RAMA UNIVERSITY

www.ramauniversity.ac.in

FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 10

Topic:10. Principal strain and Maximum Shear strain

Instructor:

Aditya Veer Gautam

Principal strain

- ✓ The maximum and minimum normal strain, called the **principal strain**, can be found from the transformation equation for the normal strain ϵ_x
- ✓ By taking the derivative of ϵ_x with respect to θ and setting it equal to zero, we obtain an equation from which we can find the values of θ at which ϵ_x is a maximum or a minimum. The equation for the derivative is

$$\frac{d\epsilon_{x_1}}{d\theta} = -(\epsilon_x - \epsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta = 0$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

- ✓ θ_p is defines the orientation of the **principal planes**, that is, the planes on which the principal strain act

Principal strain

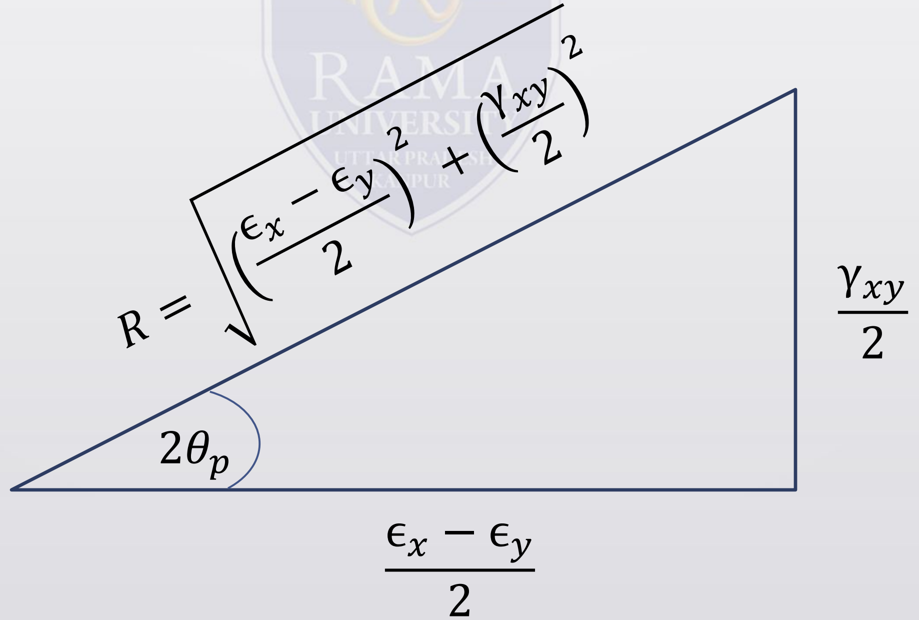
- ✓ Two values of the angle $2\theta_p$ in the range from 0 to 360° can be obtained.
- ✓ These values differ by 180°, with one value between 0 and 180° and the other between 180° and 360°. Therefore, the angle up has two values that differ by 90°, one value between 0 and 90° and the other between 90° and 180°.
- ✓ The two values of up are known as the principal angles. For one of these angles, the normal strain ϵ_{x1} is a maximum principal strain; for the other, it is a minimum principal strain.
- ✓ Because the principal angles differ by 90°, we see that the principal strain occur on mutually perpendicular planes.

Principal strain

- ✓ The principal strain can be calculated by substituting each of the θ_p two values of up into the strain-transformation equation

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

- ✓ From above equation we can constructed the following right angled triangle



Principal strain and Shear strain on principle planes

$$\epsilon_{x_1} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

- ✓ Using the triangle of previous slide we can rearrange the above equation to get principle strain

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

- ✓ If we substitute θ_p in shear strain transformation equation then we get

$$\gamma_{xy} = 0$$

The shear strain are zero on the principal planes

Maximum Shear strain

- ✓ By taking the derivative of γ_x with respect to θ and setting it equal to zero, we obtain an equation from which we can find the values of θ at which γ_x is a maximum positive and negative. The equation for the derivative is

$$\frac{d\gamma_{xy}}{d\theta} = -(\epsilon_x - \epsilon_y)\cos 2\theta - \gamma_{xy}\sin 2\theta = 0$$

$$\tan 2\theta_s = -\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}}$$

- ✓ On comparing with $\tan 2\theta_p$ we can drive the following relations

$$2\theta_s - 2\theta_p = \pm 90^0$$

The planes of maximum shear strain occur at 45° to the principal planes.

$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{max} = \frac{\epsilon_1 - \epsilon_2}{2}$$

Maximum Shear strain

the maximum shear strain is equal to one-half the difference of the principal strain

✓ The planes of maximum shear strain also contain normal strain. The normal strain acting on the planes of maximum positive shear strain can be determined by substituting the expressions for the angle

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = \frac{\epsilon_1 + \epsilon_2}{2}$$

the principal strains and principal stresses occur in the same directions.

Questions

7.7-7 The strains for an element of material in *plane strain* (see figure) are as follows: $\varepsilon_x = 480 \times 10^{-6}$, $\varepsilon_y = 140 \times 10^{-6}$, and $\gamma_{xy} = -350 \times 10^{-6}$.

Determine the principal strains and maximum shear strains, and show these strains on sketches of properly oriented elements.

7.7-8 Solve the preceding problem for the following strains: $\varepsilon_x = 120 \times 10^{-6}$, $\varepsilon_y = -450 \times 10^{-6}$, and $\gamma_{xy} = -360 \times 10^{-6}$.

Questions

7.7-9 An element of material in *plane strain* (see figure) is subjected to strains $\varepsilon_x = 480 \times 10^{-6}$, $\varepsilon_y = 70 \times 10^{-6}$, and $\gamma_{xy} = 420 \times 10^{-6}$.

Determine the following quantities: (a) the strains for an element oriented at an angle $\theta = 75°$, (b) the principal strains, and (c) the maximum shear strains. Show the results on sketches of properly oriented elements.

7.7-10 Solve the preceding problem for the following data: $\varepsilon_x = -1120 \times 10^{-6}$, $\varepsilon_y = -430 \times 10^{-6}$, $\gamma_{xy} = 780 \times 10^{-6}$, and $\theta = 45°$.

Questions

7.7-11 A brass plate with a modulus of elasticity $E = 110$ GPa and Poisson's ratio $\nu = 0.34$ is loaded in *biaxial stress* by normal stresses σ_x and σ_y (see figure). A strain gage is bonded to the plate at an angle $\phi = 35°$.

If the stress σ_x is 74 MPa and the strain measured by the gage is $\varepsilon = 390 \times 10^{-6}$, what is the maximum in-plane shear stress $(\tau_{max})_{xy}$ and shear strain $(\gamma_{max})_{xy}$? What is the maximum shear strain $(\gamma_{max})_{xz}$ in the xz plane? What is the maximum shear strain $(\gamma_{max})_{yz}$ in the yz plane?