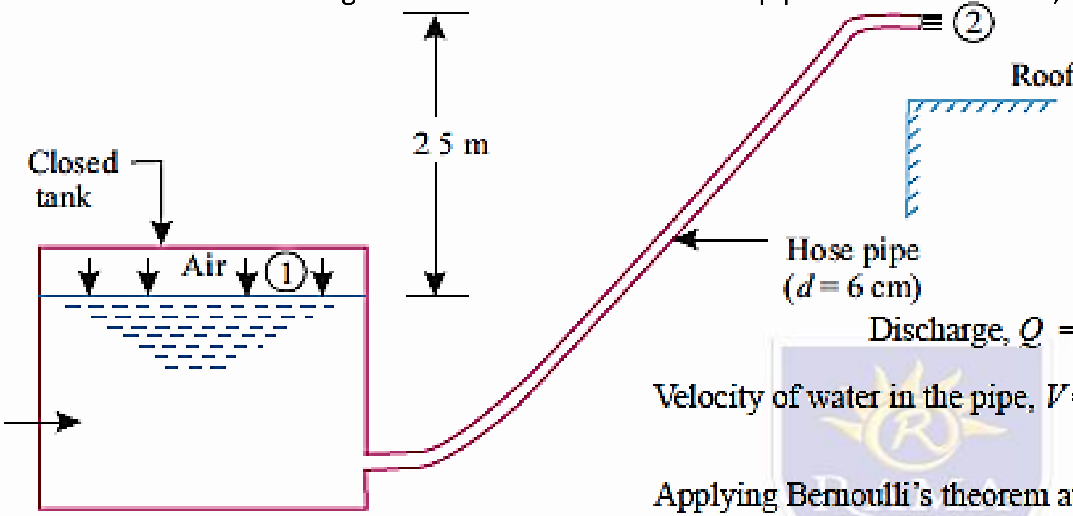
- The closed tank of a fire engine is partly filled with water, the air space abovebeing under pressure. A 6 cm bore connected to the tank discharges on the roof of a building 2.5 m above the level of water in the tank. The friction losses are 45 cm of water. Determine the air pressure which must be maintained in the tank to deliver 20 litres/sec. on the roof
- Solution. Refer to Fig. 6.19 Given: Diameter of hose pipe d = 6 cm = 0.06 m; Friction, hf = 45 cm or 0.45 m of water



$$\text{Discharge, } Q = 20 \textit{ litres/sec.} \text{ or } 0.02 \text{ m}^3\text{/s.}$$

$$\text{Velocity of water in the pipe, } V = \frac{Q}{A} = \frac{0.02}{\frac{\pi}{4} \times (0.06)^2} = 7.07 \text{ m/s.}$$

Applying Bernoulli's theorem at points 1 and 2 respectively, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_f$$

Here, $V_1 = 0;\ z_1 = 0;\ p_2 = 0;\ V_2 = 7.07 \text{ m/s};\ z_2 = 2.5 \text{ m};\ h_f = 0.45 \text{ m}$

Inserting the various values in the above equation, we get:

$$\frac{p_1}{w} + 0 + 0 = 0 + \frac{(7.07)^2}{2g} + 2.5 + 0.45$$

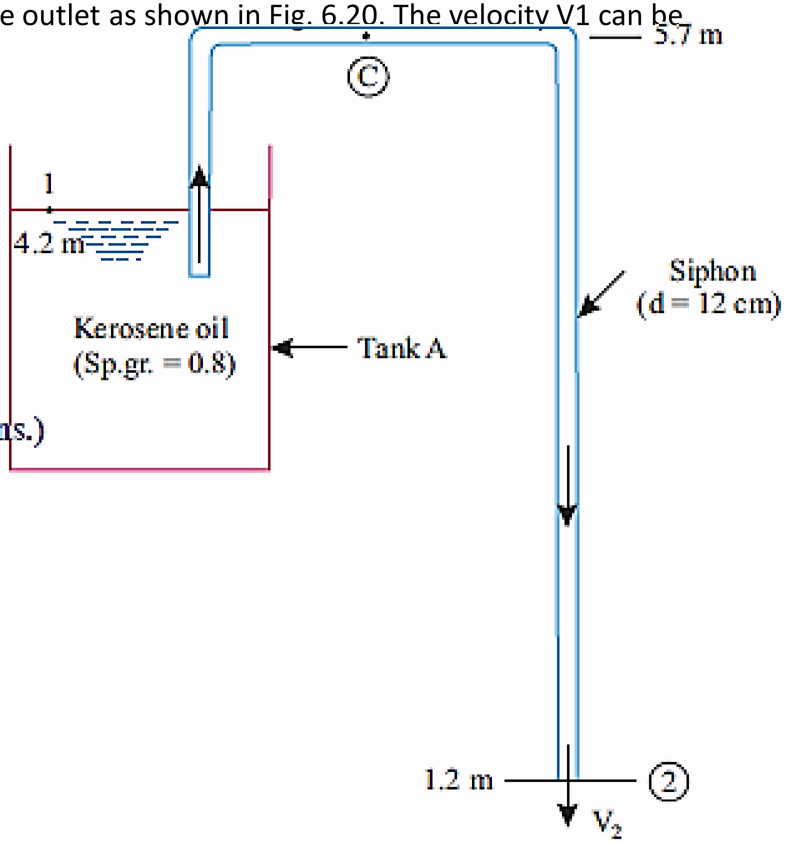
or,

$$\frac{p_1}{9.81} = \frac{(7.07)^2}{2 \times 9.81} + 2.5 + 0.45$$

$$= 5.497 \text{ m of water (where } p_1 \text{ is in kN/m}^2\text{)}$$

$$\therefore \quad p_1 = 9.81 \times 5.497 = \mathbf{53.93\ kN/m^2\ (gauge)\ (Ans.)}$$

- A siphon consisting of a pipe of 12cm diameter is used to empty kerosene oil (Sp. gr. = 0.8) from the tank A. The siphon discharges to the atmosphere at an elevation of 1.2 m. The oil surface in the tank is at an elevation of 4.2 m. The centre line of the siphon pipe at its highest
- point C is at an elevation of 5.7 m. Determine: (i) The discharge in the pipe (ii) The pressure at point C. The losses in the pipe may be assumed to be 0.45 m up to summit and 1.25m from the summit to the outlet.
- Solution. Consider points 1 and 2 at the surface of the oil in the tank A and at the outlet as shown in Fig. 6.20. The velocity V1 can be assumed to be zero. Applying Bernoulli's equation at points 1 and 2, we get:



$$\frac{p_1}{w}+\frac{V_1^2}{2g}+z_1 = \frac{p_2}{w}+\frac{V_2^2}{2g}+z_1+h_{f(1-2)} \text{ (losses)}$$

$$0+0+4.2=0+\frac{V_2^2}{2g}+1.2+(0.45+1.25)$$

$$V_2 = 5.05 \text{ m/s}$$

(i) The discharge in the pipe, Q:

$$Q=A_2V_2=\frac{\pi}{4}\times(0.12)^2\times 5.05 = \mathbf{0.057\ m^3/s\ (Ans.)}$$

(ii) The pressure at point C:

Applying Bernoulli's equation at points 1 and C, we get:

$$\frac{p_1}{w}+\frac{V_1^2}{2g}+z_1 = \frac{p_C}{w}+\frac{V_C^2}{2g}+z_C+h_{f(1-C)}$$

$$0+0+4.2 = \frac{p_C}{w}+\frac{(5.05)^2}{2\times 9.81}+5.7+0.45$$

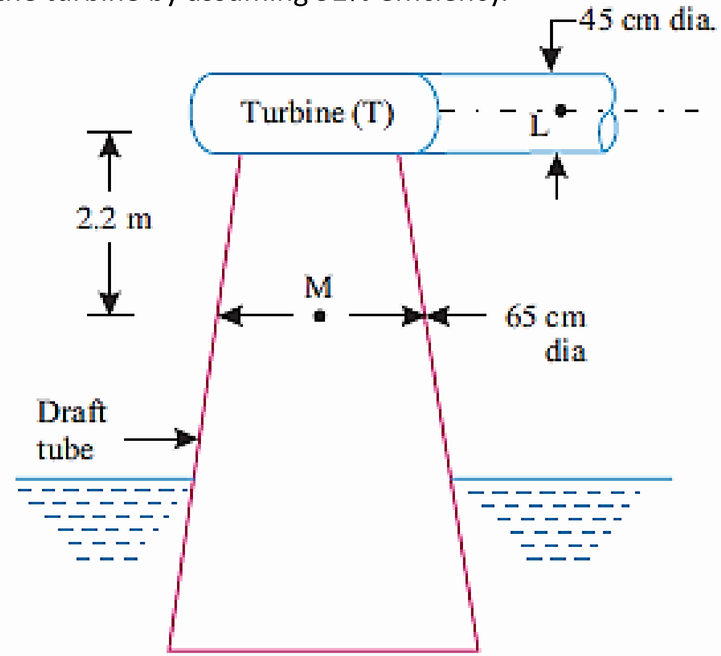
or, $$\frac{p_C}{w} = -3.25 \text{ m}$$

or, $$p_C = (0.8\times 9.81)\times(-3.25)$$
$$= -25.5 \text{ kN/m}^2 \text{ or } \mathbf{-25.5\ kPa\ (gauge)\ (Ans.)}$$

- A turbine has a supply line of diameter 45 cm and a tapering draft tube asshown in Fig. 6.22. When the flow in the pipe is 0.6 m3/s the pressure head at point L upstream of the turbine is 35 m and at a point M in the draft tube, where the diameter is 65 cm, the pressure head is – 4.1 m. Point M is 2.2 m below the point L. Determine the power output of the turbine by assuming 92% efficiency.

Solution. $V_L = \dfrac{Q}{A_L} = \dfrac{0.6}{\frac{\pi}{4} \times (0.45)^2} = 3.77 \text{ m/s}$

$$V_M = \frac{Q}{A_M} = \frac{0.6}{\frac{\pi}{4} \times (0.65)^2} = 1.81 \text{ m/s}$$



Applying Bernoulli's equation to points L and M:

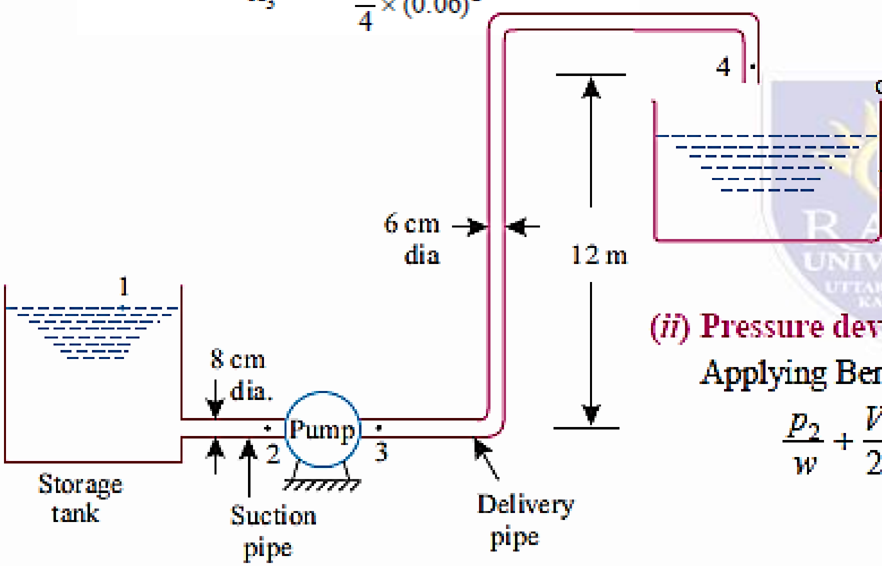
$$\frac{P_L}{w} + \frac{V_L^2}{2g} + z_L = \frac{P_M}{w} + \frac{V_M^2}{2g} + z_M + H_T$$

$$35 + \frac{(3.77)^2}{2 \times 9.81} + 2.2 = -4.1 + \frac{(1.81)^2}{2 \times 9.81} + 0 + H_T$$

- ∴ HT = 41.86 m
- Power output of the turbine, P = wQHT × η
- = 9.81 × 0.6 × 41.86 × 0.92 = 226.68 kW (Ans.)

- Fig. shows a pump drawing a solution (specific gravity =1.8) from a storage tank through an 8 cm steel pipe in which the flow velocity is 0.9 m/s. The pump discharges through a 6 cm steel pipe to an overhead tank, the end of discharge is 12 m above the level of the solution in the feed tank. If the friction losses in the entire piping system are 5.5 m and pump efficiency is 65 per cent, determine: (i) Power rating of the pump. (ii) Pressure developed by the pump.
- Solution. Given: d2 = 8 cm or 0.08 m; d3 = 6 cm or 0.06 m; V2 = 0.9 m/s, ηpump = 65%
- (i) Power rating of the pump:
- From continuity equation, we have:
- A2V2 = A3V3

$$V_3(=V_4) = \frac{A_2V_2}{A_3} = \frac{\frac{\pi}{4}\times(0.08)^2\times 0.9}{\frac{\pi}{4}\times(0.06)^2} = 1.6 \text{ m/s}$$



Applying Bernoulli's equation between points 1 and 4, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 + H_P = \frac{p_4}{w} + \frac{V_4^2}{2g} + z_4 + \text{Losses}$$

(where, H_P = Energy added by the pump per unit weight of liquid in Nm/N or m of the liquid pumped)

$$0+0+0+H_P = 0 + \frac{(1.6)^2}{2\times 9.81} + 12 + 5.5$$

or, $$H_P = 17.63 \text{ m of liquid}$$

$$\therefore \text{ Power rating of the pump} = \frac{wQH_P}{\eta_{\text{pump}}} = \frac{(9.81\times 1.8)\times\left(\frac{\pi}{4}\times 0.08^2\times 0.9\right)\times 17.63}{0.65}$$

$$= 2.167 \text{ kW (Ans.)}$$

(*ii*) Pressure developed by the pump, $(p_3 - p_2)$:

Applying Bernoulli's equation between points 2 and 3, we have:

$$\frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + H_P = \frac{p_3}{w} + \frac{V_3^2}{2g} + z_3$$

$$\left(\frac{p_3 - p_2}{w}\right) = \left(\frac{V_2^2 - V_3^2}{2g}\right) + H_P \quad \text{...(}$$

$$= \frac{(0.9)^2 - (1.6)^2}{2\times 9.81} + 17.63 = 17.54 \text{ m}$$

or, $$p_3 - p_2 = 17.54\times(9.81\times 1.8) = 309.72 \text{ kN/m}^2 \text{ or kPa (Ans.)}$$

PRACTICAL APPLICATIONS OF BERNOULLI'S EQUATION

- Bernoulli's equation is applicable in all problems of incompressible flow where there is involvement of energy considerations but here we shall discuss its applications in the following measuring devices:
- 1. Venturimeter
- 2. Orificemeter
- 3. Rotameter and elbow meter
- 4. Pitot tube.

- Venturimeter
- A venturimeter is one of the most important practical applications of Bernoulli's theorem. It is an instrument used to measure the rate of discharge in a pipeline and is often fixed permanently at different sections of the pipeline to know the discharges there.
- A venturimeter has been named after the 18th century Italian engineer Venturi.
- Types of venturimeters:
- Venturimeters may be classified as follows:
- 1. Horizontal venturimeters.
- 2. Vertical venturimeters.
- 3. Inclined venturimeters.
- Horizontal venturimeters
- A venturimeter consists of the following three parts:
- (i) A short converging part,
- (ii) Throat, and
- (iii) Diverging part.

Expression for rate of flow: venturimeters

Fig 6.29 shows a venturimeter fitted in horizontal pipe through which a fluid is flowing.

Let, D_1 = Diameter at inlet or at section 1,

A_1 = Area at inlet $\left(=\frac{\pi}{4}d_1^2\right)$

p_1 = Pressure at section 1,

V_1 = Velocity of fluid at section 1,

and D_2, A_2, p_2, and V_2 are the corresponding values at section 2.

Applying Bernoulli's equation at sections 1 and 2, we get:

$$\frac{p_1}{w}+\frac{V_1^2}{2g}+z_1 = \frac{p_2}{w}+\frac{V_2^2}{2g}+z_2 \quad \text{...}(i)$$

Here, $z_1 = z_2$ ... since the pipe is horizontal.

$$\therefore \quad \frac{p_1}{w}+\frac{V_1^2}{2g} = \frac{p_2}{w}+\frac{V_2^2}{2g}$$

or,

$$\frac{p_1-p_2}{w} = \frac{V_2^2}{2g}-\frac{V_1^2}{2g} \quad \text{...}(ii)$$

But, $\frac{p_1-p_2}{w}$ = Difference of pressure heads at sections 1 and 2 and is equal to h.

i.e.,

$$\frac{p_1-p_2}{w} = h$$

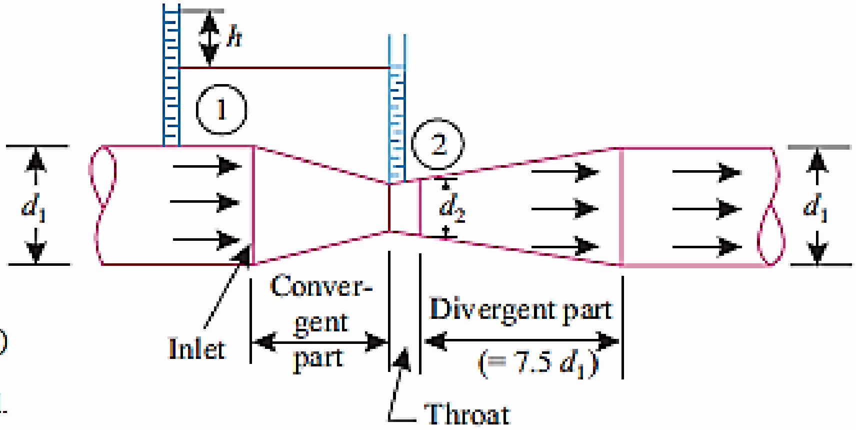
Substituting this value of $\frac{p_1-p_2}{w}$ in eqn. (*ii*), we get:

$$h = \frac{V_2^2}{2g}-\frac{V_1^2}{2g}$$

Applying continuity equation at sections 1 and 2, we have:

$$A_1V_1 = A_2V_2 \quad \text{or} \quad V_1=\frac{A_2V_2}{A_1}$$

Substituting the value of V_1 in eqn. (*iii*), we get:



$\left(\text{Throat ratio } \frac{d_2}{d_1} \text{ varies } \frac{1}{4} \text{ to } \frac{3}{4}\right)$

$$h = \frac{V_2^2}{2g}-\frac{\left(\frac{A_2V_2}{A_1}\right)^2}{2g} = \frac{V_2^2}{2g}\left(1-\frac{A_2^2}{A_1^2}\right)$$

$$h = \frac{V_2^2}{2g}\left(\frac{A_1^2-A_2^2}{A_1^2}\right) \quad \text{or} \quad V_2^2 = 2gh\left(\frac{A_1^2}{A_1^2-A_2^2}\right)$$

$$V_2 = \sqrt{2gh\left(\frac{A_1^2}{A_1^2-A_2^2}\right)} = \frac{A_1}{\sqrt{A_1^2-A_2^2}}\sqrt{2gh}$$

$$\text{Discharge, } Q = A_2V_2 = A_2\frac{A_1}{\sqrt{A_1^2-A_2^2}}\times\sqrt{2gh}$$

$$Q = \frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\times\sqrt{2gh}$$

$$Q = C\sqrt{h}$$

C = constant of venturimeter