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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 15

Topic: Thin Pressure Vessels: Spherical

Instructor:

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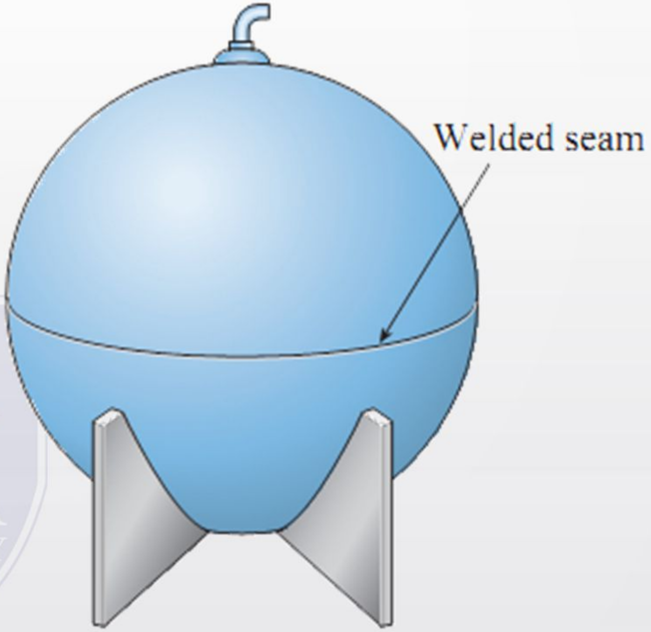
Pressure Vessels

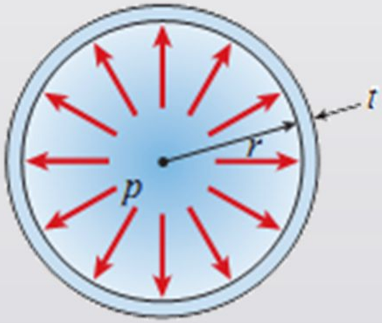
- ✓ Pressure vessels are closed structures containing liquids or gases under pressure. These are special case of **plane stress** i.e. $\sigma_{z\ or\ 3} = 0$
- ✓ When pressure vessels have ***walls*** that are ***thin*** in comparison to their overall dimensions, they are included within a more general category known as ***shell structures***.
- ✓ Pressure vessels are considered to be thin-walled when the ratio of ***radius r*** to wall ***thickness t*** is greater than 10

$$\frac{d}{t} > 20 \quad or \quad \frac{t}{d} < \frac{1}{20}$$

Spherical Pressure Vessels

Thin-walled spherical pressure vessel used for storage of propane in this oil refinery





Spherical Pressure Vessels

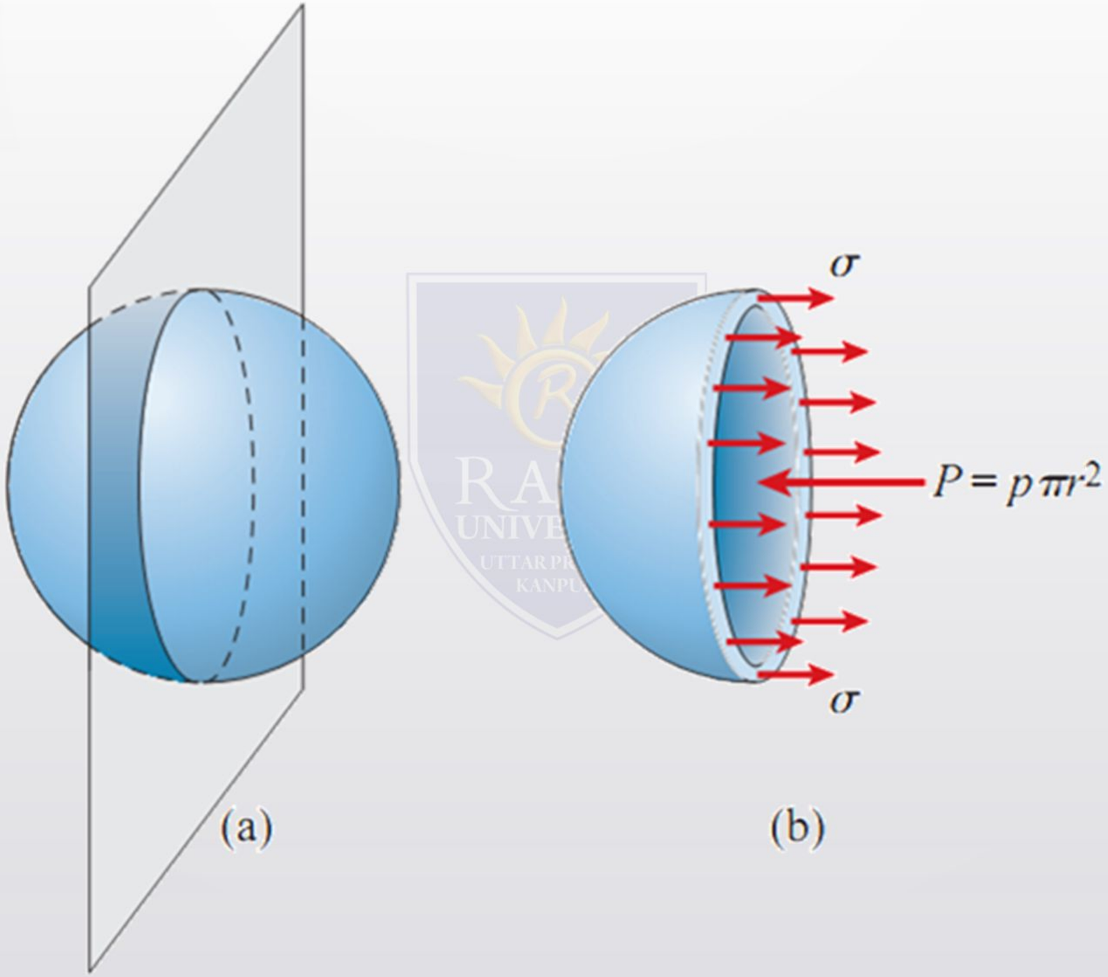
- ✓To determine the stresses in a spherical vessel, let us cut through the sphere on a vertical diametral plane and isolate half of the shell and its fluid contents as a single free body
- ✓Pressure acts horizontally against the plane circular area of fluid remaining inside the hemisphere. Since the pressure is uniform, the resultant pressure force

$$P = p(\pi r^2)$$

Where **r** is inner radius

p is gauge pressure

Spherical Pressure Vessels



Spherical Pressure Vessels

- ✓ Because of the symmetry of the vessel and its loading the tensile stress σ is uniform around the circumference. Furthermore, since the wall is thin, we can assume with good accuracy that the stress is uniformly distributed across the thickness ***t***. The accuracy of this approximation increases as the shell becomes thinner and decreases as it becomes thicker

- ✓ The resultant of the tensile stresses σ in the wall is a horizontal force equal to the stress σ times the area over which it acts

$$\sigma(2\pi r_m t)$$

$$r_m = r + \frac{t}{2}$$

r_m is mean radius

Spherical Pressure Vessels

- ✓ Equilibrium of forces in the horizontal direction
- ✓ Force which has a tendency to split the shell = resisting force due to stress

$$p \times \pi r^2 = \sigma(2\pi r_m t)$$

$$\sigma = \frac{pr^2}{2r_m t}$$

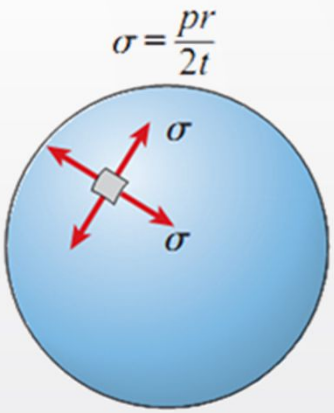
Replacing $r_m = r$

$$\sigma = \frac{pr}{2t} = \frac{pd}{4t}$$

The wall of a pressurized spherical vessel is subjected to uniform tensile stresses σ in all directions.

Stresses that act tangentially to the curved surface of a shell Hoop or Circumferential or membrane stress

Spherical Pressure Vessels: Stresses at the Outer Surface



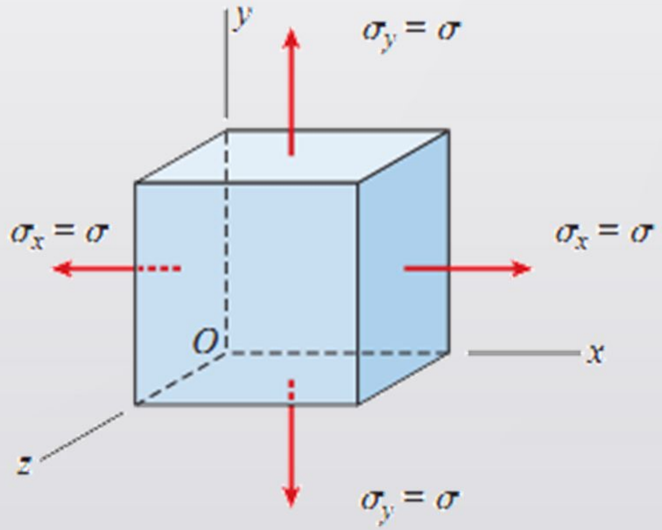
- ✓ The outer surface of a spherical pressure vessel is usually free of any loads. Therefore, the element shown below is in biaxial stress.
- ✓ The x and y axes are tangential to the surface of the sphere, and the z axis is perpendicular to the surface.

$$\sigma_x = \sigma_y = \sigma$$

- ✓ If we analyse the element of by using the transformation equations for plane stress

$$\sigma_{x_1} = \sigma \; , \tau_{x_1 y_1} = 0$$

$$\sigma_1 = \sigma_2 = \frac{pd}{4t} = \frac{pr}{2t}$$



Spherical Pressure Vessels: Stresses at the Outer Surface

- ✓ The normal stresses remain constant and there are no shear stresses. Every plane is a principal plane and every direction is a principal direction
- ✓ Maximum in-plane shear stress

$$\tau_{max} = 0$$

- ✓ Maximum out-of-plane shear stress

$$\tau_{max} = \frac{\sigma}{2}$$

Spherical Pressure Vessels: Stresses at the Inner Surface

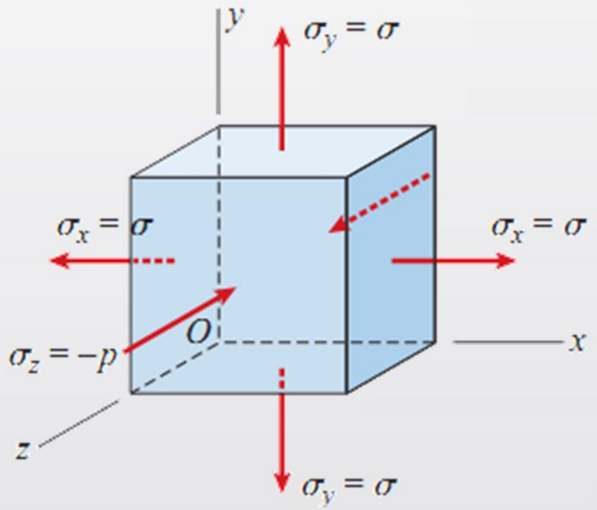
$$\sigma_1 = \sigma_2 = \frac{pd}{4t} = \frac{pr}{2t}$$

$$\sigma_3 = -p$$

✓ Maximum out-of-plane shear stress

$$\tau_{max} = \frac{\sigma + p}{2} = \frac{pr}{2t} + \frac{\mathrm{p}}{2} = \frac{\mathrm{p}}{2}\left(\frac{\mathrm{r}}{2\mathrm{t}} + 1\right)$$

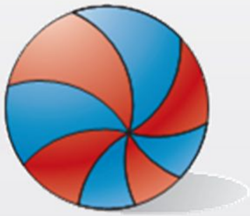
✓ When the vessel is thin-walled and the ratio r/t is large, we can disre-gard the number 1 in comparison with the term r/2t.



Questions

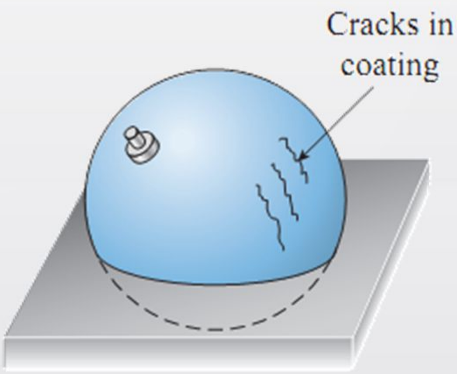
7.2-4 A rubber ball (see figure) is inflated to a pressure of 60 kPa. At that pressure the diameter of the ball is 230 mm and the wall thickness is 1.2 mm. The rubber has modulus of elasticity $E = 3.5$ MPa and Poisson's ratio $\nu = 0.45$.

Determine the maximum stress and strain in the ball.



7.2-6 A spherical steel pressure vessel (diameter 480 mm, thickness 8.0 mm) is coated with brittle lacquer that cracks when the strain reaches 150×10^{-6} (see figure).

What internal pressure p will cause the lacquer to develop cracks? (Assume $E = 205$ GPa and $\nu = 0.30$.)

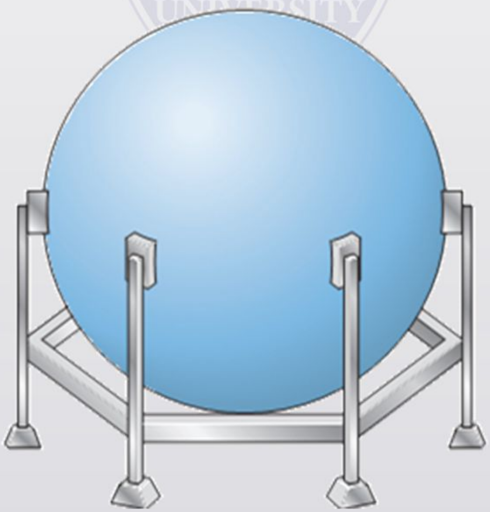


Questions

8.2-1 A large spherical tank (see figure) contains gas at a pressure of 3.5 MPa. The tank is 20 m in diameter and is constructed of high-strength steel having a yield stress in tension of 550 MPa.

(a) Determine the required thickness of the wall of the tank if a factor of safety of 3.5 with respect to yielding is required.

(b) If the tank wall thickness is 100 mm, what is the maximum permissible internal pressure?



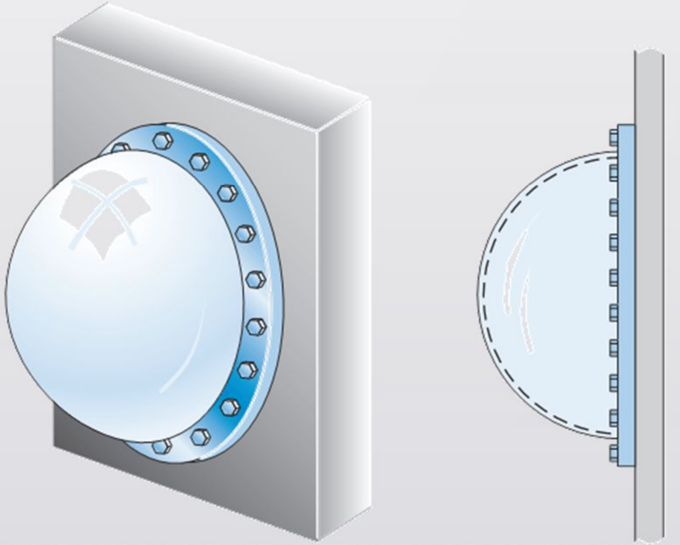
Questions

8.2-3 A hemispherical window (or *viewport*) in a decompression chamber (see figure) is subjected to an internal air pressure of 575 kPa. The port is attached to the wall of the chamber by 14 bolts.

(a) Find the tensile force F in each bolt and the tensile stress σ in the viewport if the radius of the hemisphere is 190 mm and its thickness is 32 mm.

(b) If the yield stress for each of the 14 bolts is 345 MPa and the factor of safety is 3.0, find the required bolt diameter.

(c) If the stress in the viewport is limited to 1.85 MPa, find the required radius of the hemisphere.



Questions

8.2-7 A spherical tank of diameter 1.2 m and wall thickness 50 mm contains compressed air at a pressure of 17 MPa. The tank is constructed of two hemispheres joined by a welded seam (see figure).

(a) What is the tensile load f (N per mm of length of weld) carried by the weld?

(b) What is the maximum shear stress τ_{max} in the wall of the tank?

(c) What is the maximum normal strain ε in the wall? (For steel, assume $E = 210$ GPa and $\nu = 0.29$.)