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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 12

Topic:12. State of Plane Strain Transformation Equations for Plane Strain

Instructor:

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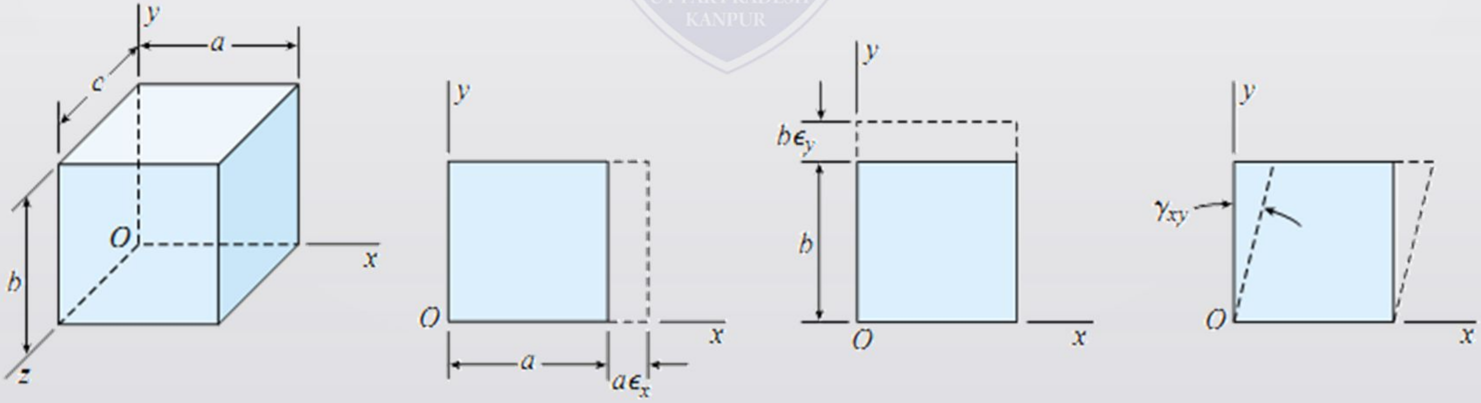
Plane Strain

- ✓ Consider a small element of material having sides of lengths a, b, and c in the x, y, and z directions, respectively.
- ✓ If the only deformations are those in the xy plane, then three
- ✓ strain components may exist
  - ✓ the normal strain ϵ_x in the x direction
  - ✓ The normal strain ϵ_y in the y direction, and
  - ✓ The shear strain γ_{xy}

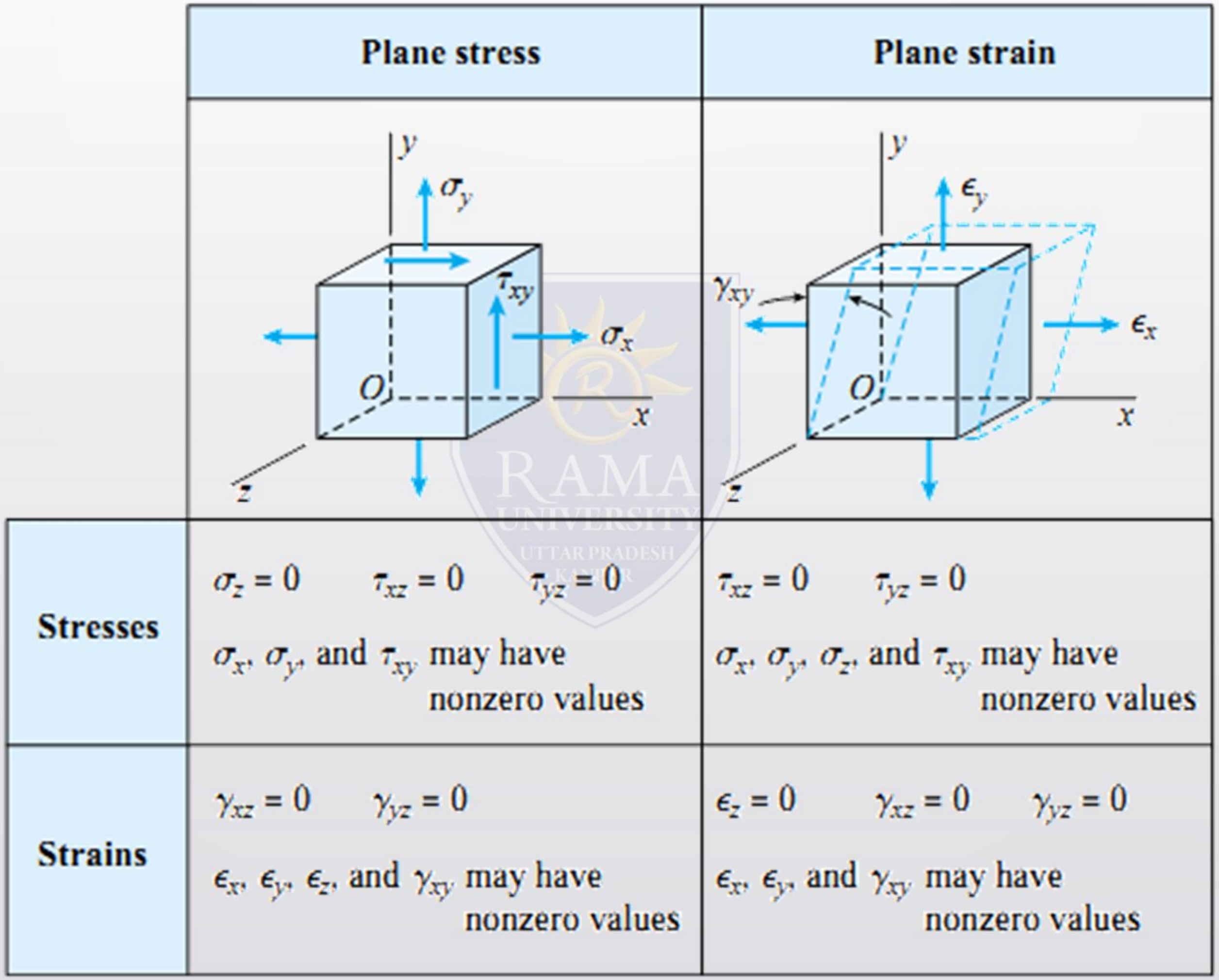


(a) (b) (c) (d)

Plane Strain

- ✓ If the only deformations are those in the **xy** plane, then three strain components may exist—$\epsilon_x, \epsilon_y, \gamma_{xy}$ $(\epsilon_z = 0, \gamma_{zy} = \gamma_{zx} = 0)$ An element of material subjected to these strains (and only these strains) is said to be in a state of plane strain
  - ✓ Under ordinary conditions plane stress and plane strain do not occur simultaneously
  - ✓ When $\sigma_x = -\sigma_y$ Plane Stress and Plane Strain occur simultaneously
- ✓ the transformation equations for plane stress can also be used for the stresses in plane strain because σ_z does not enter the equations of equilibrium used in derivation
- ✓ Similarly the transformation equations for plane strain can also be used for the strains in plane stress

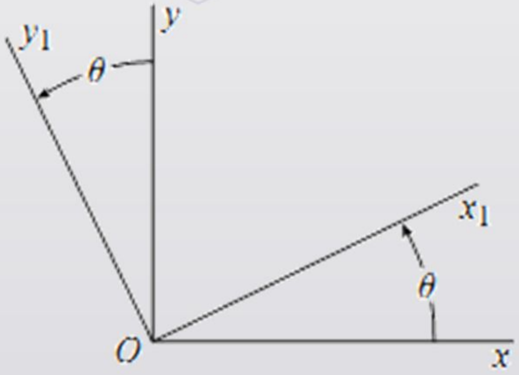
Place Stress Vs Place Strain



	Plane stress	Plane strain
Stresses	$\sigma_z = 0$ $\tau_{xz} = 0$ $\tau_{yz} = 0$ σ_x, σ_y, and τ_{xy} may have nonzero values	$\tau_{xz} = 0$ $\tau_{yz} = 0$ σ_x, σ_y, σ_z, and τ_{xy} may have nonzero values
Strains	$\gamma_{xz} = 0$ $\gamma_{yz} = 0$ ϵ_x, ϵ_y, ϵ_z, and γ_{xy} may have nonzero values	$\epsilon_z = 0$ $\gamma_{xz} = 0$ $\gamma_{yz} = 0$ ϵ_x, ϵ_y, and γ_{xy} may have nonzero values

Transformation Equation for Plane Strain

- ✓ In the derivation of the transformation equations for plane strain, we will use the coordinate axes shown below.
- ✓ We will assume that the normal strains and the shear strain associated with the xy axes are known.
- ✓ The objectives of our analysis are to determine the normal strain and the shear strain associated with the axes, which are rotated counterclockwise through an angle θ from the xy axes

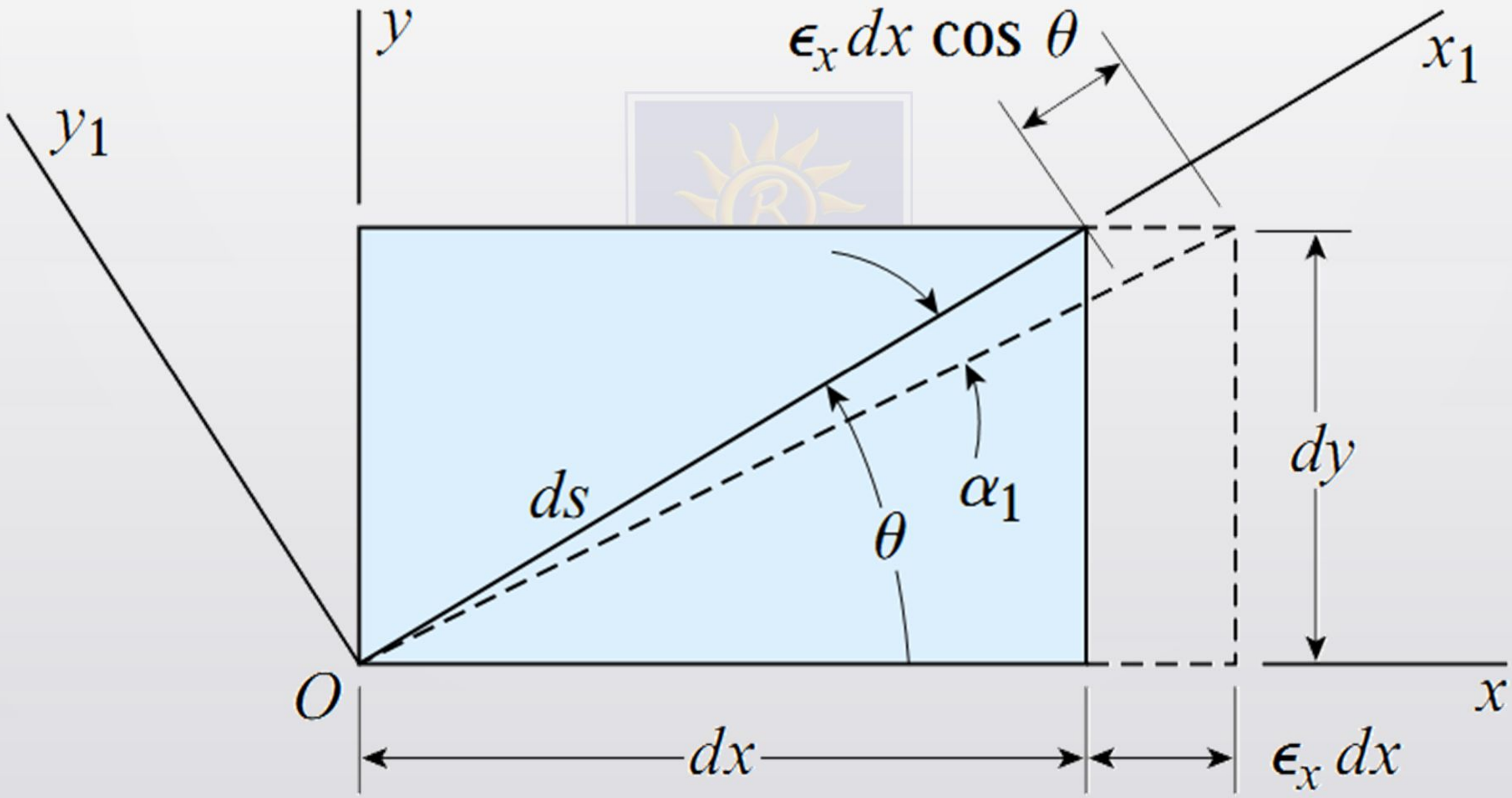


Transformation Equation for Plane Strain: Normal Strain

- ✓ First we will find out Normal strain ϵ_{x_1} .
- ✓ To determine the normal strain ϵ_{x_1} in the x_1 direction, we consider a small element of material selected so that the x_1 axis is along a **diagonal** of the z face of the element and the x and y axes are along the sides of the element
- ✓ The Procedure required 4 steps
  1. Find out the elongation in diagonal due o ϵ_x
  2. Find out the elongation in diagonal due o ϵ_y
  3. Find out the elongation in diagonal due o γ_{xy}
  4. Get the total elongation to get the strain along diagonal

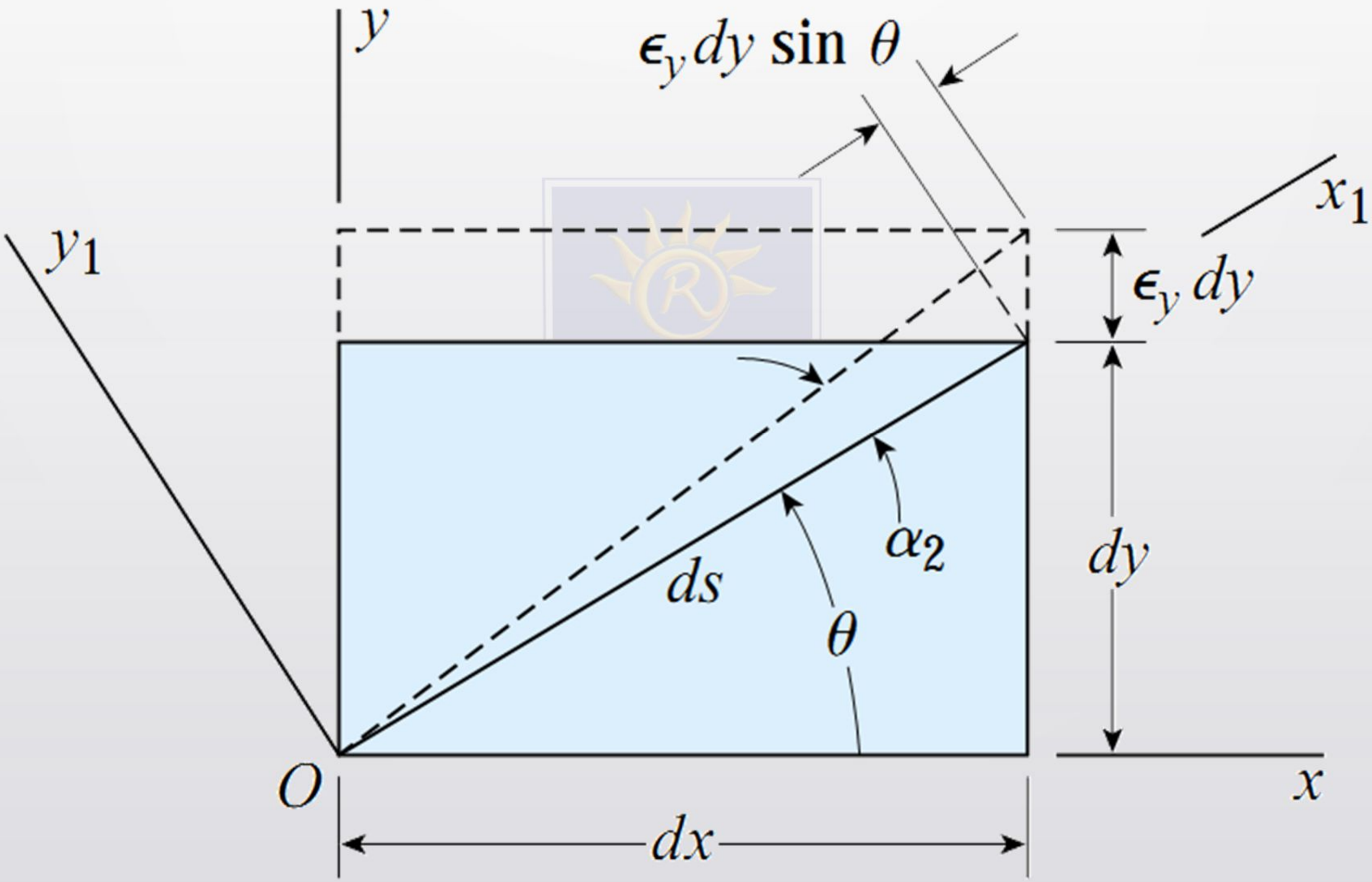
Transformation Equation for Plane Strain : Normal Strain

- ✓ Step 1



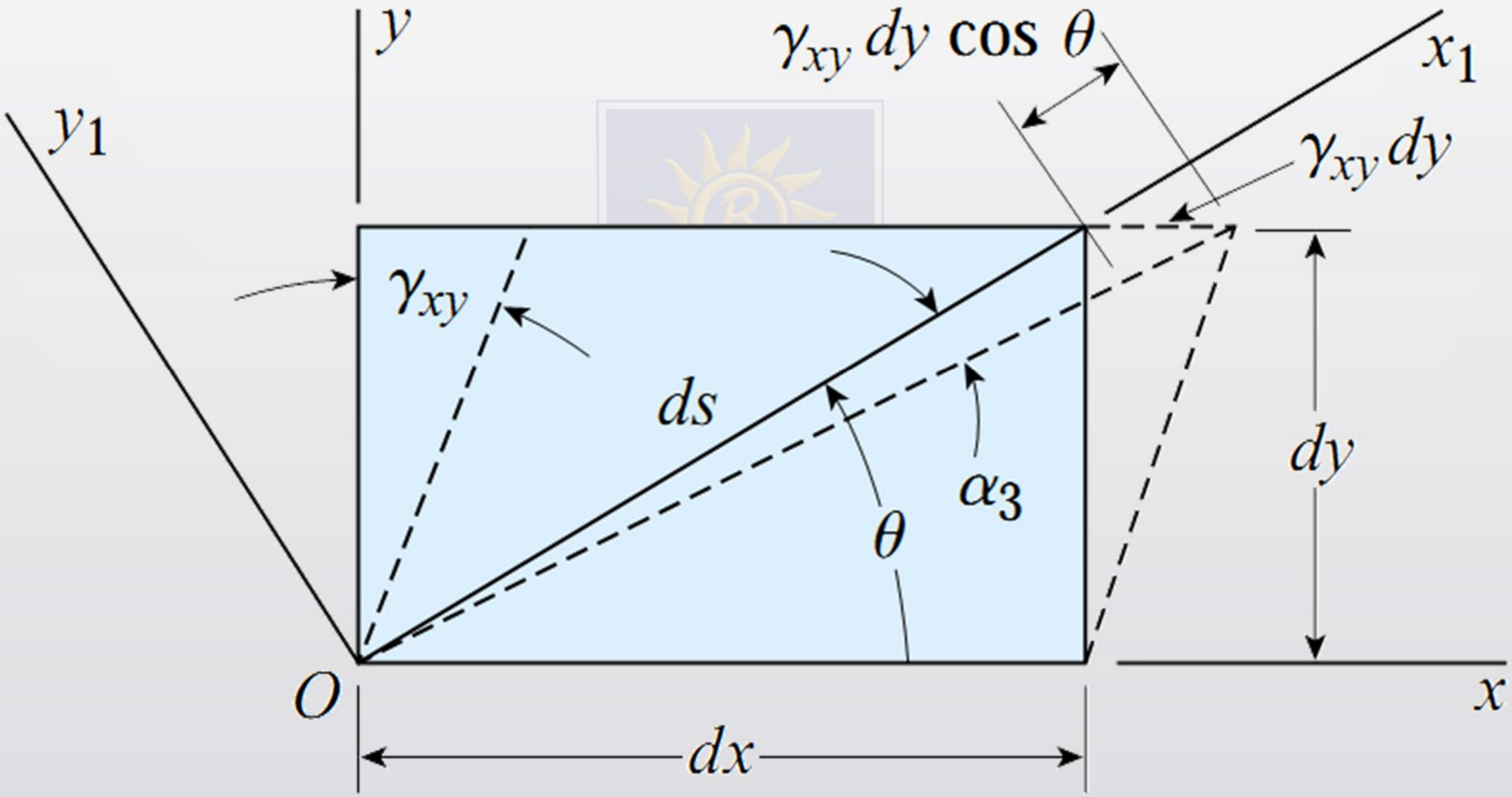
Transformation Equation for Plane Strain : Normal Strain

- ✓ Step 2



Transformation Equation for Plane Strain : Normal Strain

- ✓ Step 3



Transformation Equation for Plane Strain : Normal Strain

- ✓ Step 4

The total increase Δd in the length of the diagonal is the sum of the preceding three expressions; thus,

$$\Delta d = \epsilon_x dx \cos\theta + \epsilon_y dy \sin\theta + \gamma_{xy} dy \cos\theta$$

therefore strain ϵ_{x_1}

$$\epsilon_{x_1} = \frac{\Delta d}{ds} = \epsilon_x \frac{dx}{ds}\cos\theta + \epsilon_y \frac{dy}{ds}\sin\theta + \gamma_{xy}\frac{dy}{ds}\cos\theta$$

but

$$\frac{dx}{ds} = \cos\theta \ , \frac{dy}{ds} = \sin\theta$$

Hence

$$\epsilon_{x_1} = \epsilon_x \cos^2\theta + \epsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

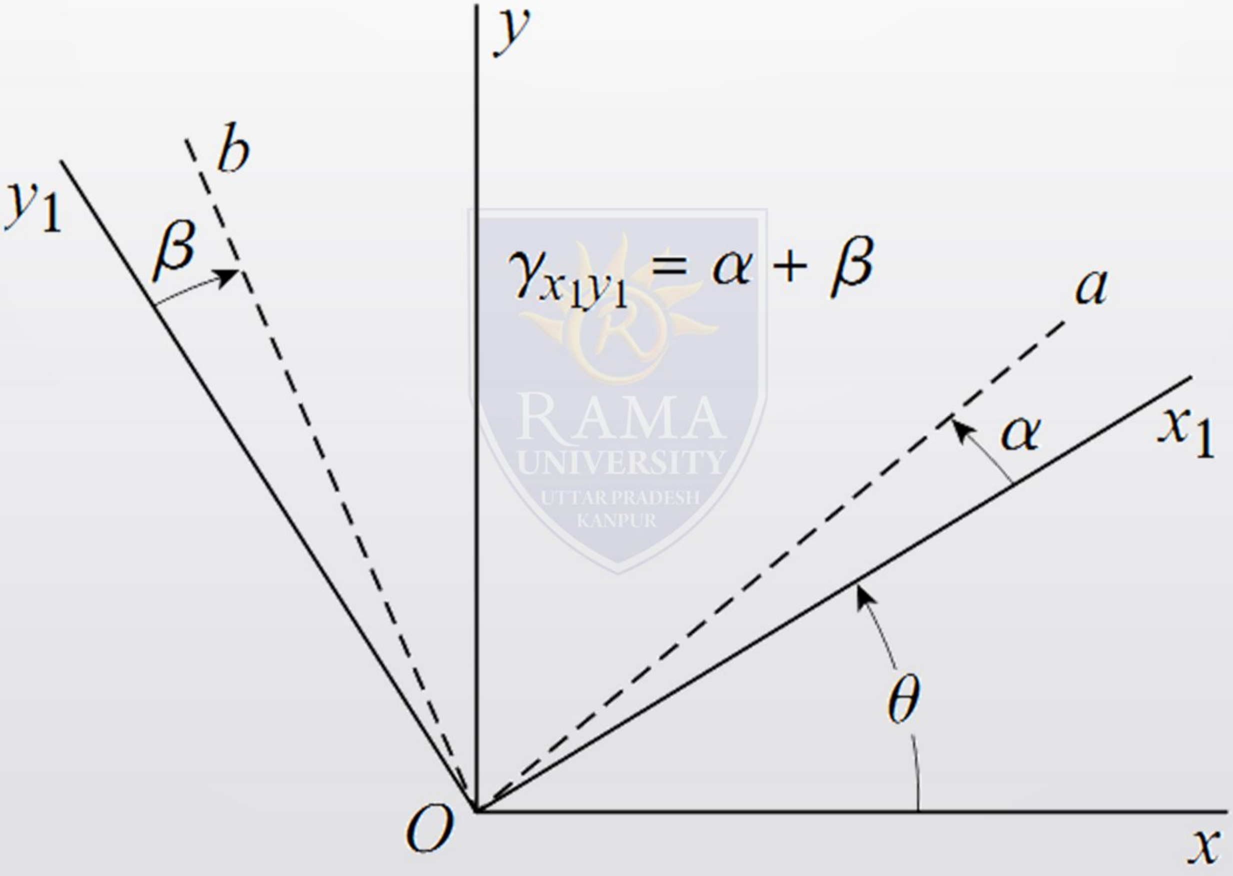
If we $\theta = 90^0 + \theta$ in the above equation we will get the value of ϵ_{y_1}

Transformation Equation for Plane Strain : Shear Strain

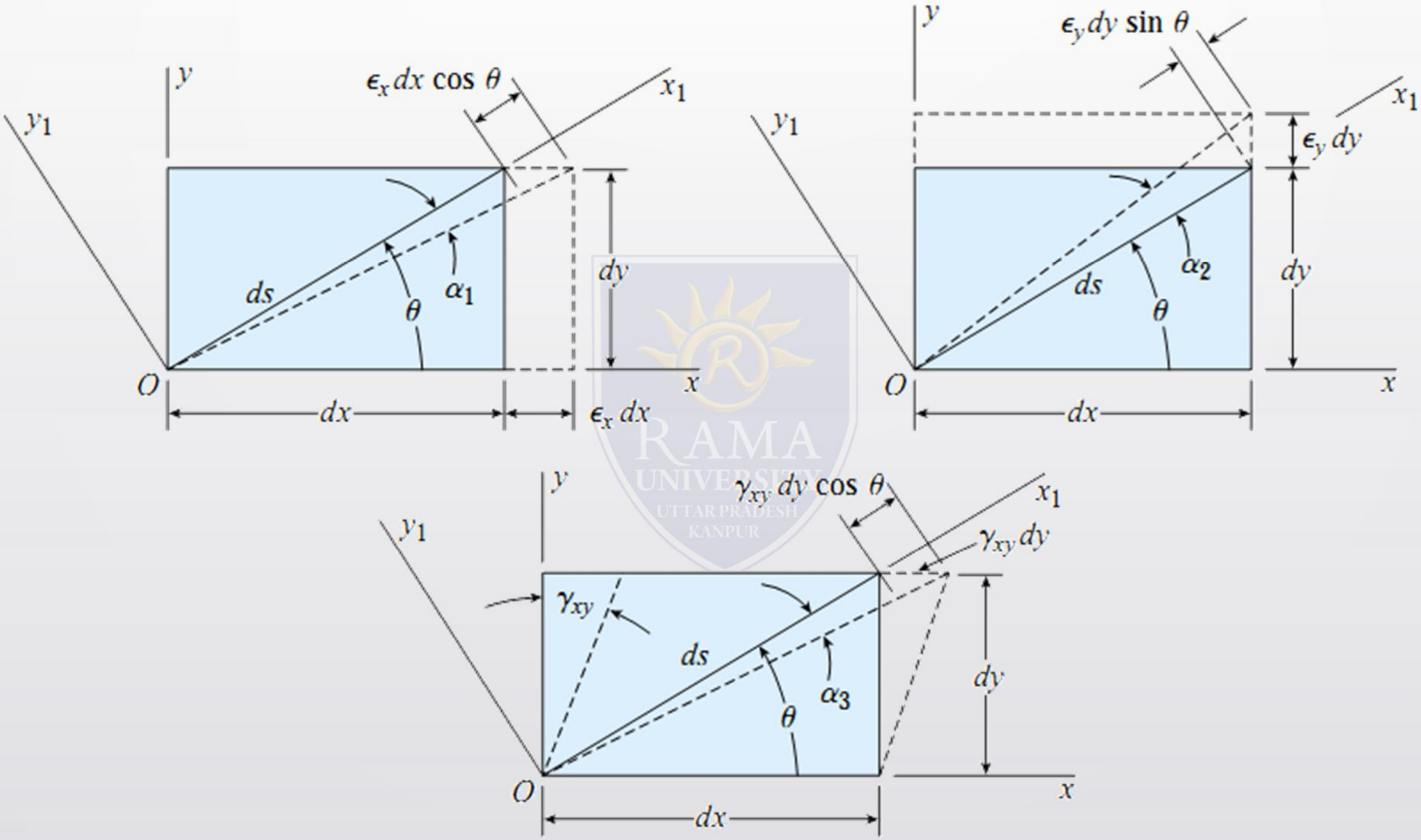
- ✓ This strain is equal to the decrease in angle between lines in the material that were initially along the $\mathbf{x}_1$ and $\mathbf{y}_l$ axes.
- ✓ From figure, let Oa represent a line that initially was along the $\mathbf{x}_l$ axis (that is, along the diagonal of the element.
- ✓ The deformations caused by the strains ϵ_x , $\epsilon_y and\ \gamma_{xy}$ cause line Oa to rotate through a counterclockwise angle $\boldsymbol{\alpha}$ from the $\mathbf{x}_l$ axis to the position shown.
- ✓ Similarly, line Ob was originally along the $\mathbf{y}_l$ axis, but because of the deformations it rotates through a clockwise angle β .
- ✓ The shear strain $\gamma_{x_1y_1}$ is the decrease in angle between the two lines that originally were at right angles; therefore

$$\gamma_{x_1y_1} = \alpha + \beta$$

Transformation Equation for Plane Strain : Shear Strain



Transformation Equation for Plane Strain : Shear Strain



Transformation Equation for Plane Strain : Shear Strain

$$\alpha_1 = \epsilon_x \frac{dx}{ds} \sin\theta$$

$$\alpha_2 = \epsilon_y \frac{dy}{ds} \cos\theta$$

$$\alpha_3 = \gamma_{xy} \frac{dy}{ds} \sin\theta$$

$$\alpha = -\alpha_1 + \alpha_2 - \alpha_3$$

$$\alpha = -\epsilon_x \frac{dx}{ds} \sin\theta + \epsilon_y \frac{dy}{ds} \cos\theta - \gamma_{xy} \frac{dy}{ds} \sin\theta$$

$$\alpha = -\left(\epsilon_x - \epsilon_y\right) \sin\theta \cos\theta - \gamma_{xy} \sin^2\theta$$

Similarly

$$\beta = -\left(\epsilon_x - \epsilon_y\right) \sin\theta \cos\theta + \gamma_{xy} \cos^2\theta$$

$$\gamma_{x_1 y_1} = -2\left(\epsilon_x - \epsilon_y\right) \sin\theta \cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)$$

Transformation Equations

$$\epsilon_{x_1} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$\frac{\gamma_{x_1 y_1}}{2} = -\frac{\epsilon_x - \epsilon_y}{2} \sin 2\theta + \frac{\gamma_{xy}}{2} \cos 2\theta$$

$$\epsilon_{y_1} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$\epsilon_{x_1} + \epsilon_{y_1} = \epsilon_x + \epsilon_y$$

Question

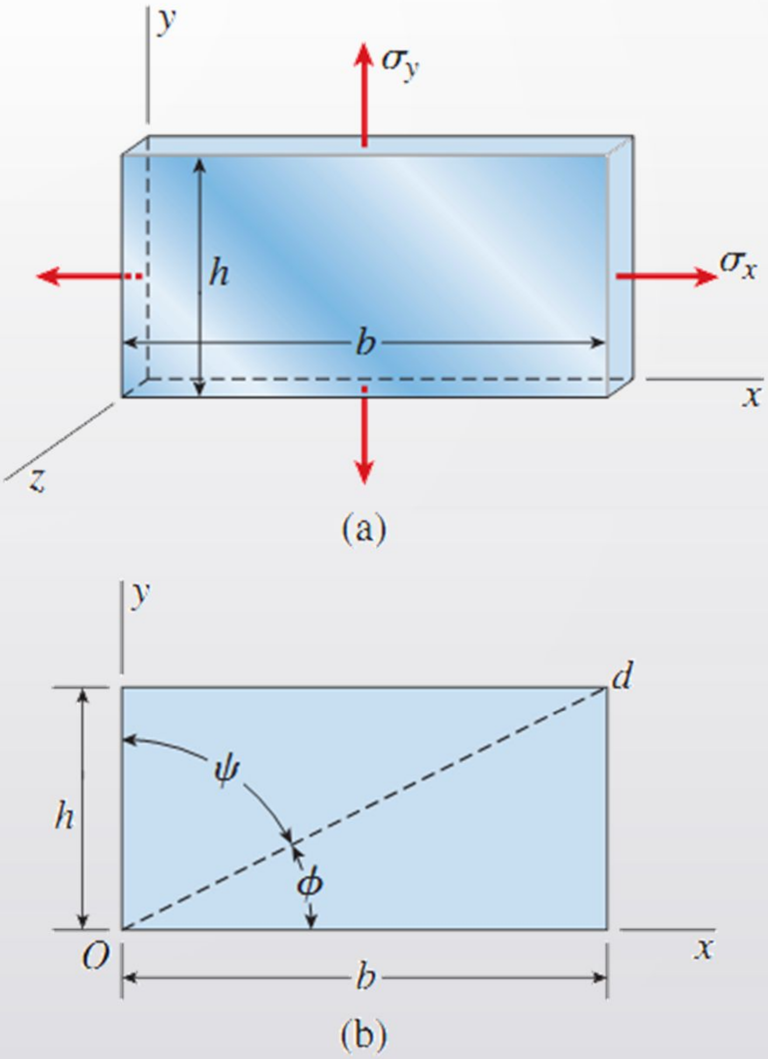
7.7-1 A thin rectangular plate in *biaxial stress* is subjected to stresses σ_x and σ_y, as shown in part a of the figure. The width and height of the plate are $b = 190$ mm and $h = 63$ mm, respectively. Measurements show that the normal strains in the x and y directions are $\varepsilon_x = 285 \times 10^{-6}$ and $\varepsilon_y = -190 \times 10^{-6}$, respectively.

With reference to part b of the figure, which shows a two-dimensional view of the plate, determine the following quantities.

(a) The increase Δd in the length of diagonal Od.

(b) The change $\Delta\phi$ in the angle ϕ between diagonal Od and the x axis.

(c) The change $\Delta\psi$ in the angle ψ between diagonal Od and the y axis.



Question

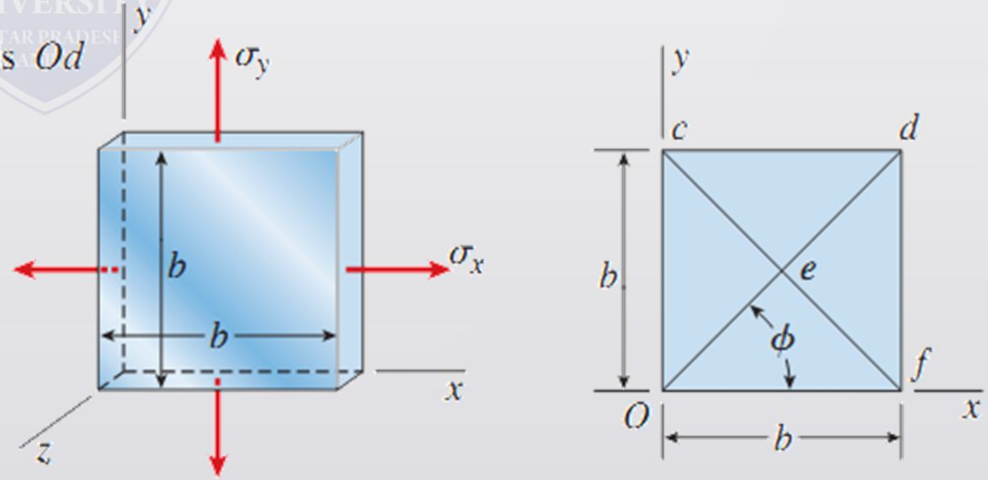
7.7-3 A thin square plate in *biaxial stress* is subjected to stresses σ_x and σ_y, as shown in part a of the figure. The width of the plate is $b = 300$ mm. Measurements show that the normal strains in the x and y directions are $\varepsilon_x = 427 \times 10^{-6}$ and $\varepsilon_y = 113 \times 10^{-6}$, respectively.

With reference to part b of the figure, which shows a two-dimensional view of the plate, determine the following quantities.

(a) The increase Δd in the length of diagonal Od.

(b) The change $\Delta\phi$ in the angle ϕ between diagonal Od and the x axis.

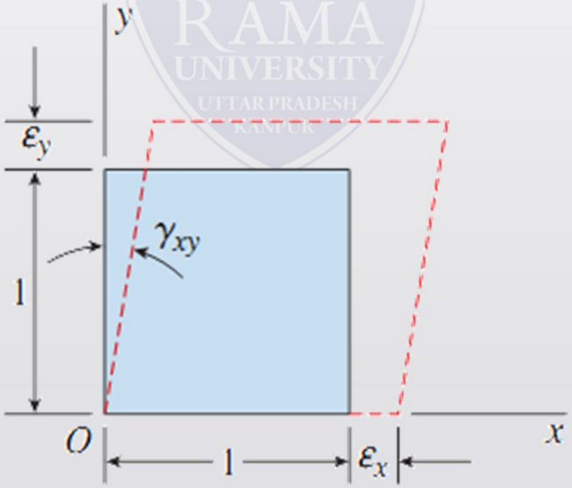
(c) The shear strain γ associated with diagonals Od and cf (that is, find the decrease in angle ced).



Question

7.7-5 An element of material subjected to *plane strain* (see figure) has strains of $\varepsilon_x = 280 \times 10^{-6}$, $\varepsilon_y = 420 \times 10^{-6}$, and $\gamma_{xy} = 150 \times 10^{-6}$.

Calculate the strains for an element oriented at an angle $\theta = 35°$. Show these strains on a sketch of a properly oriented element.



Question

if $b = 225$ mm,

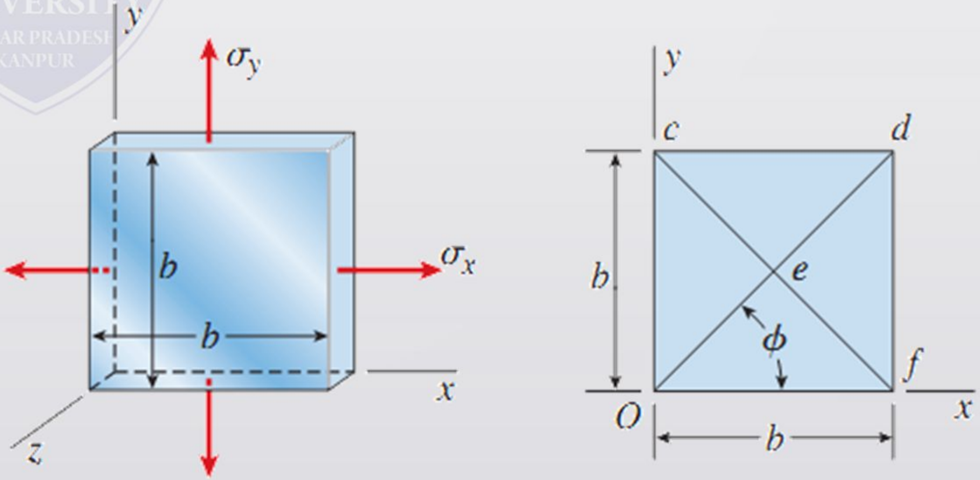
$\varepsilon_x = 845 \times 10^{-6}$, and $\varepsilon_y = 211 \times 10^{-6}$.

With reference to part b of the figure, which shows a two-dimensional view of the plate, determine the following quantities.

(a) The increase Δd in the length of diagonal Od.

(b) The change $\Delta\phi$ in the angle ϕ between diagonal Od and the x axis.

(c) The shear strain γ associated with diagonals Od and cf (that is, find the decrease in angle ced).



Question

$\varepsilon_x = 190 \times 10^{-6}$, $\varepsilon_y = -230 \times 10^{-6}$, $\gamma_{xy} = 160 \times 10^{-6}$, and $\theta = 40^\circ$.

Calculate the strains for an element oriented at an angle $\theta = 35^\circ$. Show these strains on a sketch of a properly oriented element.