A_1V_1 = 0.0707 \times 3.026 = \mathbf{0.2139 \text{ m}^3/\text{s}}$ **(Ans.)**

BERNOULLI'S EQUATION FOR REAL FLUID

- Bernoulli's equation earlier derived was based on the assumption that fluid is non-viscous and therefore frictionless. Practically, all fluids are real (and not ideal) and therefore are viscous as such there are always some losses in fluid flows. These losses have, therefore, to be taken into consideration in the application of Bernoulli's equation which gets modified (between sections 1 and 2) for real fluids as follows:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_L \quad ...(6.4)$$

where, h_L = Loss of energy between sections 1and 2.



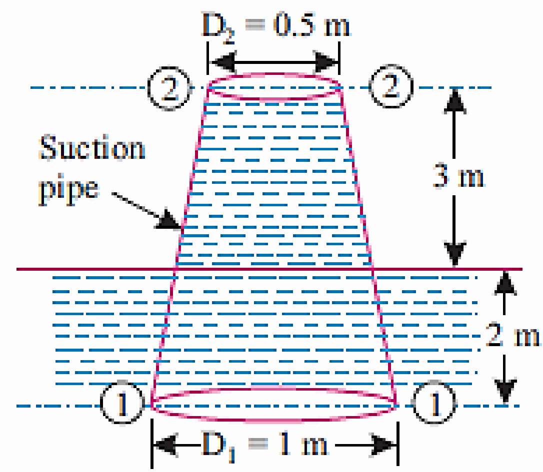
Fluid Properties

- A drainage pump has tapered suction pipe. The pipe is running full of water. The pipe diameters at the inlet and at the upper end are 1 m and 0.5 m respectively. The free water surface is 2 m above the centre of the inlet and centre of upper end is 3 m above the top of free water surface. The pressure at the tip end of the pipe is 25 cm of mercury and it is known that loss of head by friction between top and the bottom section is one-tenth of the velocity head at the top section. Compute the discharge in litre/sec. Neglect loss of head at the entrance of the tapered pipe. (UPTU)

Solution. *Given:* $D_1 = 1\text{m}; D_2 = 1.5\text{ m};$

$$p_1 = 76\text{ cm of Hg} = \frac{76}{100} \times 13.6 = 10.336 \text{ of water};$$

$$p_2 = 25\text{ cm of Hg} = \frac{25}{100} \times 13.6 = 3.4\text{ m of water}; \; h_f = \frac{1}{10}\frac{V_2^2}{2g}.$$



Discharge, Q:

Refer to Fig. 6.18. Applying continuity equation for the flow through pipe, we get:

$$A_1V_1 = A_2V_2$$

$$\frac{\pi}{4}D_1^2V_1 = \frac{\pi}{4}D_2^2V_2$$

or, $$D_1^2V_1 = D_2^2V_2$$

or, $$1^2 \times V_1 = (0.5)^2\, V_2$$

or, $$V_2 = 4V_1$$

Now, applying Bernoulli's equation at 1-1 and 2-2,

get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_f$$

$$10.336 + \frac{V_1^2}{g} + 0 = 3.4 + \frac{16V_1^2}{2g} + 5 + \frac{1}{10} \times \frac{16V_1^2}{2g}$$

$$\frac{16V_1^2}{2g} + \frac{1.6V_1^2}{2g} - \frac{V_1^2}{2g} = 10.336 - 3.4 - 5 = 1.936$$

or, $$16.6\; V_1^2 = 2 \times 9.81 \times 1.936 = 37.98$$

$\therefore$ $$V_1 = 1.513 \text{ m/s}$$

Discharge $Q = A_1V_1 = \frac{\pi}{4} \times 1^2 \times 1.513 = 1.188 \text{ m}^3\text{/s} = \mathbf{1188 \text{ litres/sec.}}$ **(Ans.)**