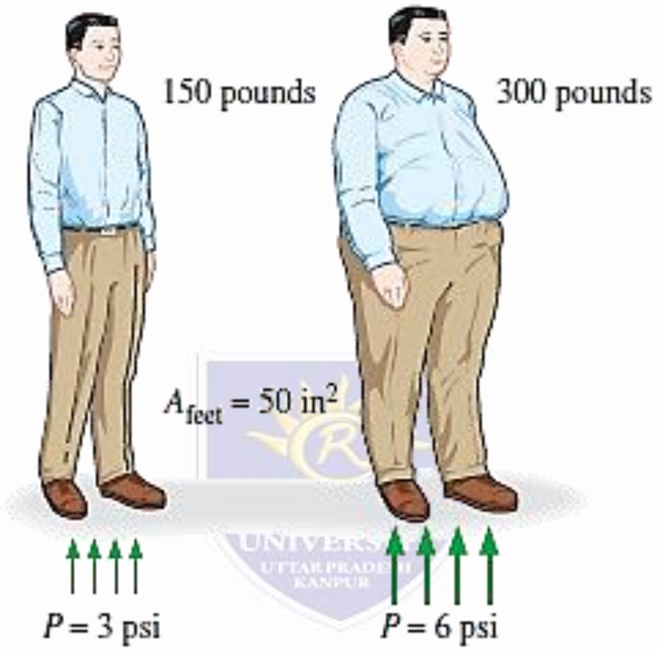
Fluid Static - Concept of Pressure



$$P = \sigma_n = \frac{W}{A_{feet}} = \frac{150\ lbf}{50\ in^2} = 3\ psi$$

The normal stress (or "pressure") on the feet of a chubby person is much greater than on the feet of a slim person.

Fluid Properties

- Pressure is defined as a normal force exerted by a fluid per unit area. We speak of pressure only when we deal with a gas or a liquid.
- The counterpart of pressure in solids is normal stress. Since pressure is defined as force per unit area, it has the unit of newtons per square meter (N/m2), which is called a pascal (Pa).
- 1Pa 5 1 N/m2
- 1 bar 5 105 Pa 5 0.1 MPa 5 100 kPa
- 1 atm 5 101,325 Pa 5 101.325 kPa 5 1.01325 bars
- 1 kgf/cm2 5 9.807 N/cm2 5 9.807 3 104 N/m2 5 9.807 3 104 Pa
- 5 0.9807 bar
- 5 0.9679 atm

- Pressure is also used on solid surfaces as synonymous to normal stress, which is the force acting perpendicular to the surface per unit area. For example, a 150-pound person with a total foot imprint area of 50 in2 exerts a pressure of 150 lbf/50 in2 5 3.0 psi on the floor
- This also explains how a person can walk on fresh snow without sinking by wearing large snowshoes, and how a person cuts with little effort when using a sharp knife.

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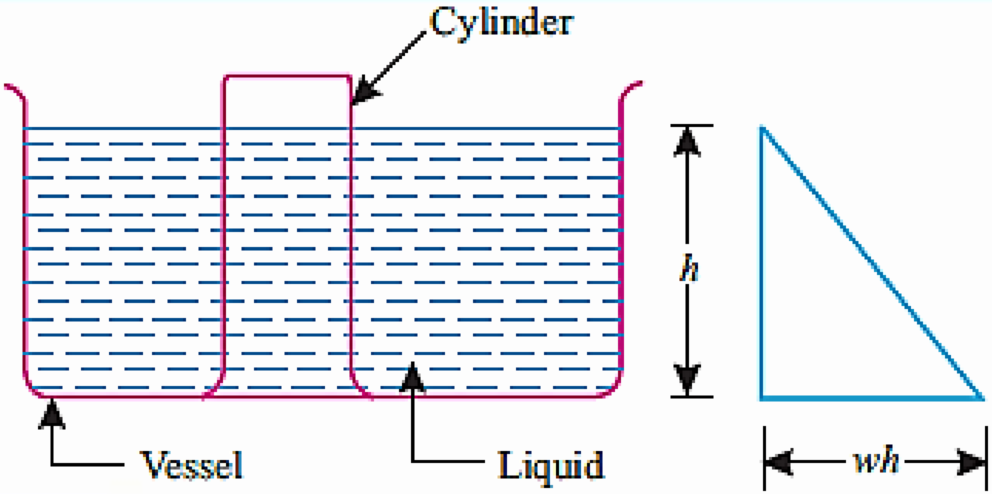
- PRESSURE OF A LIQUID
- When a fluid is contained in a vessel, it exerts force
- at all points on the sides and bottom and top of the
- container. The force per unit area is called pressure.
- If, P = The force, and
- A = Area on which the force acts; then intensity of pressure, p = P/A,
- The pressure of a fluid on a surface will always act bnormal to the surface



- PRESSURE HEAD OF A LIQUID
- Let, h = Height of liquid in the cylinder,
- A = Area of the cylinder base,
- w = Specific weight of the liquid,
- and, p = Intensity of pressure.
- Now, Total pressure on the base of the cylinder
  = Weight of liquid in the cylinder
- i.e., p. A. = wAh

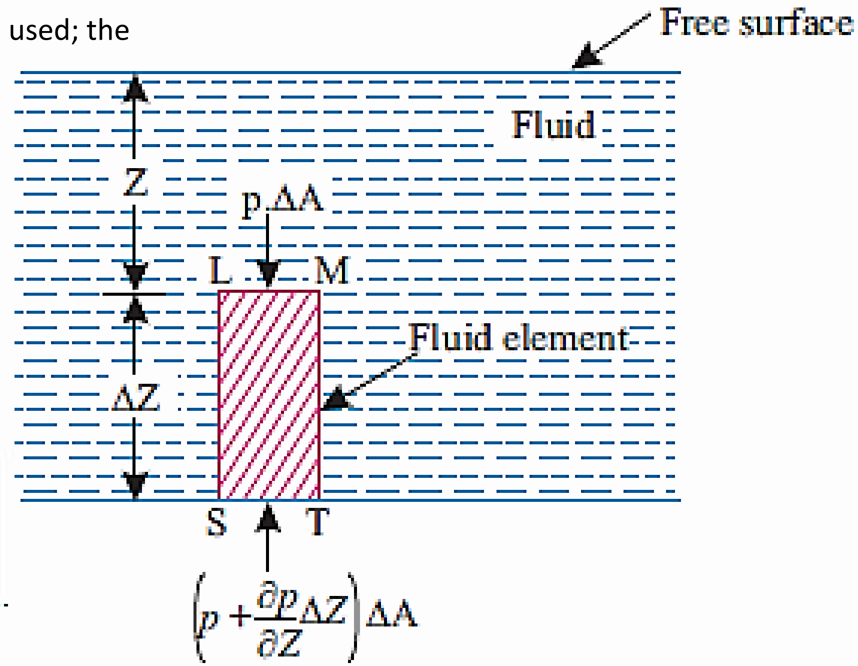
$$p = \frac{wAh}{A} = wh \qquad i.e.,\ p = wh$$

- As p = wh, the intensity of pressure in a liquid due to its depth will vary directly with depth. As the pressure at any point in a liquid depends on height of the free surface above that point, it is sometimes convenient to express a liquid pressure by the height of the free surface which would cause the pressure,

$$h = \frac{p}{w}$$

- The height of the free surface above any point is known as the static head at that point. In this
- case, static head is h.
- Hence, the intensity of pressure of a liquid may be expressed in the following two ways:
- 1. As a force per unit area (i.e., N/mm2, N/m2), and
- 2. As an equivalent static head (i.e., metres, mm or cm of liquid).

- Pressure variation in fluid at rest:
- to determine the pressure at any point in a fluid at rest “hydrostatic law” is used; the
- law states as follows:
- “The rate of increase of pressure in a
- vertically downward direction must be equal to
- the specific weight of the fluid at that point.”
- The proof of the law is as follows. Refer to Fig.
- Let, p = Intensity of pressure on face LM,
- Δ A = Cross-sectional area of the element,
- Z = Distance of the fluid element from
- free surface, and ΔZ = Height of the fluid element.
- The forces acting on the element are:
- (i) Pressure force on the face L M = p × ΔA ...(acting downward



(ii) Pressure force on the face $ST = \left(p + \frac{\partial p}{\partial Z} \times \Delta Z\right) \times \Delta A$ ...

(iii) Weight of the fluid element $=$ Weight density × volume

$= w \times (\Delta A \times \Delta Z)$

(iv) Pressure forces on surfaces MT and LS ..... are equal and opposite.

For equilibrium of the fluid element, we have:

$$p \times \Delta A - \left[p + \frac{\partial p}{\partial Z} \times \Delta Z\right] \times \Delta A + w \times (\Delta A \times \Delta Z) = 0$$

or, $$p \times \Delta A - p \times \Delta A - \frac{\partial p}{\partial Z} \times \Delta Z \times \Delta A + w \times \Delta A \times \Delta Z = 0$$

or, $$\frac{\partial p}{\partial Z} \Delta Z \times \Delta A + w \times \Delta A \times \Delta Z = 0$$

or, $$\frac{\partial p}{\partial Z} = w \text{ (cancelling } \Delta Z \times \Delta A \text{ from both the sides)}$$

or, $$\frac{\partial p}{\partial Z} = \rho \times g \qquad (\because w = \rho \times g)$$

states *that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point*. This is “**hydrostatic law**”.

$$\int dp = \int \rho g \, dZ$$

or, $$p = \rho g \, Z (= wZ)$$

where, p is the pressure above atmospheric pressure.

From eqn. (2.4), we have:

$$Z = \frac{p}{\rho g}\left(= \frac{p}{w}\right)$$

Here Z is known as *pressure head*.

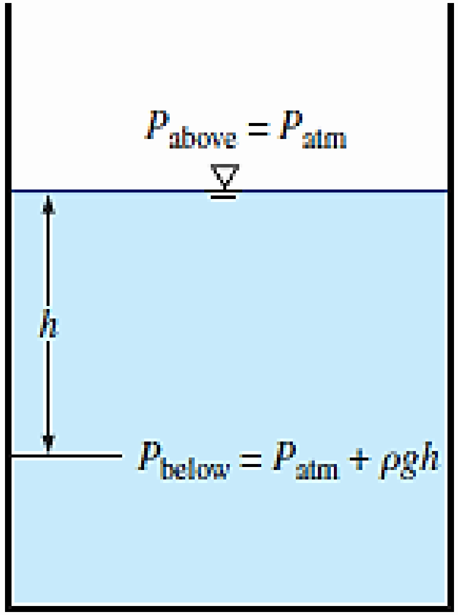
Fluid Properties

- Find the pressure at a depth of 15 m below the free surface of water in a reservoir.
- Solution. Depth of water, h = 15 m
- Specific weight of water, w = 9.81 kN/m3
- Pressure p:
- We know that, p = wh = 9.81 × 15 = 147.15 kN/m2
- i.e., p = 147.15 kN/m2 = 147.15 kPa (Ans.)

- Find the height of water column corresponding to a pressure of 54 kN/m2.

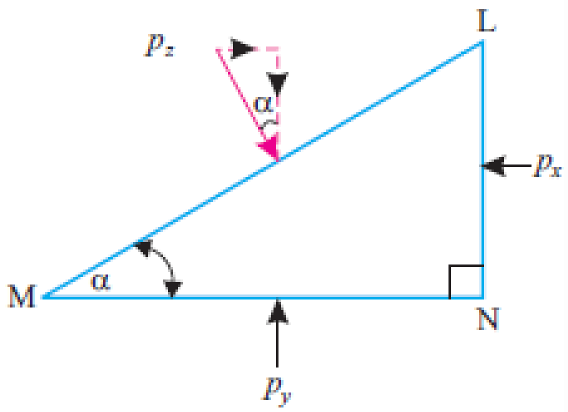
- Solution. Intensity of pressure, p = 54 kN/m2
- Specific weight of water, w = 9.81 kN/m3
- Height of water column, h:
- Using the relation: p = wh;

$$h = \frac{p}{w} = \frac{54}{9.81} = \mathbf{5.5\ m\ (Ans.)}$$



Fluid Static –Pascal Law

- PASCAL'S LAW
- The Pascal's law states as follows :
- "The intensity of pressure at any point in a liquid
- at rest, is the same in all directions".
- Proof. Let us consider a very small wedge shaped
- element LMN of a liquid, as shown in Fig. 2.3.
- Let, px = Intensity of horizontal pressure on
- the element of liquid,
- py = Intensity of vertical pressure on the
- element of liquid,
- pz = Intensity of pressure on the diagonal
- of the right angled triangular element,
- α = Angle of the element of the liquid,
- Px = Total pressure on the vertical side LN of the liquid,
- Py = Total pressure on the horizontal side MN of the liquid, and
- Pz = Total pressure on the diagonal LM of the liquid.
- Now, Px = px × LN ...(i)
- and, Py = py × MN ...(ii)
- and, Pz = pz × LM ...(iii)
- As the element of the liquid is at rest, therefore the sum of horizontal and vertical components
- of the liquid pressures must be equal to zero.
- Resolving the forces horizontally:
- Pz sin α = Px



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