- A U-tube is made up of two capillaries of bores 1.2 m and 2.4 mm respectively. The tube is held vertical and partially filled with liquid of surface tension 0.06 N/m and zero contact angle. If the estimated difference in the level of two menisci is 15 mm, determine the mass density of the liquid.

Solution. *Given:* Bores of the capillaries:

$$d_1 = 1.2 \text{ mm} = 0.0012 \text{ m}$$

$$d_2 = 2.4 \text{ mm} = 0.0024 \text{ m}$$

Difference of level, $h_1 - h_2 = 15 \text{ mm} = 0.015 \text{ m}$; Angle of contact, $\theta = 0$

Mass density of the liquid, ρ:

$$h_1 = \frac{4\sigma\cos\theta}{wd_1}, \quad \text{and} \quad h_2 = \frac{4\sigma\cos\theta}{wd_2}$$

[where w (= ρg) = weight density of the liquid)]

$$\therefore \quad h_1 - h_2 = \frac{4\sigma}{w}\left[\frac{1}{d_1} - \frac{1}{d_2}\right] \qquad (\because \theta = 0)$$

$$0.015 = \frac{4 \times 0.06}{\rho \times 9.81}\left[\frac{1}{0.0012} - \frac{1}{0.0024}\right] = \frac{0.02446}{\rho} \times 416.67$$

$$\therefore \quad \rho = \frac{0.02446 \times 416.67}{0.015} = 679.45 \text{ kg/m}^3 \text{ (Ans.)}$$

- single column U-tube manometer, made of glass tubing having a nominal inside diameter of 2.4 mm, has been used to measure pressure in a pipe or vessel containing air. If the limb opened to atmosphere is 10 percent oversize, find the error in mm of mercury in the measurement of air pressure due to surface tension effects. It is stated that mercury is the manometric fluid for which surface tension σ = 0.52 N/m and angle of contact α = 140°
- Solution. Given: d1 = 2.4 mm; d2 = 2.4 × 1.1 = 2.64 mm; σ = 0.52 N/m; α = 140°. Error in measurement due to surface tension effects:
- The surface tension manifests the phenomenon of capillary action due to which rise or depression of manometric liquid in a tube is given by

$$h = \frac{4\sigma\cos\theta}{wd_1}$$

Now,

$$h_1 = \frac{4 \times 0.52 \times \cos 140^\circ}{(13.6 \times 9810) \times (2.4 \times 10^{-3})} = 4.97 \times 10^{-3}\ \text{m}$$

(Negative sign indicates capillary depression)

$$h_2 = \frac{4 \times 0.52 \times \cos 140^\circ}{(13.6 \times 9810) \times (2.64 \times 10^{-3})} = -4.52 \times 10^{-3}\ \text{m}$$

Hence, error in measurement due to surface tension effects

$$= (4.97 - 4.52) \times 10^{-3} = 0.45 \times 10^{-3}\ \text{m} = \mathbf{0.45\ mm\ (Ans.)}$$

Calculate the capillary effect in millimeters in a glass tube of 4 mm diameter, when immersed in (i) water and (ii) mercury. The temperature of the liquid is 20°C and the values of surface tension of water and mercury at 20°C in contact with air are 0.0735 N/m and 0.51 N/m respectively. The contact angle for water θ = 0° and for mercury θ = 130°. Take specific weight of water at 20°C as equal to 9790 N/m3.

Solution. Given: Diameter of glass tube, d = 4 mm = 0.004 m Surface tension at 20°C, σ: σwater = 0.0735 N/m, σmercury = 0.051 N/m Specific weight of water at 20°C = 9790 N/m3
The rise or depression h of a liquid in a capillary tube is given by

$$h = \frac{4\sigma\cos\theta}{wd}$$

where, σ = surface tension, θ = angle of contact, and w = specific weight.

(i) Capillary effect for water:

$$h = \frac{4 \times 0.0735 \times \cos 0^\circ}{9790 \times 0.004} \qquad (\because \theta_{water} = 0^\circ \text{ ...given})$$

$$= 7.51 \times 10^{-3} \text{ m} = \mathbf{7.51\ mm\ (rise)\ (Ans.)}$$

(ii) Capillary effect for mercury

$$h = \frac{4 \times 0.051 \times \cos 130^\circ}{(13.6 \times 9790) \times 0.004} \qquad (\because \theta_{mercury} = 130^\circ \text{ ... given})$$

or, $= -2.46 \times 10^{-3} \text{ m} = -2.46 \text{ mm}$

i.e., $h = \mathbf{2.46\ mm\ (depression\ )\ (Ans.)}$

When the pressure of liquid is increased from 3.5 MN/m2 to MN/m2 its volume is found to decrease by 0.08 percent. What is the bulk modulus of elasticity of the liquid?

Solution. Initial pressure $= 3.5 \text{ MN/m}^2$

Final pressure $= 6.5 \text{ MN/m}^2$

$\therefore$ Increase in pressure, $dp = 6.5 - 3.5 = 3.0 \text{ MN/m}^2$

Decrease in volume $= 0.08$ percent $\therefore -\frac{dV}{V} = \frac{0.08}{100}$

Bulk modulus (K) is given by:

$$K = \frac{dp}{-\frac{dV}{V}} = \frac{3 \times 10^6}{\frac{0.08}{100}} = 3.75 \times 10^9 \text{ N/m}^2 \text{ or } 3.75 \text{ GN/m}^2$$

Hence, $K = \mathbf{3.75\ GN/m^2\ (Ans.)}$

Fluid Properties

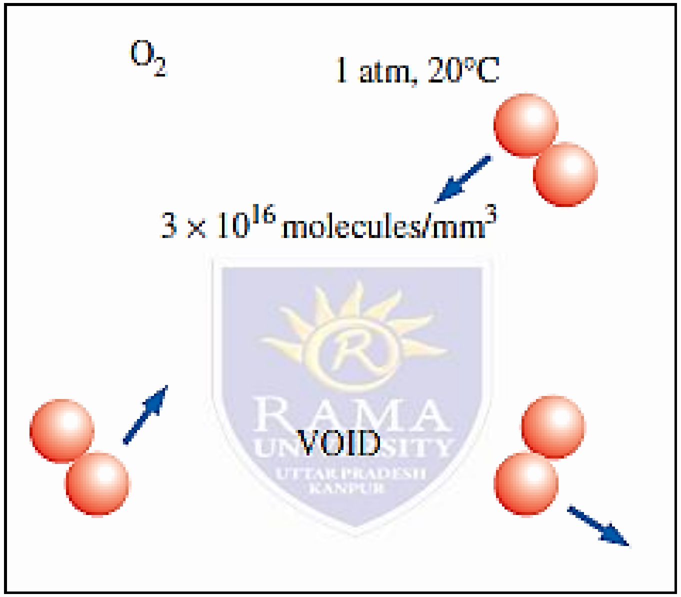
Sr. No.	Characteristics	Symbol	Definition	Dimensions	Units
1.	*Mass density*	ρ	Mass per unit volume, $\frac{m}{V}$	ML^{-3}	kg/m^3
2.	*Weight density* (or *specific weight*)	w	Weight per unit volume, $\frac{w}{V}$	FL^{-3}	N/m^3
3.	*Specific volume*	v	Volume per unit mass $\frac{V}{m} = \frac{1}{\rho}$	L^3M^{-1}	m^3/kg
4.	*Specific gravity*	S	$\frac{\text{Specific weight of liquid}}{\text{Specific weight of pure water}}$ $= \frac{w_{liquid}}{w_{water}}$		
5.	*Dynamic viscosity*	μ	Newton's law: $\tau = \mu.\frac{du}{dy}$	FTL^{-2}	N.s/m^2 poise, centipoise
6.	*Kinematic viscosity*	v	$v = \frac{\mu}{\rho}$	L^2T^{-1}	m^2/s stoke, centistoke
7.	*Bulk modulus*	K	$K = -\frac{\Delta p}{dV / V}$	FL^{-2}	N/m^2
8.	*Surface tension*	σ	Force per unit length	FL^{-1}	N/m
9.	*Vapour pressure*	p	$p_v = \frac{F}{A}$	FL^{-2}	N/m^2

Properties of Some Common Fluids at 20°C and Atmospheric Pressure

Fluid	Mass density $\rho (kN/m^3)$	Specific weight $w (kN/m^3)$	Dynamic viscosity μ		Kinematic viscosity ν		Modulus of elasticity $E (N/m^2)$	Surface tension in contact with air, σ (N/m)	Vapour pressure (N/m^2)
			Poise	kg/ms	Stoke	m^2/s			
Air	1.208	0.01185	1.85×10^{-4}	1.85×10^{-5}	1.53×10^{-1}	1.53×10^{-5}	–	–	–
Benzene	860	8.434	0.007	7.00×1^{-4}	8.14×10^{-3}	8.14×10^{-7}	1.0356×10^{9}	0.0255	1.000×10^{4}
Castor oil	960	9.414	9.800	9.80×10^{-1}	1.00×10^{1}	1.00×10^{3}	1.441×10^{9}	0.0392	–
Carbon tetrachloride	1594	15.632	0.010	1.00×10^{-3}	6.04×10^{-3}	6.04×10^{-7}	1.104×10^{9}	0.0265	1.275×10^{4}
Ethyl alcohol	789	7.737	0.012	1.20×10^{-3}	1.52×10^{-2}	1.52×10^{-6}	1.118×10^{9}	0.0216	5.786×10^{3}
Glycerine	1260	12.356	8.350	8.35×10^{-1}	6.63	6.63×10^{-4}	4.354×10^{9}	0.0637	1.373×10^{-2}
Kerosene	800	7.845	0.020	2.00×10^{-3}	2.50×10^{-2}	2.50×10^{-6}	–	0.0235	–
Mercury	13550	132.880	0.016	1.60×10^{-3}	1.18×10^{-3}	1.18×10^{-7}	2.431×10^{10}	0.510	1.726×10^{-1}

Lecture -08 –Continuum

- A fluid is composed of molecules which may be widely spaced apart, especially in the gas phase. Yet it is convenient to disregard the atomic nature of the fluid and view it as continuous, homogeneous matter with no holes, that is, a continuum.
- The continuum idealization allows us to treat properties as point functions and to assume that the properties vary continually in space
- with no jump discontinuities



- there are about 3 3 1016 molecules of oxygen in the tiny volume of 1 mm3 at 1 atm pressure and 20°C (Fig. 2–3). The continuum model
- is applicable as long as the characteristic length of the system (such as its diameter) is much larger than the mean free path of the molecules. At very low pressure, e.g., at very high elevations, the mean free path may become 100 km). For such cases the rarefied gas flow theory should be used, and the impact of individual molecules should be considered.
- Treatment of matter as a continuous (without holes) distribution of finite mass differential volume elements. Each volume element must contain huge numbers of molecules so that the macroscopic effect of the molecules can be modeled without considering individual molecules.