FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

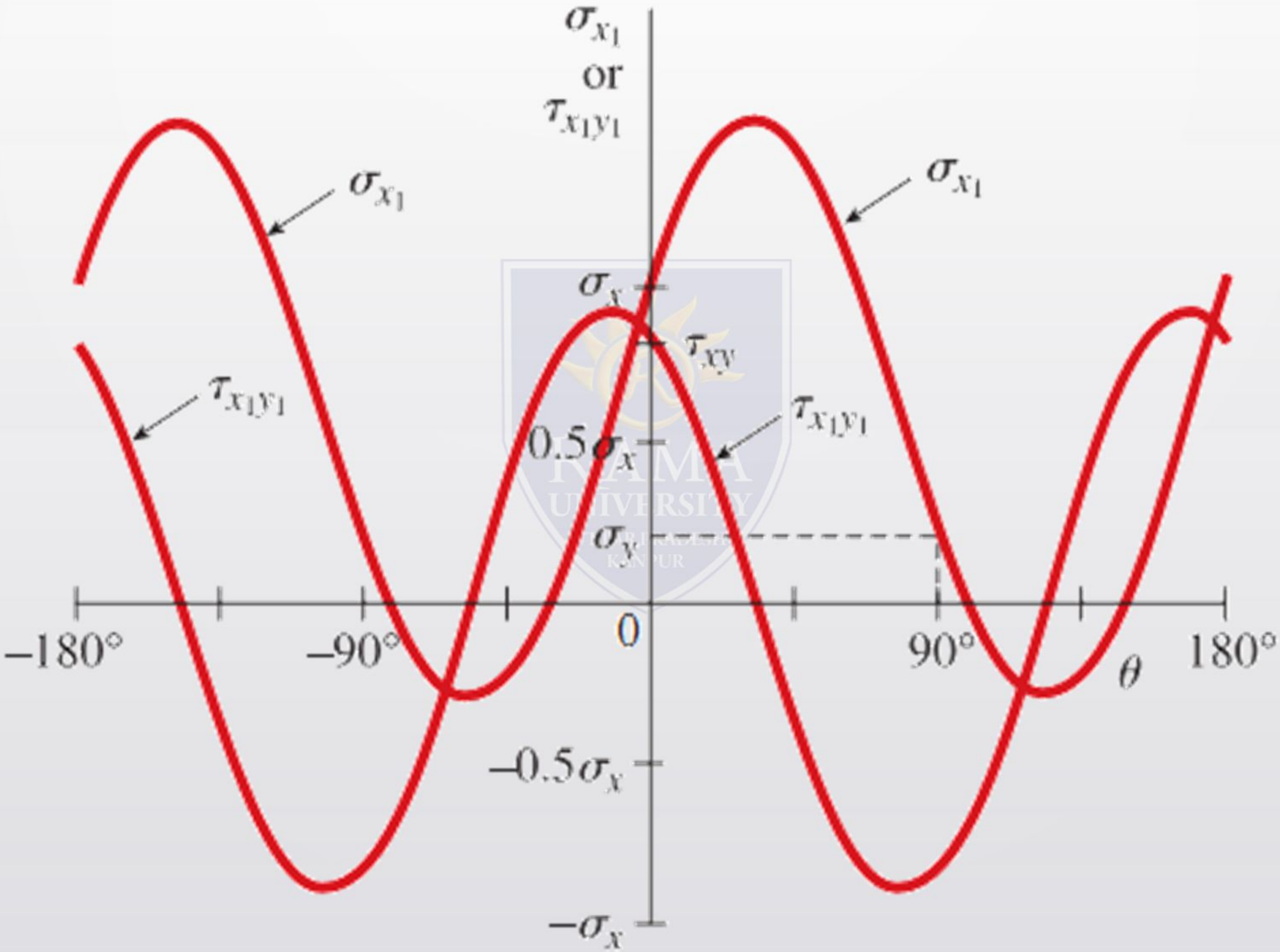
Lecture 10

Topic:10. Principal Stresses and Maximum Shear Stresses

Instructor:

Aditya Veer Gautam

Plane Stress variation



Special Cases of Plane Stress

- ✓ From the figure, we see that both the normal and shear stresses reach maximum and minimum values at 90° intervals.
- ✓ Not surprisingly, these maximum and minimum values are usually needed for design purposes.
- ✓ For instance, fatigue failures of structures such as machines and aircraft are often associated with the maximum stresses, and hence their magnitudes and orientations should be determined as part of the design process

Principal Stresses

- ✓ The maximum and minimum normal stresses, called the **principal stresses**, can be found from the transformation equation for the normal stress σ_x
- ✓ By taking the derivative of σ_x with respect to θ and setting it equal to zero, we obtain an equation from which we can find the values of θ at which σ_x is a maximum or a minimum. The equation for the derivative is

$$\frac{d\sigma_{x_1}}{d\theta} = -(\sigma_x - \sigma_y)\sin 2\theta + 2\tau_{xy}\cos 2\theta = 0$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

- ✓ θ_p is defines the orientation of the **principal planes**, that is, the planes on which the principal stresses act

Principal Stresses

- ✓ Two values of the angle $2\theta_p$ in the range from 0 to 360° can be obtained.
- ✓ These values differ by 180°, with one value between 0 and 180° and the other between 180° and 360°. Therefore, the angle up has two values that differ by 90°, one value between 0 and 90° and the other between 90° and 180°.
- ✓ The two values of up are known as the principal angles. For one of these angles, the normal stress σ_{x1} is a maximum principal stress; for the other, it is a minimum principal stress.
- ✓ Because the principal angles differ by 90°, we see that the principal stresses occur on mutually perpendicular planes.

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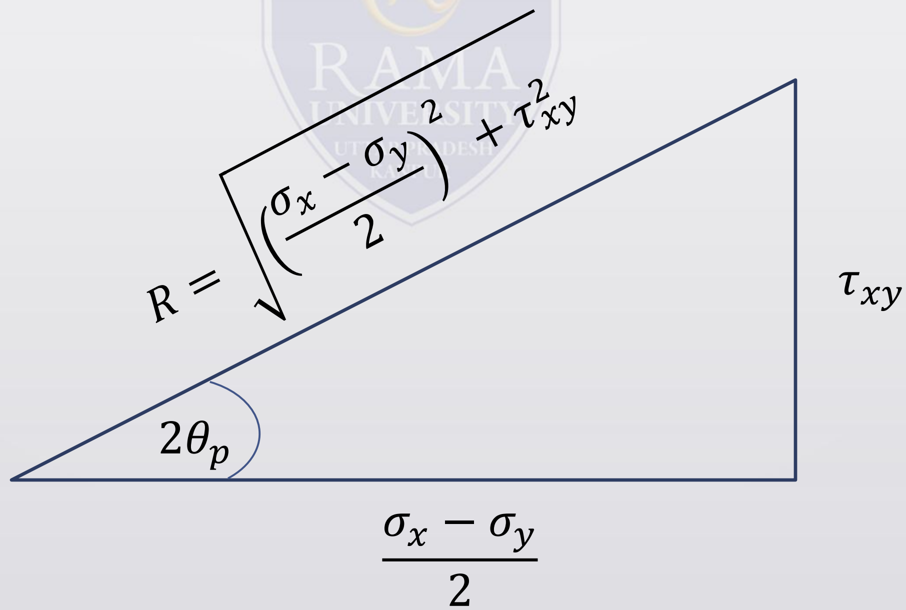
Principal Stresses

- ✓ The principal stresses can be calculated by substituting each of the θ_p two values of up into the stress-transformation equation

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

- ✓ From above equation we can constructed the following right angled triangle



Principal Stresses and Shear Stresses on principle planes

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

- ✓ Using the triangle of previous slide we can rearrange the above equation to get principle stresses

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

- ✓ If we substitute θ_p in shear stress transformation equation then we get

$$\tau_{xy} = 0$$

The shear stresses are zero on the principal planes

Maximum Shear Stresses

- ✓ By taking the derivative of τ_x with respect to θ and setting it equal to zero, we obtain an equation from which we can find the values of θ at which τ_x is a maximum positive and negative. The equation for the derivative is

$$\frac{d\tau_{xy}}{d\theta} = -(\sigma_x - \sigma_y)\cos 2\theta - 2\tau_{xy}\sin 2\theta = 0$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

- ✓ On comparing with $\tan 2\theta_p$ we can drive the following relations

$$2\theta_s - 2\theta_p = \pm 90^0$$

The planes of maximum shear stress occur at 45° to the principal planes.

$$\tau_{max/min} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

Maximum Shear Stresses

the maximum shear stress is equal to one-half the difference of the principal stresses

- ✓ The planes of maximum shear stress also contain normal stresses. The normal stress acting on the planes of maximum positive shear stress can be determined by substituting the expressions for the angle

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_1 + \sigma_2}{2}$$

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Review Questions

Q1 An element in plane stress (see figure) is subjected to stresses σ_x , σ_y *and* τ_{xy}

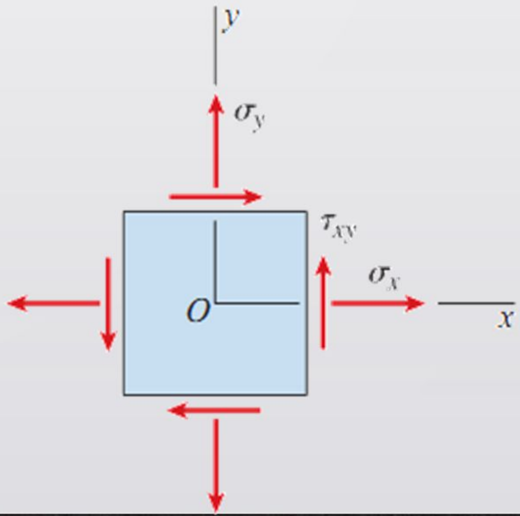
(a) Determine the principal stresses and show them on a sketch of a properly oriented element.

(b) Determine the maximum shear stresses and associated normal stresses and show them on a sketch of a properly oriented element.

i. $\sigma_x = 2150$ kPa, $\sigma_y = 375$ kPa, $\tau_{xy} = -460$ kPa

ii. $\sigma_x = 16.5$ MPa, $\sigma_y = -91$ MPa, $\tau_{xy} = -39$ MPa

iii. $\sigma_x = -108$ MPa, $\sigma_y = 58$ MPa, $\tau_{xy} = -58$ MPa



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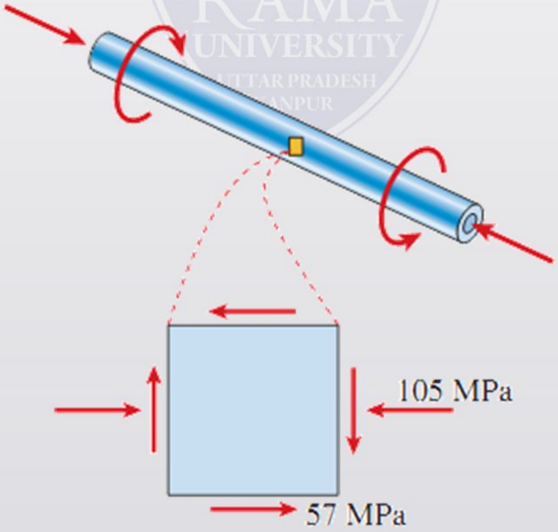
Review Questions

7.3-10 A propeller shaft subjected to combined torsion and axial thrust is designed to resist a shear stress of 57 MPa and a compressive stress of 105 MPa (see figure).

(a) Determine the principal stresses and show them on a sketch of a properly oriented element.

(b) Determine the maximum shear stresses and associated normal stresses and show them on a sketch of a properly oriented element.



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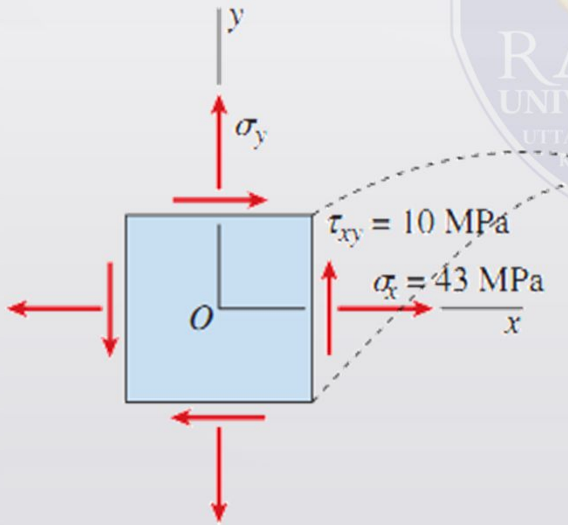
Review Questions

7.3-17 At a point on the web of a girder on a gantry crane, the stresses acting on the x face of a stress element are σ_x = 43 MPa and τ_{xy} = 10 MPa (see figure).

What is the allowable range of values for the stress σ_y if the maximum shear stress is limited to τ_0 = 15 MPa?



Review Questions

7.3-19 The stresses at a point on the down tube of a bicycle frame are $\sigma_x = 33$ MPa and $\tau_{xy} = -13$ MPa (see figure). It is known that one of the principal stresses equals 44 MPa in tension.

(a) Determine the stress σ_y.

(b) Determine the other principal stress and the orientation of the principal planes, then show the principal stresses on a sketch of a properly oriented element.

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Review Questions

7.3-20 An element in *plane stress* on the surface of an automobile drive shaft (see figure) is subjected to stresses of $\sigma_x = -45$ MPa and $\tau_{xy} = 39$ MPa (see figure). It is known that one of the principal stresses equals 41 MPa in tension.

(a) Determine the stress σ_y.

(b) Determine the other principal stress and the orientation of the principal planes, then show the principal stresses on a sketch of a properly oriented element.

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