FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 14

Topic: Mohr's Circle for Plane strain, Strain rosette

Instructor:

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Strain Transformation Equations

$$\epsilon_{x_1} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \gamma_{xy}\sin 2\theta$$

$$\epsilon_{y_1} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \gamma_{xy}\sin 2\theta$$

$$\gamma_{x_1 y_1} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \gamma_{xy}\cos 2\theta$$

Plane strain

- ✓ When the material is in plane strain in the **xy** plane, only the **x** and **y** faces of the element are subjected to strain, and all strain act parallel to the **x** and **y** axes
- ✓ Only $\epsilon_x, \epsilon_y, \gamma_{xy}$ $(\epsilon_z = 0, \gamma_{zy} = \gamma_{xz} = 0)$ are acting this condition is very common because it exists at the surface of any strained body, except at points where external loads act on the surface.

Equations of Mohr's Circle

- ✓ Parametric form of strain transformation equations

$$\epsilon_{x_1} - \frac{\epsilon_x + \epsilon_y}{2} = \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{x_1y_1}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

- ✓ Eliminate 2θ from both equation , square both side of each equation and add

$$\left(\epsilon_{x_1} - \frac{\epsilon_x + \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{x_1y_1}}{2}\right)^2 = \left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2$$

Equations of Mohr's Circle

- ✓ Substitute

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2}, R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

- ✓ So final equation is $\left(\epsilon_{x1} - \epsilon_{avg}\right)^2 + \gamma_{x_1 y_1}^2 = R^2$
- ✓ Comparing with equation of circle $(x - x_1)^2 + (y - y_1)^2 = r^2$

$$x_1 = \epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2}$$

$$r = R$$

$$y_1 = 0$$

$$x = \epsilon_1$$

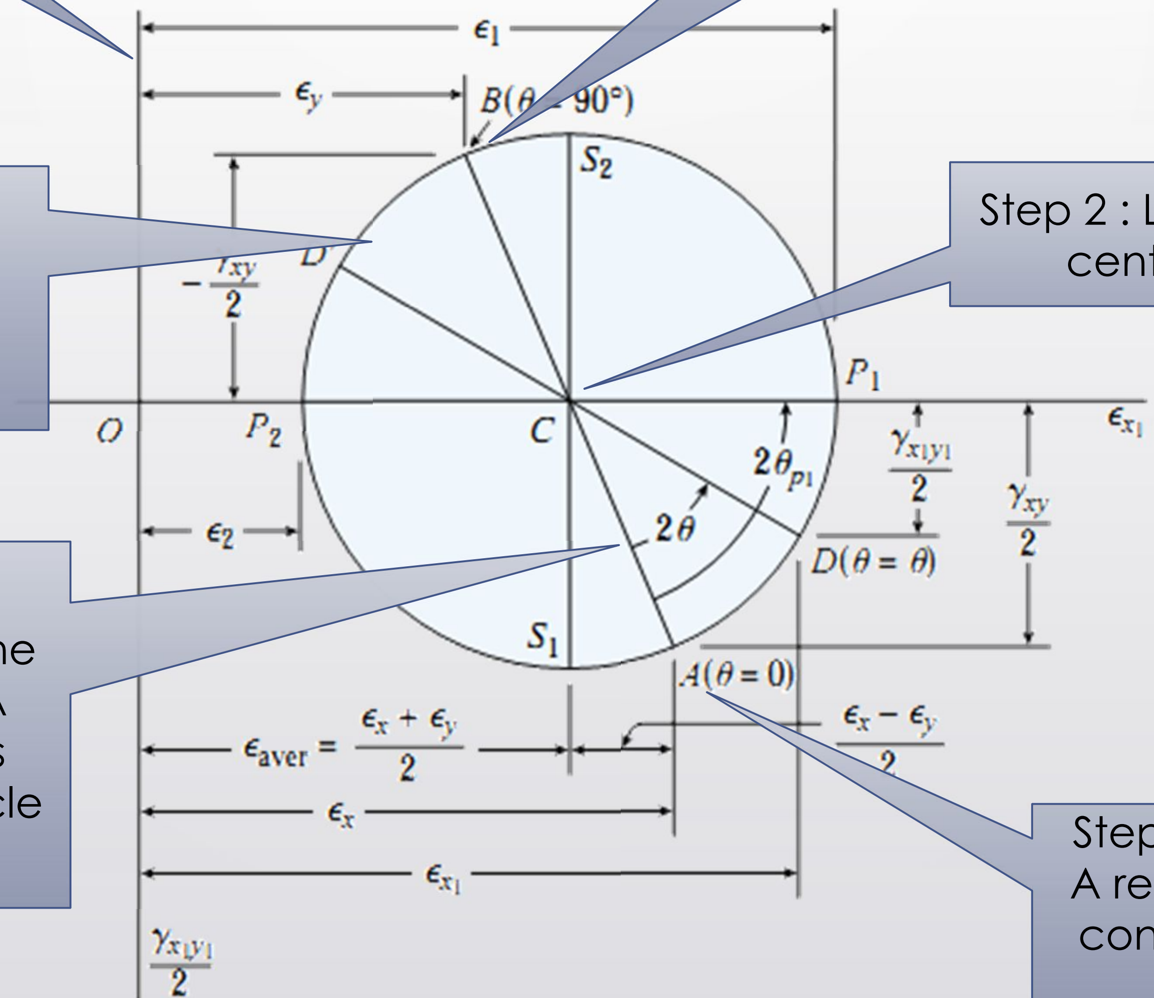
$$y = \frac{\gamma_{x_1 y_1}}{2}$$

Step 1 : Draw coordinate axes

Step 4 : Locate point B representing strain condition on y face

Step 6 : Draw Circle with C centre and AB diameter

Step 2 : Locate centre

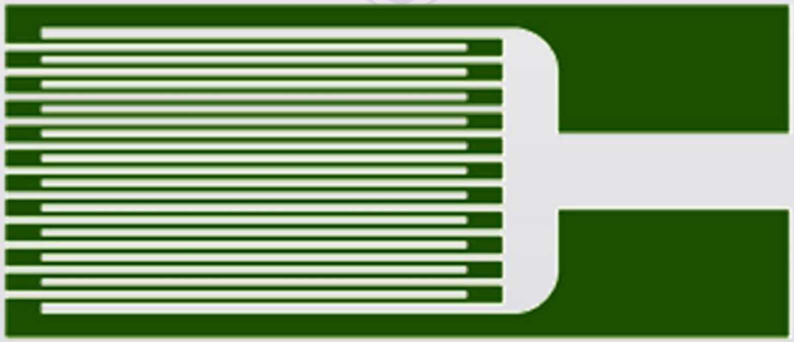


Step 5 : Draw line joining point A and B which is diameter of circle

Step 3: Locate point A representing strain condition on x face

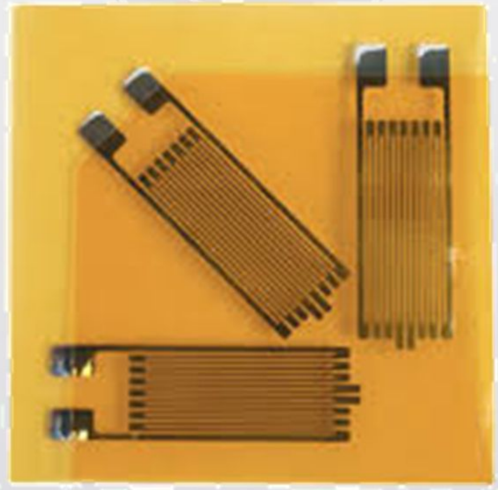
Strain Measurement

- ✓ An electrical-resistance strain gage is a device for measuring normal strains on the surface of a stressed object.
- ✓ The gages are bonded securely to the surface of the object so that they change in length in proportion to the strains in the object itself.
- ✓ Each gage consists of a fine metal grid that is stretched or shortened when the object is strained at the point where the gage is attached



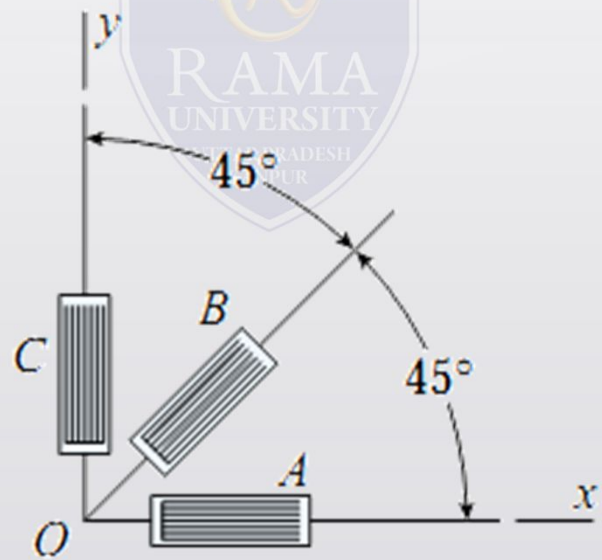
Strain Measurement: Strain Rosette

- ✓ Since each gage measures the normal strain in only one direction, and since the directions of the principal stresses are usually unknown, it is necessary to use three gages in combination, with each gage measuring the strain in a different direction. From three such measurements, it is possible to calculate the strains in any direction



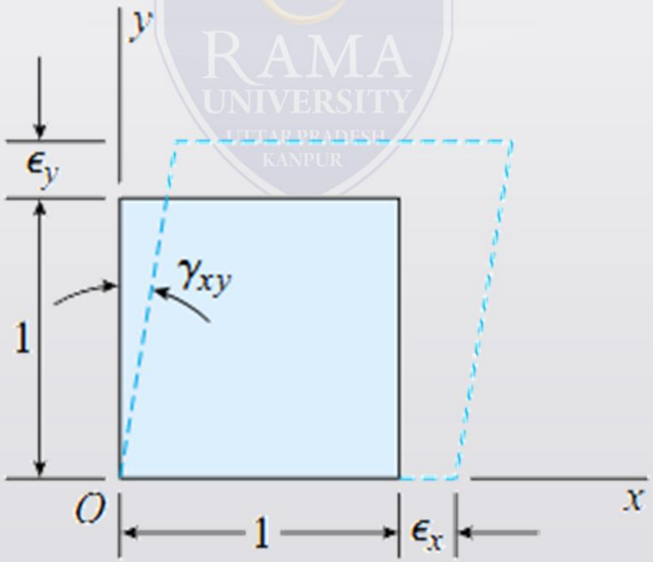
Questions

Q 1 During a test of an airplane wing, the strain gage readings from a 45° rosette (see figure) are as follows: gage A, 520 X 10^{-6}; gage B, 360 X 10^{-6}; and gage C -80 X 10^{-6} Determine the principal strains and maximum shear strains, and show them on sketches of properly oriented elements.



Question

Q2 An element of material subjected to plane strain (see figure) has strains as follows: ϵ_x= 220 X10^{-6} , ϵ_y =480 X10^{-6}, and γ_{xy}= 180 X10^{-6} . Calculate the strains for an element oriented at an angle θ = 50° using Mohr Circle and show these strains on a sketch of a properly oriented element

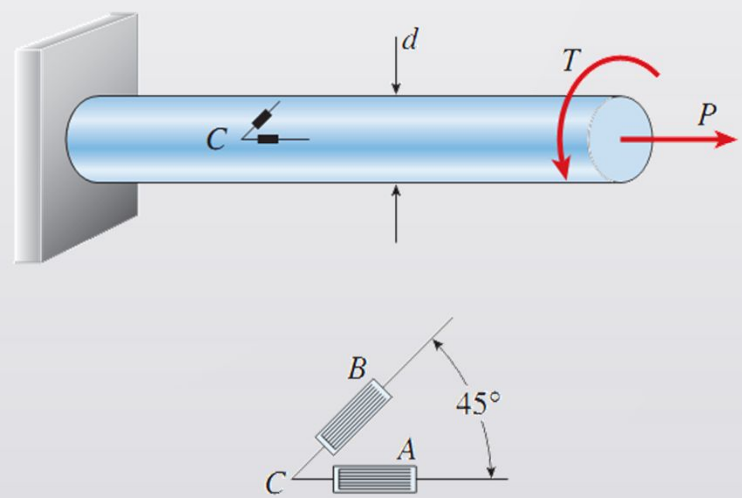


Question

7.7-17 A solid circular bar with a diameter of $d = 32$ mm is subjected to an axial force P and a torque T (see figure). Strain gages A and B mounted on the surface of the bar give readings $\varepsilon_A = 140 \times 10^{-6}$ and $\varepsilon_B = -60 \times 10^{-6}$. The bar is made of steel having $E = 210$ GPa and $\nu = 0.29$.

(a) Determine the axial force P and the torque T.

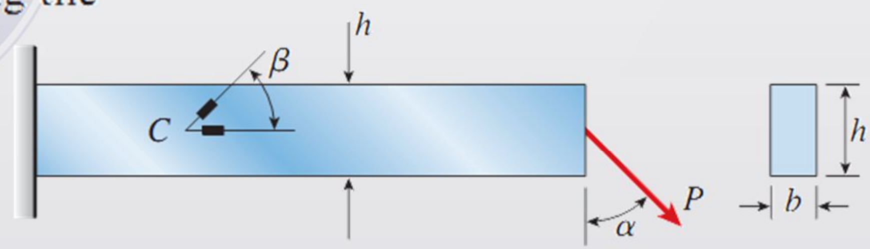
(b) Determine the maximum shear strain γ_{max} and the maximum shear stress τ_{max} in the bar.

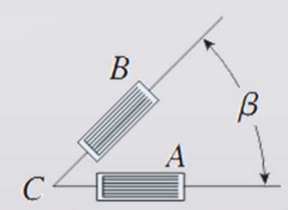


Question

7.7-18 A cantilever beam of rectangular cross section (width $b = 20$ mm, height $h = 175$ mm) is loaded by a force P that acts at the mid-height of the beam and is inclined at an angle α to the vertical (see figure). Two strain gages are placed at point C, which also is at the mid-height of the beam. Gage A measures the strain in the horizontal direction, and gage B measures the strain at an angle $\beta = 60°$ to the horizontal. The measured strains are $\varepsilon_A = 145 \times 10^{-6}$ and $\varepsilon_B = -165 \times 10^{-6}$.

Determine the force P and the angle α, assuming the material is steel with $E = 200$ GPa and $\nu = 1/3$.





Question

7.7-21 On the surface of a structural component in a space vehicle, the strains are monitored by means of three strain gages arranged as shown in the figure. During a certain maneuver, the following strains were recorded: $\varepsilon_a = 1100 \times 10^{-6}$, $\varepsilon_b = 200 \times 10^{-6}$, and $\varepsilon_c = 200 \times 10^{-6}$.

Determine the principal strains and principal stresses in the material, which is a magnesium alloy for which $E = 41$ GPa and $\nu = 0.35$. (Show the principal strains and principal stresses on sketches of properly oriented elements.)