Velocity at section 1-1, $V_1 = \frac{Q}{A_1} = \frac{0.25}{0.07068} = 3.54$ m/s

Velocity at section 2-2, $V_2 = V_1 = 3.54$ m/s $\quad (\because A_1 = A_2)$

Force along X-axis:

$$F_x = \frac{wQ}{g}[V_1 - (-V_2 \cos 45°)] + p_1 A_1 + p_2 A_2 \cos 45°$$

$$= \frac{9.81 \times 0.25}{9.81}[3.54 - (-3.54 \times 0.707)]$$

$$+ (400 \times 0.07068) + (400 \times 0.07068 \times 0.707)$$

$$= 0.25 \times (3.54 + 3.54 \times 0.707) + 28.27 + 19.98$$

$$= 49.76 \text{ kN } (\rightarrow)$$

Force along Y-axis:

$$F_y = \frac{wQ}{g}[0 - V_2 \sin 45°] - p_2 A_2 \sin 45°$$

$$= \frac{9.81 \times 0.25}{9.81}(0 - 3.54 \times 0.707) - 400 \times 0.07068 \times 0.707$$

$$= -0.625 - 19.98 = -20.6 \text{ kN}(\downarrow)$$

The *magnitude of the resultant force,*

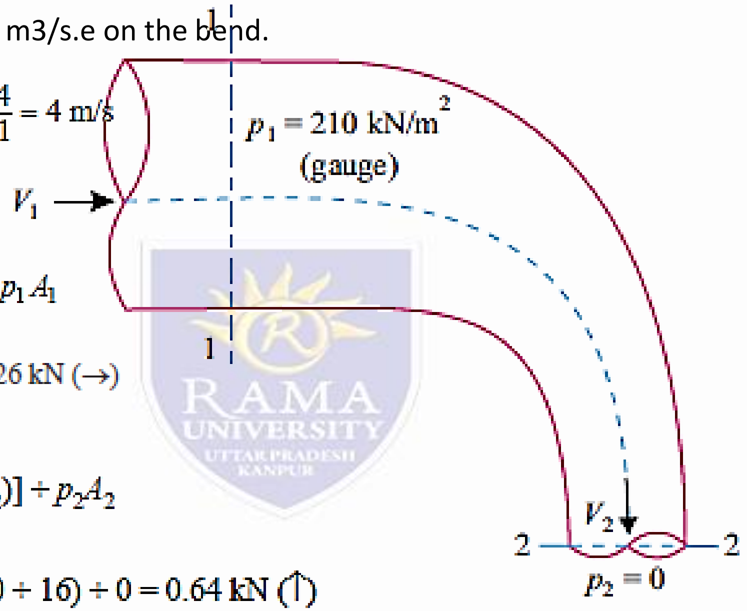
$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{49.76^2 + 20.6^2} = \mathbf{53.85 \text{ kN (Ans.)}}$$

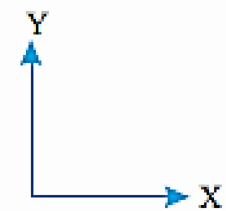
The *direction of* F_R with X- axis is given as:

$$\tan\theta = \frac{F_y}{F_x} = \frac{20.6}{49.76} = 0.414$$

$$\therefore \quad \theta = \tan^{-1} 0.414 = \mathbf{22.5° \text{ (Ans.)}}$$

- Fig. 6.52 shows a 90° reducer-bend through which water flows. The pressure at the inlet is 210 kN/m2 (gauge) where the cross-sectional area is 0.01 m2. At the exit section, the area is 0.0025 m2 and the velocity is 16 m/s. The pressure at the exit is atmospheric. Determine the magnitude and direction of the resultant forc
- Solution. Area at section 1-1, A1 = 0.01 m2
- Area at section 2-2, A2 = 0.0025 m2
- Velocity at the exit, V2 = 16 m/s.
- Discharge, Q = A2 V2 = 0.0025 × 16 = 0.04 m3/s.e on the bend.





$$\therefore \quad V_1 = \frac{Q}{A_1} = \frac{0.04}{0.01} = 4 \text{ m/s}$$

Assume the bend is horizontal and in *XY* plane.

Force along X-axis:

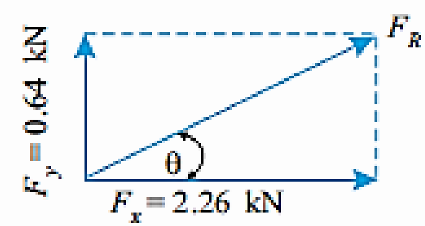
$$F_x = \frac{wQ}{g}(V_1 - 0) + p_1 A_1$$

$$= \frac{9.81 \times 0.04}{9.81}(4 - 0) + 210 \times 0.01 = 0.16 + 2.1 = 2.26 \text{ kN} (\rightarrow)$$

Force along Y-axis:

$$F_y = \frac{wQ}{g}[0 - (-V_2)] + p_2 A_2$$

$$= \frac{9.81 \times 0.04}{9.81}(0 + 16) + 0 = 0.64 \text{ kN} (\uparrow)$$



Magnitude of the resultant force acting on the bend,

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{2.26^2 + 0.64^2} = \mathbf{2.35 \text{ kN (Ans.)}}$$

Direction of the resultant force with the X-axis,

$$\tan\theta = \frac{F_y}{F_x} = \frac{0.64}{2.26} = 0.2832 \quad \therefore \theta = \tan^{-1} 0.2832 = 15.8°$$

$$\therefore \quad \theta = \mathbf{15.8° \text{ (Ans.)}}$$

Fluid Properties

- A 0.4 m × 0.3 m, 90° vertical bend carries 0.5 m3/s oil of specific gravity 0.85 with a pressure of 118 kN/m2 at inlet to the bend. The volume of the bend is 0.1 m3. Find the magnitude and direction of the force on the bend. Neglect friction and assume both inlet and outlet sections to be at same horizontal level. Also assume that water enters the bend at 45° to the horizontal.

Solution. *Given*: $D_1 = 0.4$ m, $\therefore A_1 = \frac{\pi}{4} \times 0.4^2 = 0.12566$ m^2; $D_2 = 0.3$ m;

$\therefore A_2 = \frac{\pi}{4} \times 0.3^2 = 0.07068$ m^2; $Q = 0.5$ m^3/s; $S_{oil} = 0.85$; $p_1 = 118$ kN/m^2; Volume of bend = 0.1 m^3

$$V_1 = \frac{Q}{A_1} = \frac{0.5}{0.12566} = 3.98 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.5}{0.07068} = 7.074 \text{ m/s}$$

Applying Bernoulli's equation between sections (1) and (2), we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + \text{losses}$$

Since, $z_1 = z_2$ and losses are negligible (Given),

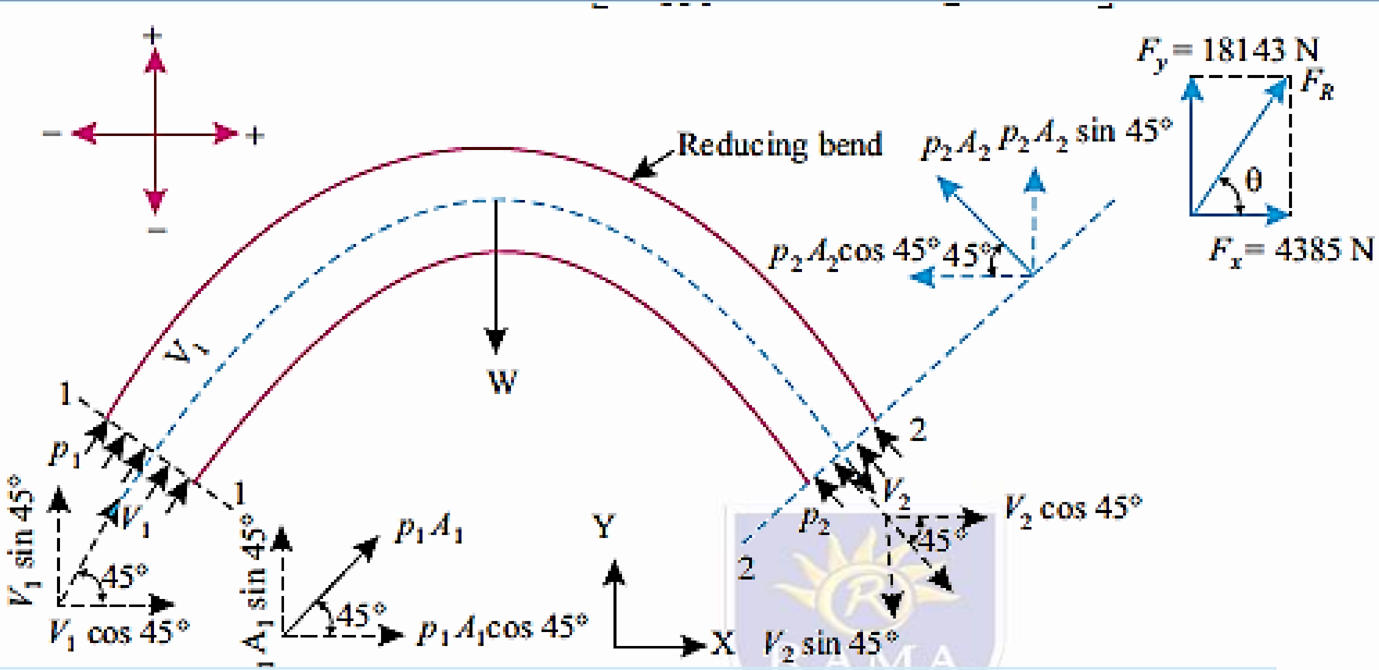
$$\therefore \quad \frac{p_1}{w} + \frac{V_1^2}{2g} = \frac{p_2}{w} + \frac{V_2^2}{2g}$$

Substituting the values, we get:

$$\frac{118 \times 10^3}{(1000 \times 0.85 \times 9.81)} + \frac{(3.98)^2}{2 \times 9.81} = \frac{p_2}{(1000 \times 0.85 \times 9.81)} + \frac{(7.074)^2}{2 \times 9.81}$$

or,
$$\frac{118 \times 10^3}{850} + \frac{(3.98)^2}{2} = \frac{p_2}{850} + \frac{(7.074)^2}{2}$$

or,
$$p_2 = 850\left[\frac{118 \times 10^3}{850} + \frac{(3.98)^2 - (7.074)^2}{2}\right] = 103464 \text{ N/m}^2$$



Magnitude and direction of the force (resultant), F_R:

Force along X-axis:

$$F_x = \frac{wQ}{g}[V_{1x} - V_{2x}] + (p_1A_1)_x + (p_2A_2)_x$$

where,

$$V_{1x} = V_1 \cos 45° = 3.98 \cos 45° = 2.814 \text{ m/s},$$

$$V_{2x} \cos 45° = 7.074 \times \cos 45° = 5.0 \text{ m/s}$$

$$(p_1A_1)_x = p_1A_1 \cos 45° = 118 \times 10^3 \times 0.12566 \cos 45° = 10484.89 \text{ N},$$

$$(p_2A_2)_x = -p_2A_2 \cos 45° = -103464 \times 0.07068 \cos 45° = -5170.96 \text{ N}$$

$$\therefore F_x = \frac{(1000 \times 0.85 \times 9.81)0.5}{9.81}[2.814 - 5.0] + 10484.89 + (-5170.96) \simeq 4385 \text{ N } (\rightarrow)$$

Force along Y-axis:

$$F_y = \frac{wQ}{g}[V_{1y} - V_{2y}] + (p_1 A_1)_y + (p_2 A_2)_y - W$$

$$V_{1y} = V_1 \sin 45° = 3.98 \sin 45° = 2.814 \text{ m/s}, \; V_{2y} = -V_2 \sin 45°$$

$$= -7.074 \times \sin 45° = -5.0 \text{ m/s}$$

$$(p_1 A_1)_y = p_1 A_1 \sin 45° = 118 \times 10^3 \times 0.12566 \sin 45° = 10484.89 \text{ N}$$

$$(p_2 A_2)_y = p_2 A_2 \sin 45° = 103464 \times 0.07068 \times \sin 45° = 5170.96 \text{ N}$$

$$W = 0.1(0.85 \times 1000) \times 9.81 = 833.85 \text{ N}$$

$$\therefore F_y = \frac{(1000 \times 0.85 \times 9.81)0.5}{9.81}[2.814 - (-5.0)] + 10484.89 + 5170.96 - 833.85 = 18143 \text{ N} (\uparrow)$$

$\therefore$ Resultant force on the bend,

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(4385)^2 + (18143)^2} = 18665 \text{ (Ans.)}$$

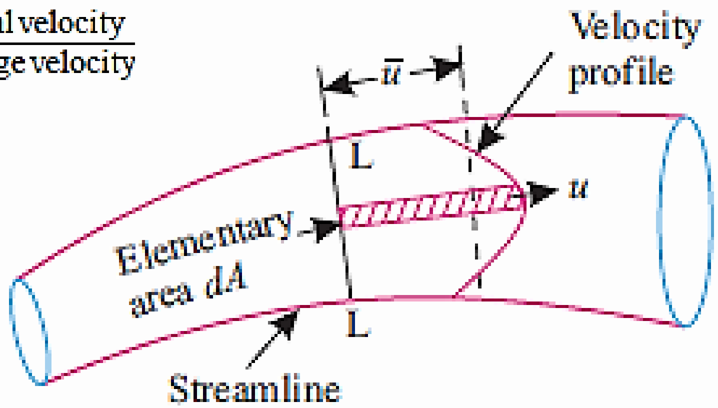
Inclination of F_x to the X-direction is,

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{18143}{4385}\right) = 76.4° \text{ (Ans.)}$$

KINETIC ENERGY AND MOMENTUM CORRECTION FACTORS (CORIOLIS CO-EFFICIENTS)

- While deriving Bernoulli's equation, it is assumed that the velocity distribution across a single stream tube is uniform. But if there is an appreciable variation in the velocity distribution (on account of viscous and boundary resistance) correction factors α and β have to be applied to obtain the exact amount of kinetic energy or momentum available at a given cross-section.
- Kinetic energy correction factor (α):
- 'Kinetic energy correction factor' is defined as the ratio of the kinetic energy of flow per second based on actual velocity across a section to the kinetic energy of flow per second based on average velocity across the same section. It is denoted by α.

Mathematically, $$\alpha = \frac{\text{Kinetic energy per second based on actual velocity}}{\text{Kinetic energy per second based on average velocity}}$$



- Let, u = Average velocity at the section LL,
- u = Local or point or actual velocity,
- dA = Elementary area, and
- A = Area of cross-section.
- For the velocity variation across the section LL of the
- stream tube the total K.E. for the entire section is given as:

$$K.E. = \frac{1}{2} m\bar{u}^2 = \frac{1}{2}(\rho A\bar{u})\bar{u}^2 = \frac{1}{2}\rho A\bar{u}^3$$

True K.E. for the entire cross-section

$$= \int \frac{1}{2} dm.u^2 = \int \frac{1}{2}(\rho.dA.u)u^2 = \frac{\rho}{2}\int_A u^3\, dA$$

$$\alpha = \frac{\frac{\rho}{2}\int_A u^3\, dA}{\frac{\rho}{2} A\bar{u}^3} = \frac{1}{A}\int \left(\frac{u}{\bar{u}}\right)^3 dA$$

α = 1 for uniform velocity distribution and tends to become greater than 1 as the distribution of velocity becomes less and less uniform.

- α = 1.02 to 1.15 for turbulent flows.
- α = 2 for laminar flow.
- It may be noted that in most of the fluid mechanics computations, α is taken as 1 without introducing much error, since the velocity is a small percentage of the total head.