RAMA UNIVERSITY

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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

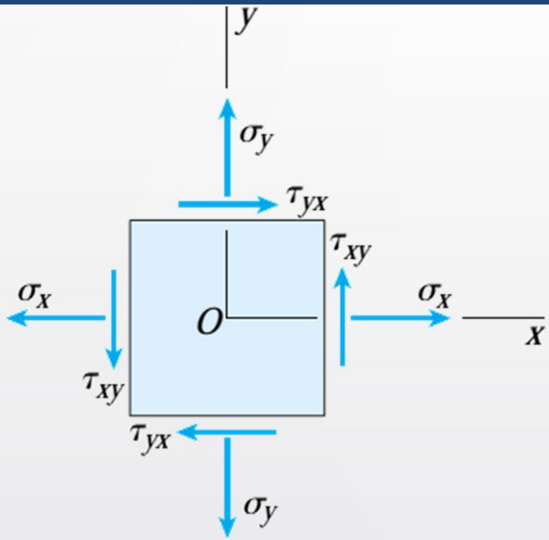
Lecture 11

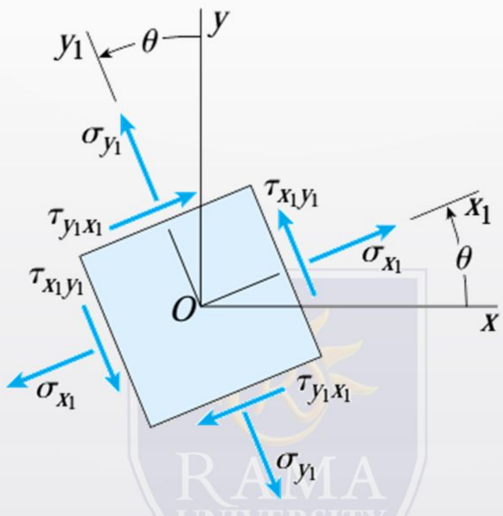
Topic: Mohr's Circle for Plane Stress

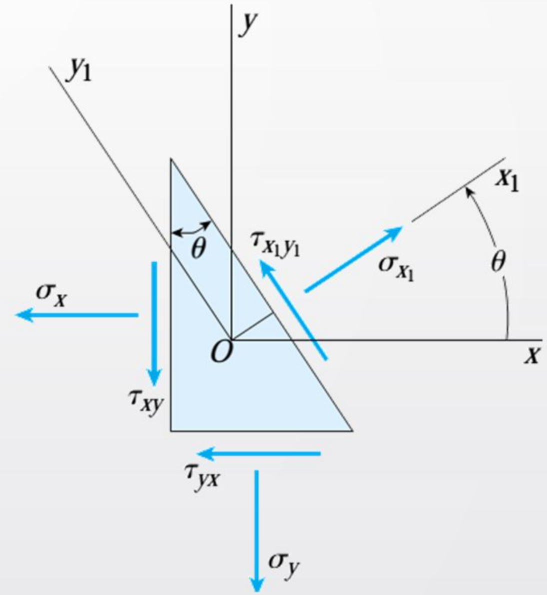
Instructor:

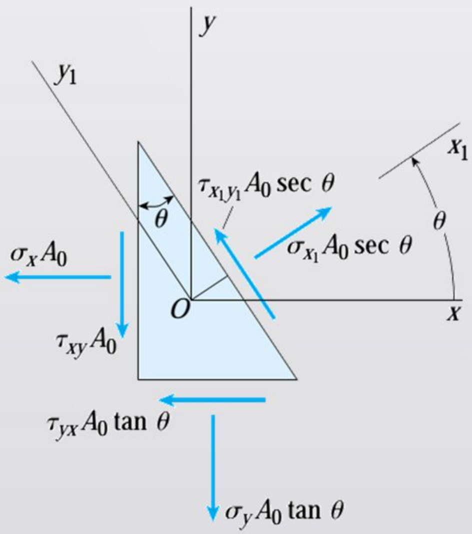
Aditya Veer Gautam

Stress Transformation Equations









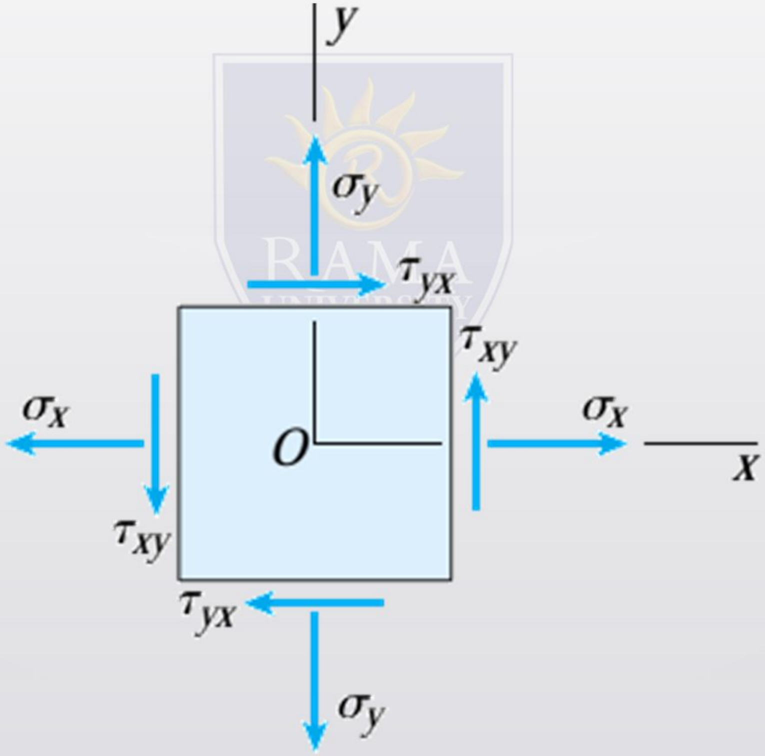
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y_1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x_1 y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

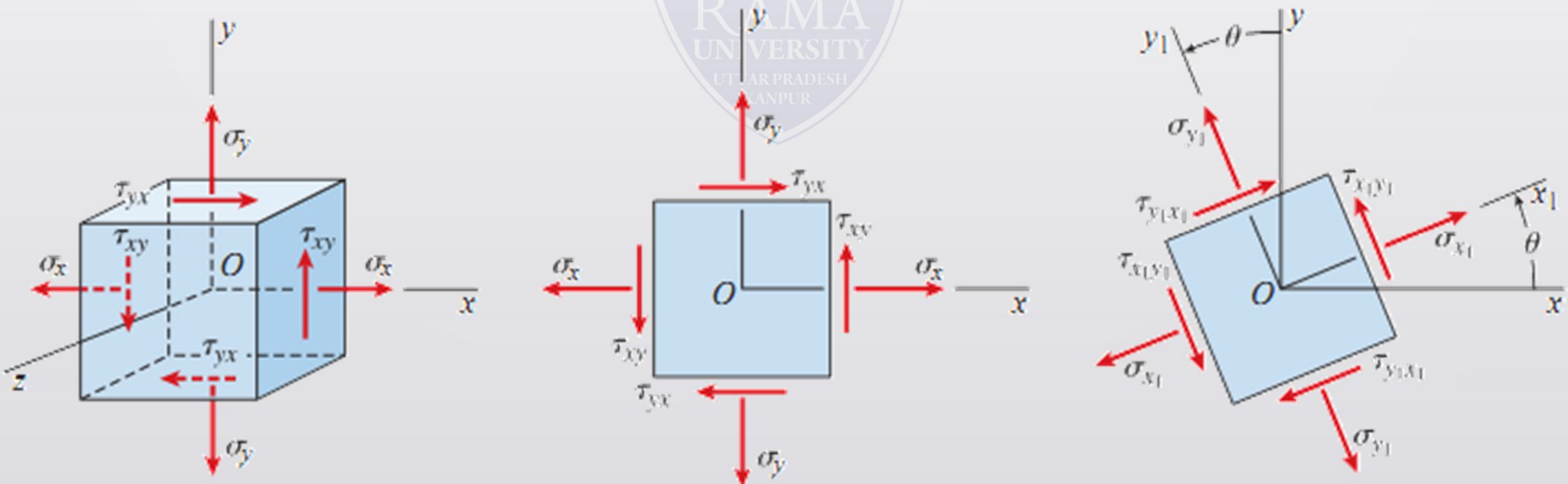
Shear Stress Sign Convention

- ✓ A shear stress is positive when it acts on a positive face of an element in the positive direction of an axis and it is negative when it acts on a positive face of and element in the negative direction of an axis



Plane Stress

- ✓ When the material is in plane stress in the **xy** plane, only the **x** and **y** faces of the element are subjected to stresses, and all stresses act parallel to the **x** and **y** axes
- ✓ Only $\sigma_x, \sigma_y, \tau_{xy}$ $(\sigma_z = 0, \tau_{zy} = \tau_{xz} = 0)$ are acting this condition is very common because it exists at the surface of any stressed body, except at points where external loads act on the surface.



Equations of Mohr's Circle

✓ Parametric form of stress transformation equations

$$\sigma_{x_1} - \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x_1 y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

✓ Eliminate 2θ from both equation , square both side of each equation and add

$$\left(\sigma_{x_1} - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{x_1 y_1}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

Equations of Mohr's Circle

- ✓ Substitute

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}, R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

- ✓ So final equation is $\left(\sigma_{x1} - \sigma_{avg}\right)^2 + \tau_{x_1y_1}^2 = R^2$
- ✓ Comparing with equation of circle $(x - x_1)^2 + (y - y_1)^2 = r^2$

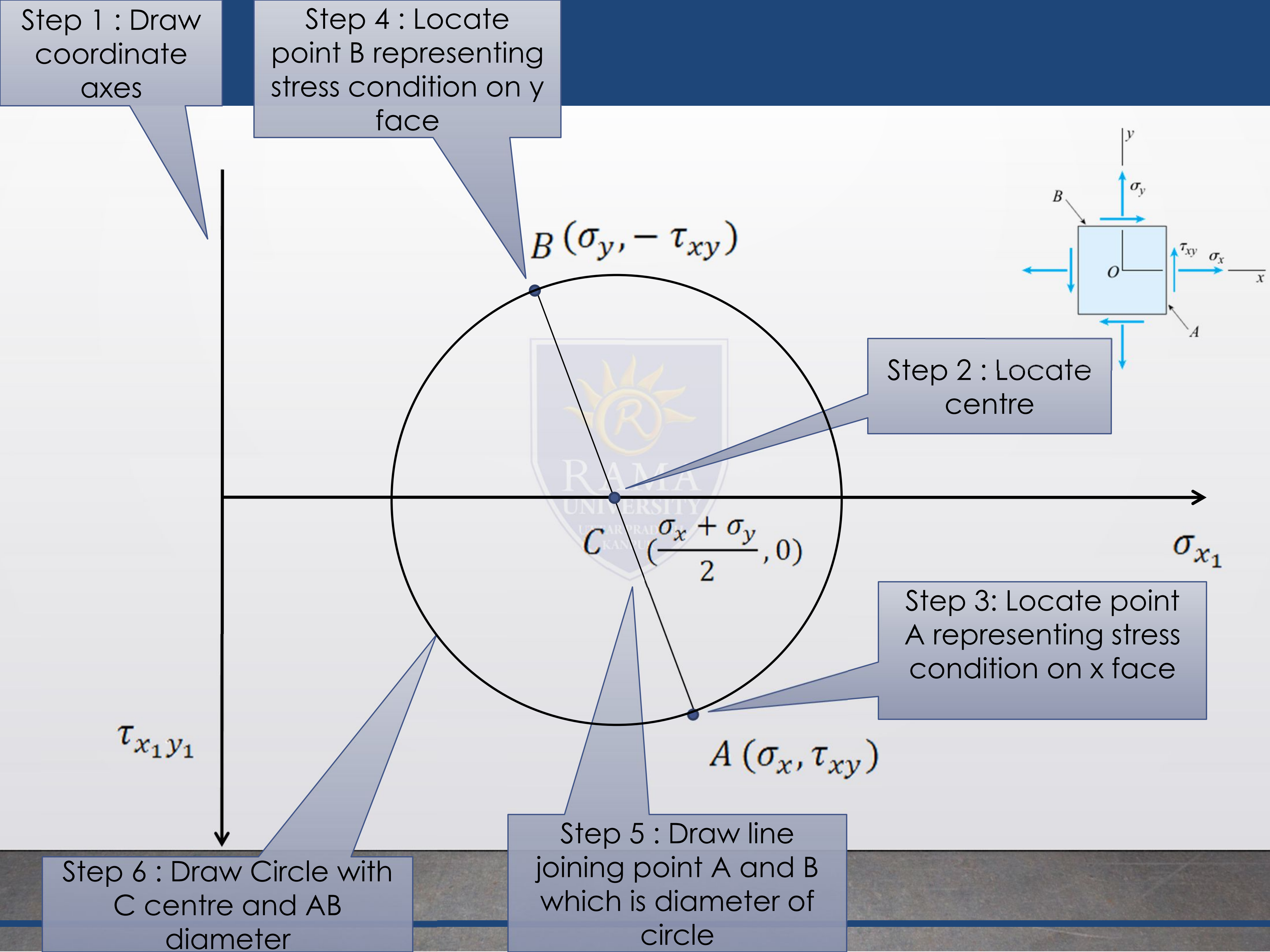
$$x_1 = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$r = R$$

$$y_1 = 0$$

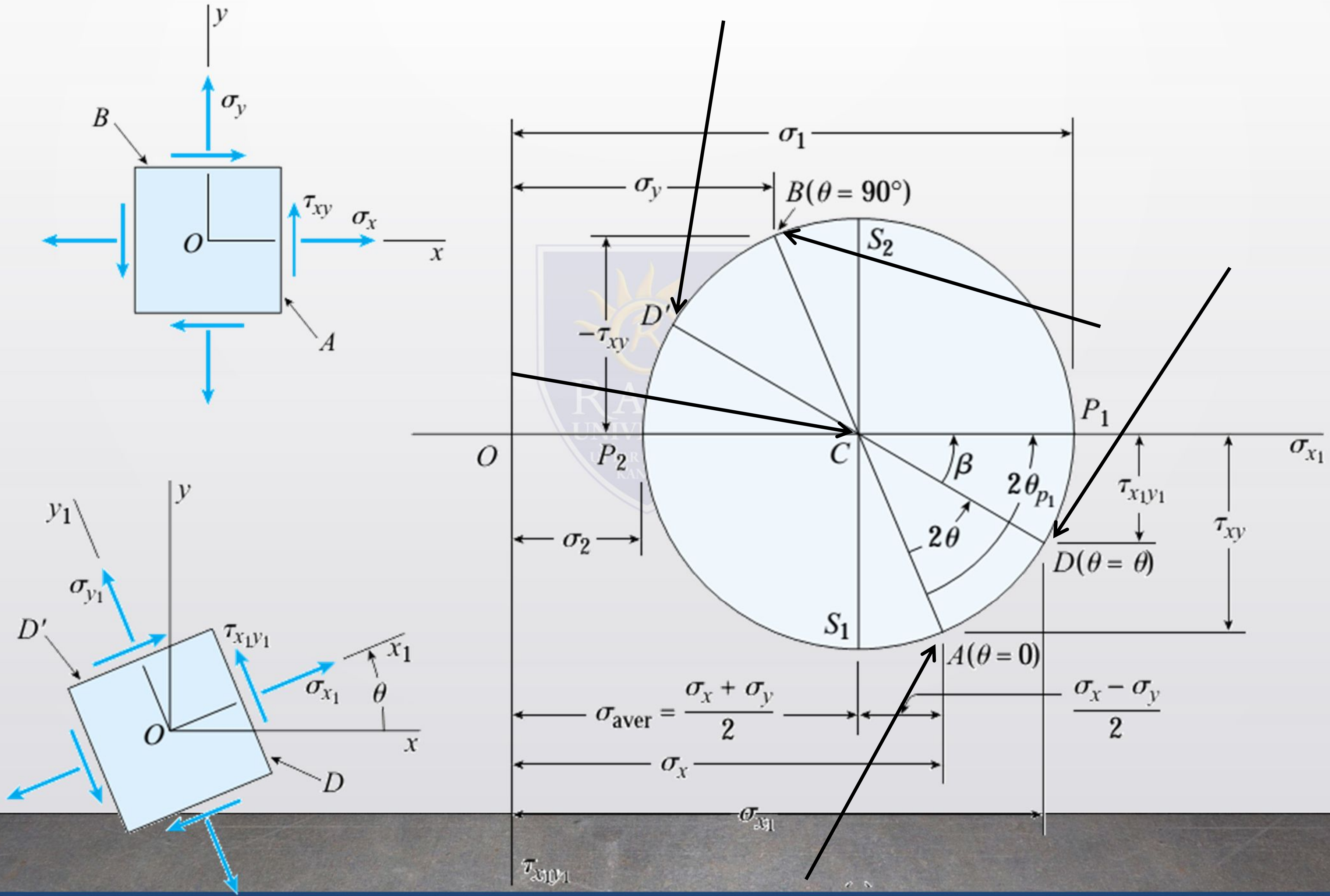
$$x = \sigma_1$$

$$y = \tau_{x_1y_1}$$



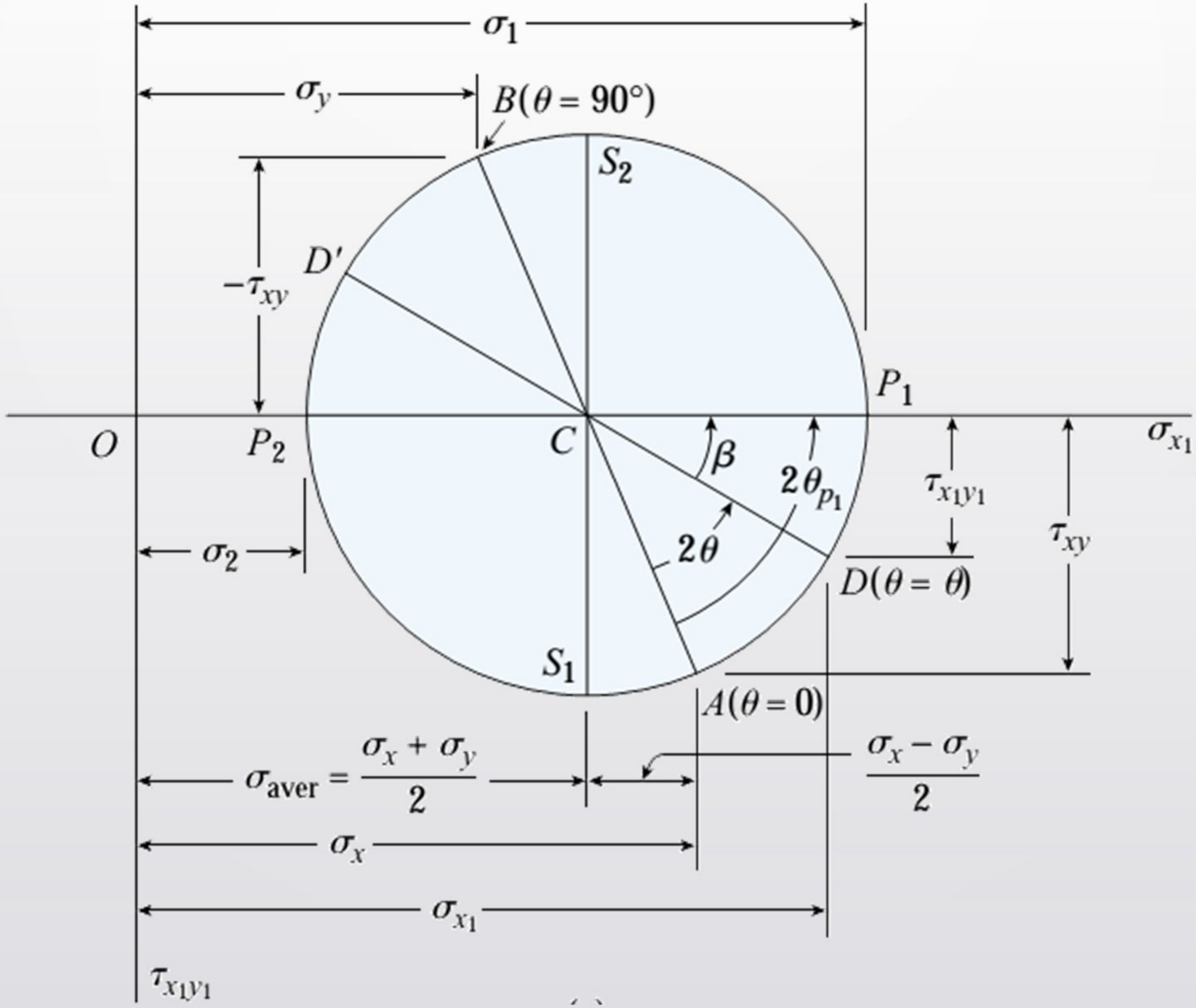
Step 1 : Draw coordinate axes
Step 4 : Locate point B representing stress condition on y face
$B\,(\sigma_y, -\tau_{xy})$
Step 2 : Locate centre
$C\,(\frac{\sigma_x+\sigma_y}{2}, 0)$
σ_{x_1}
Step 3: Locate point A representing stress condition on x face
$A\,(\sigma_x, \tau_{xy})$
$\tau_{x_1 y_1}$
Step 6 : Draw Circle with C centre and AB diameter
Step 5 : Draw line joining point A and B which is diameter of circle
y
σ_y
B
τ_{xy}
σ_x
O
x
A

Plotting of Mohr's Circle



Application

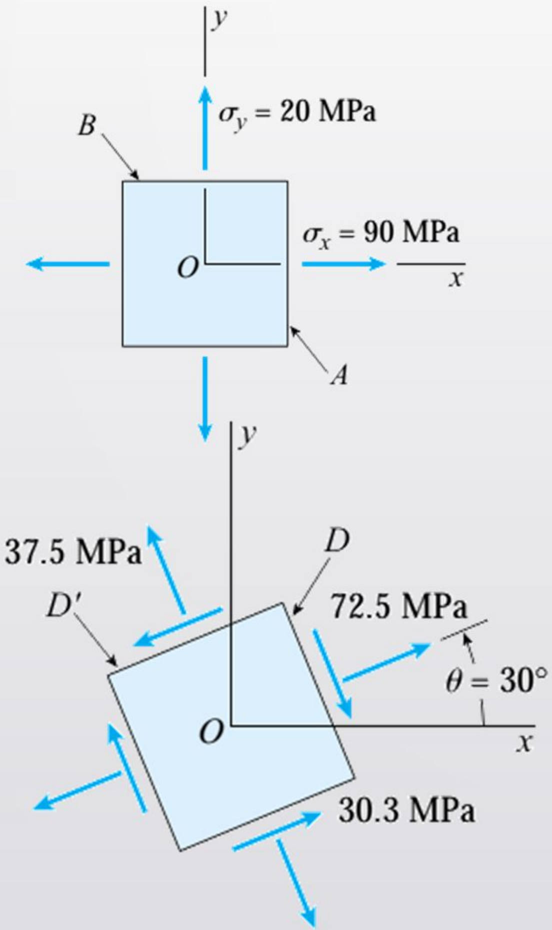
- It provide a visualization of
  - Relation between normal and shear stress
  - Principle stress and principle plane
  - Maximum in plane shear stress
- Note: It is usually preferable to obtain stresses by numerical calculation for better accuracy

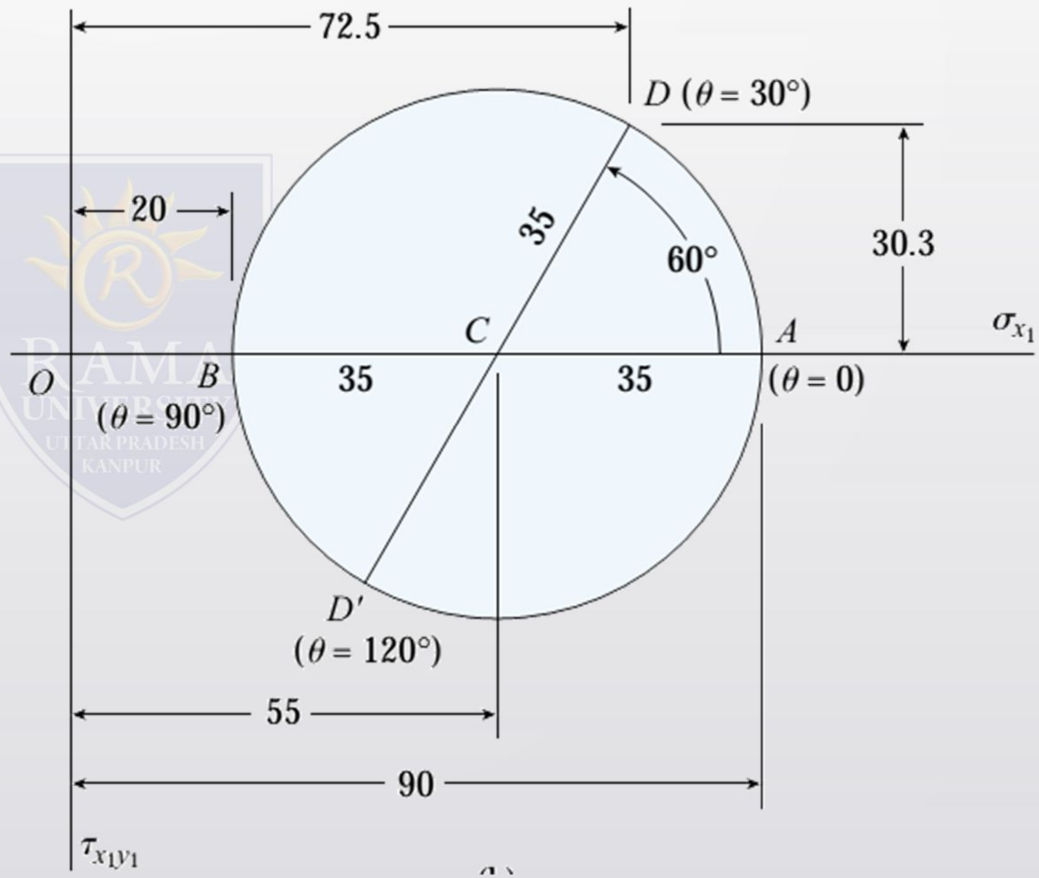


σ_1
σ_y
$B(\theta = 90°)$
S_2
D'
$-\tau_{xy}$
P_1
O
P_2
C
β
$2\theta_{p_1}$
σ_{x_1}
$\tau_{x_1y_1}$
τ_{xy}
σ_2
2θ
$D(\theta = \theta)$
S_1
$A(\theta = 0)$
$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2}$
$\frac{\sigma_x - \sigma_y}{2}$
σ_x
σ_{x_1}
$\tau_{x_1y_1}$

Example 1

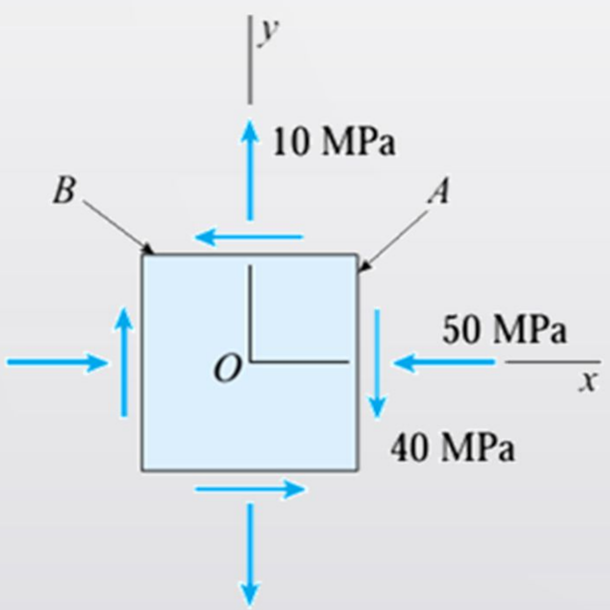
Using Mohr's circle, determine the stresses acting on an element inclined at an angle 30°

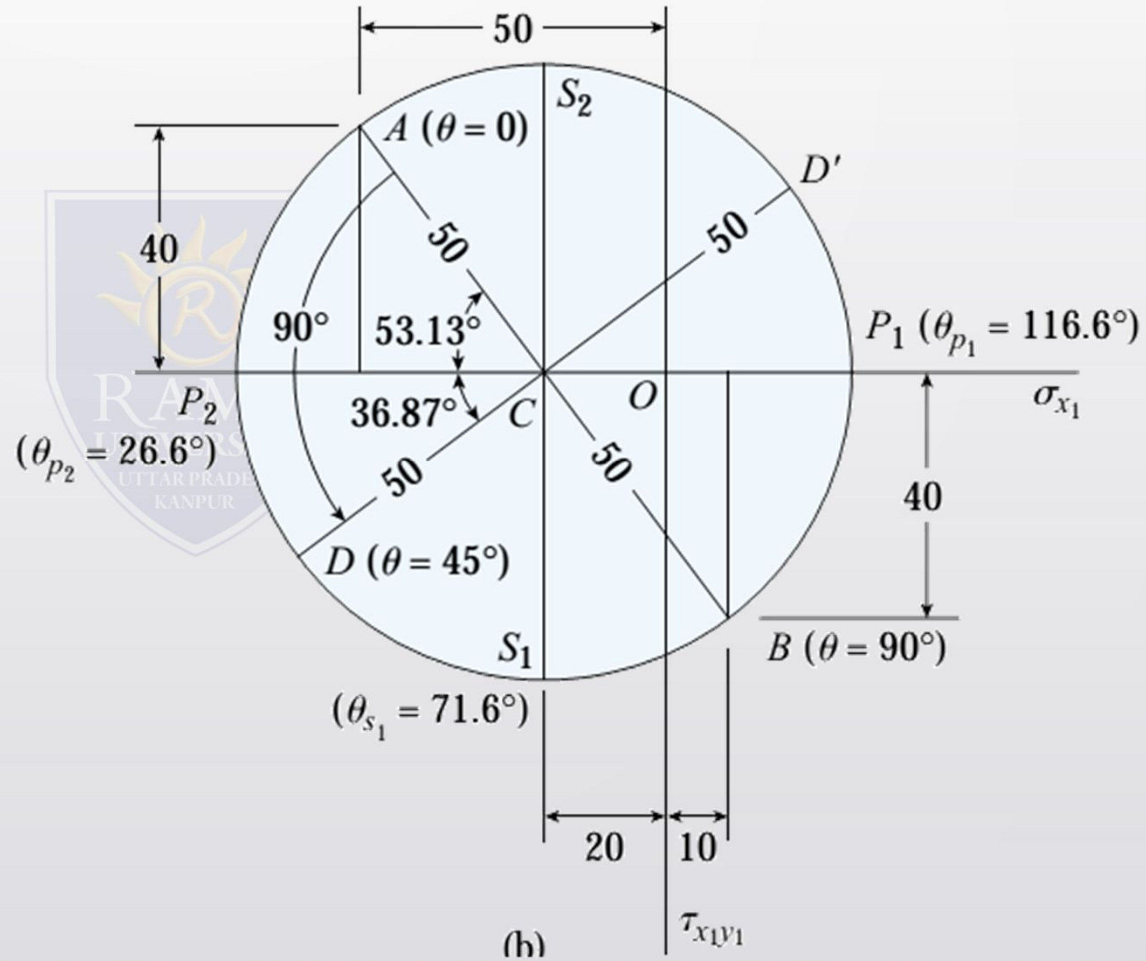




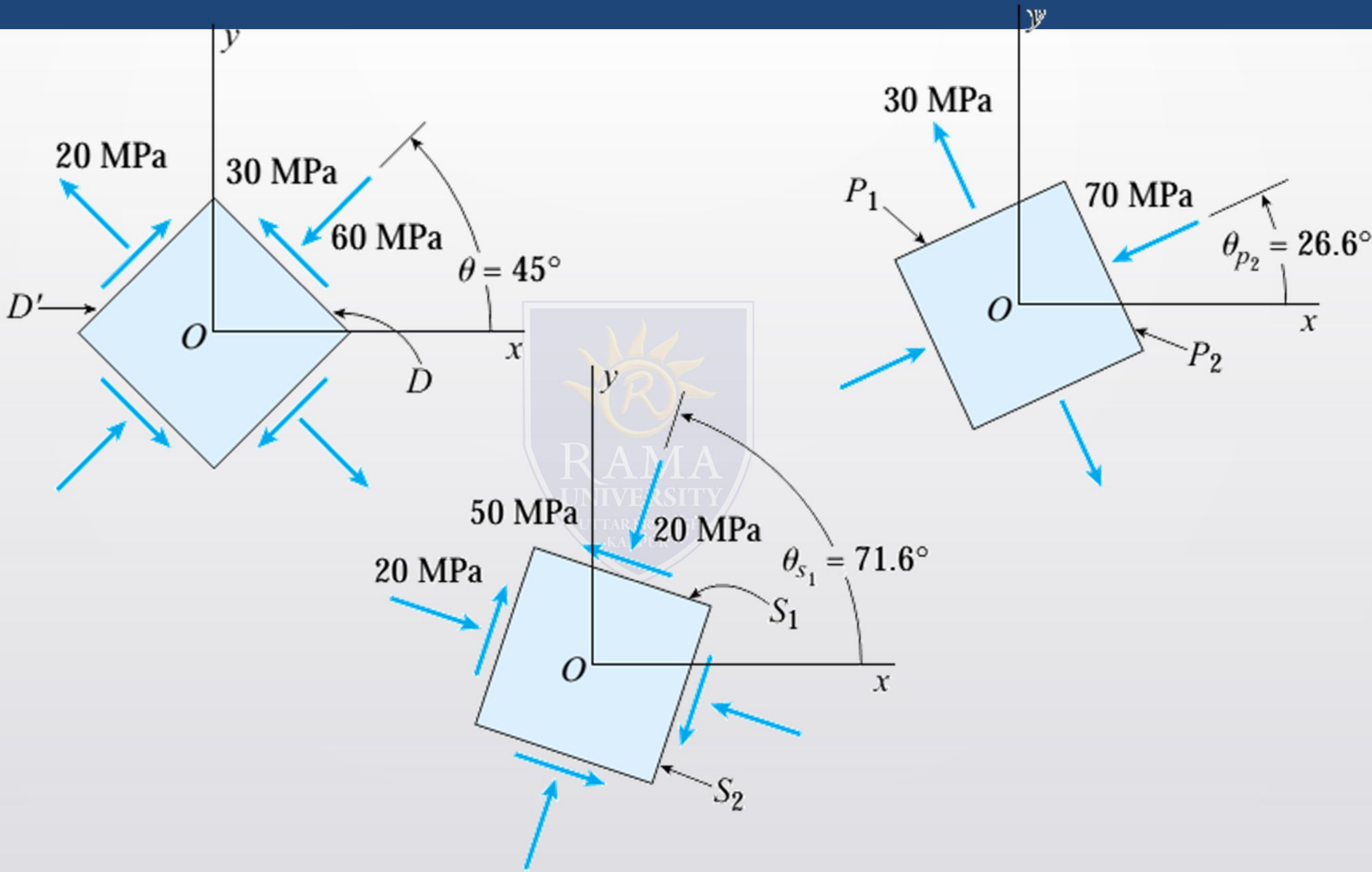
Example 2

Using Mohr's circle, determine the following quantities: (a) the stresses acting on an element inclined at an angle 45°, (b) the principal stresses, and (c) the maximum shear stresses





Example 2



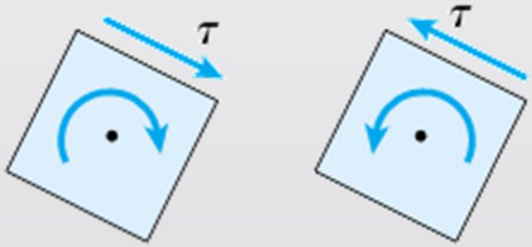
20 MPa
30 MPa
60 MPa
θ = 45°
D′
O
D
x
y
30 MPa
P1
70 MPa
θp2 = 26.6°
O
x
y
P2
50 MPa
20 MPa
20 MPa
θs1 = 71.6°
S1
O
x
y
S2

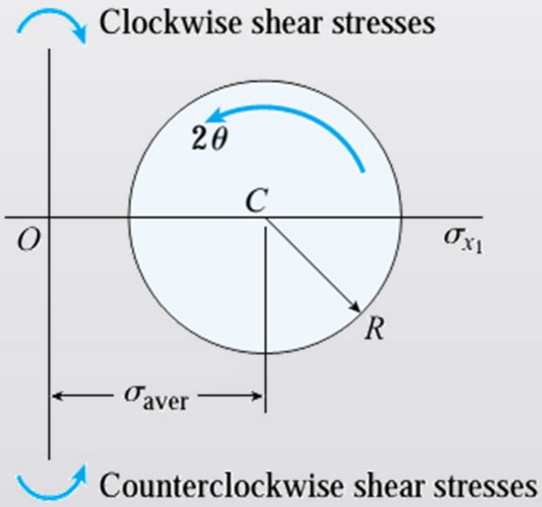
Frequently asked questions…

- Why co-ordinate of centre circle is ? $(\frac{\sigma_x + \sigma_y}{2}, 0)$
    - Comes from equations of circle
- Why downward shear stress is taken positive ?
    - There is no standard rule
    - But using this standard means counterclockwise 2θ is positive and matches with original derivation of stress transformation equations
- Is it correct to draw Mohr's Circle with upward shear stress positive.
    - Yes there is no harm its just another form except that now clockwise rotation will be positive

Frequently asked questions …

- Why coordinate of point B is $(\sigma_y, -\tau_{xy})$
  - Its related to sign conventions and stress transformation equations
- Is there a simpler sign convention for shear stress that can used for Mohr's circle
  - In order to remove the problem of positive or negative shear stress you can use clockwise and counterclockwise notation





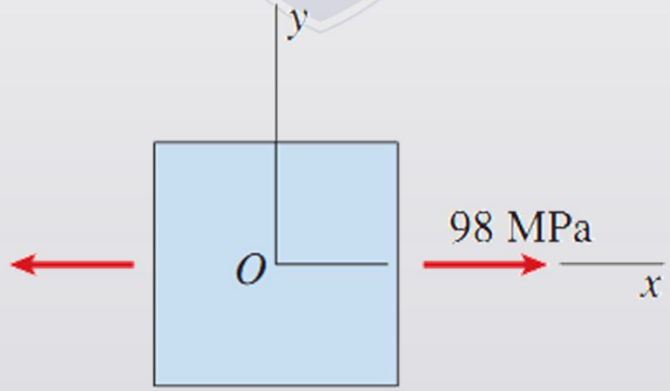
Question

7.4-1 An element in *uniaxial stress* is subjected to tensile stresses $\sigma_x = 98$ MPa, as shown in the figure. Using Mohr's circle, determine the following.

(a) The stresses acting on an element oriented at a counterclockwise angle $\theta = 29°$ from the x axis.

(b) The maximum shear stresses and associated normal stresses.

Show all results on sketches of properly oriented elements.



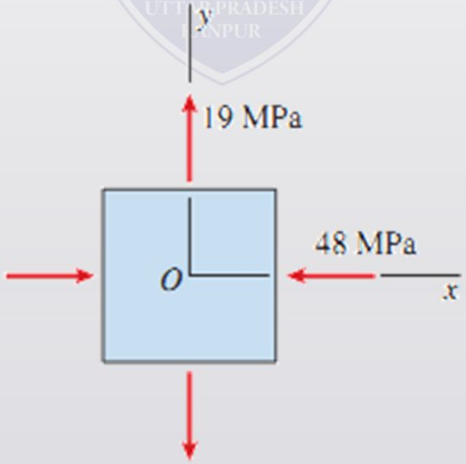
Question

7.4-4 An element on the top surface of the fuel tanker in Prob. 7.2-1 is in *biaxial stress* and is subjected to stresses $\sigma_x = -48$ MPa and $\sigma_y = 19$ MPa, as shown in the figure. Using Mohr's circle, determine the following.

(a) The stresses acting on an element oriented at a counterclockwise angle $\theta = 25°$ from the x axis.

(b) The maximum shear stresses and associated normal stresses.

Show all results on sketches of properly oriented elements.



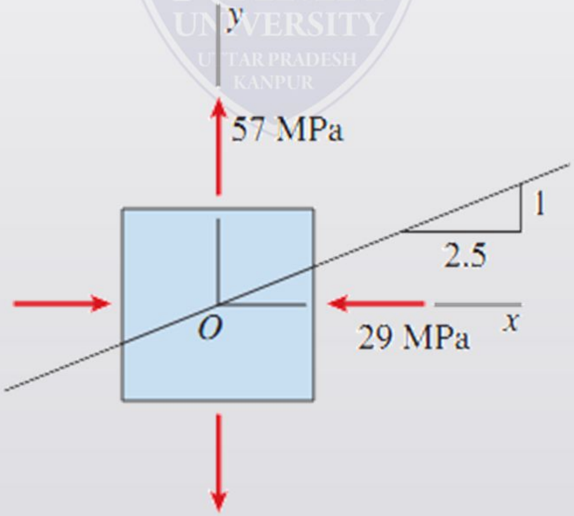
Question

7.4-6 An element in *biaxial stress* is subjected to stresses $\sigma_x = -29$ MPa and $\sigma_y = 57$ MPa, as shown in the figure. Using Mohr's circle, determine the following.

(a) The stresses acting on an element oriented at a slope of 1 on 2.5 (see figure).

(b) The maximum shear stresses and associated normal stresses.

Show all results on sketches of properly oriented elements.



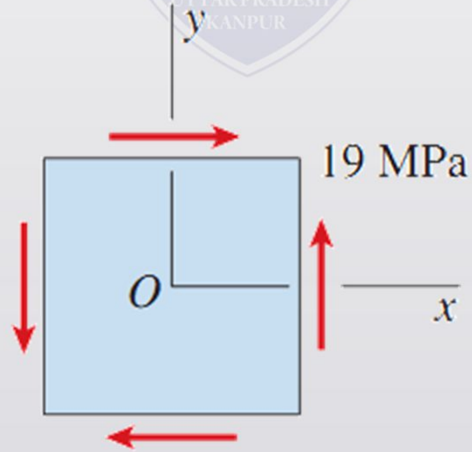
Question

7.4-7 An element on the surface of a drive shaft is in *pure shear* and is subjected to stresses $\tau_{xy} = 19$ MPa, as shown in the figure. Using Mohr's circle, determine the following.

(a) The stresses acting on an element oriented at a counterclockwise angle $\theta = 52°$ from the x axis.

(b) The principal stresses.

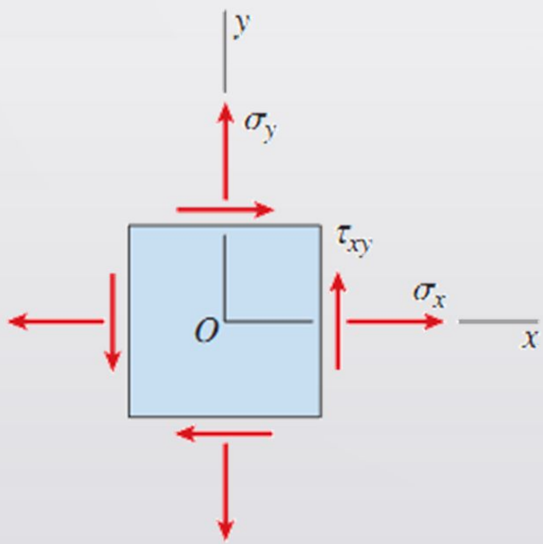
Show all results on sketches of properly oriented elements.



Question

7.4-10 through 7.4-15 An element in *plane stress* is subjected to stresses σ_x, σ_y, and τ_{xy} (see figure).

Using Mohr's circle, determine the stresses acting on an element oriented at an angle θ from the x axis. Show these stresses on a sketch of an element oriented at the angle θ. (*Note:* The angle θ is positive when counterclockwise and negative when clockwise.)



7.4-10 $\sigma_x = 27$ MPa, $\sigma_y = 14$ MPa, $\tau_{xy} = 6$ MPa, $\theta = 40°$

7.4-11 $\sigma_x = 24$ MPa, $\sigma_y = 84$ MPa, $\tau_{xy} = -23$ MPa, $\theta = -51°$

7.4-12 $\sigma_x = -47$ MPa, $\sigma_y = -186$ MPa, $\tau_{xy} = -29$ MPa, $\theta = -33°$

7.4-13 $\sigma_x = -12$ MPa, $\sigma_y = -5$ MPa, $\tau_{xy} = 2.5$ MPa, $\theta = 14°$

7.4-14 $\sigma_x = 33$ MPa, $\sigma_y = -9$ MPa, $\tau_{xy} = 29$ MPa, $\theta = 35°$

7.4-15 $\sigma_x = -39$ MPa, $\sigma_y = 7$ MPa, $\tau_{xy} = -15$ MPa, $\theta = 65°$