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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 15

Topic: Thin Pressure Vessels: Cylindrical

Instructor:

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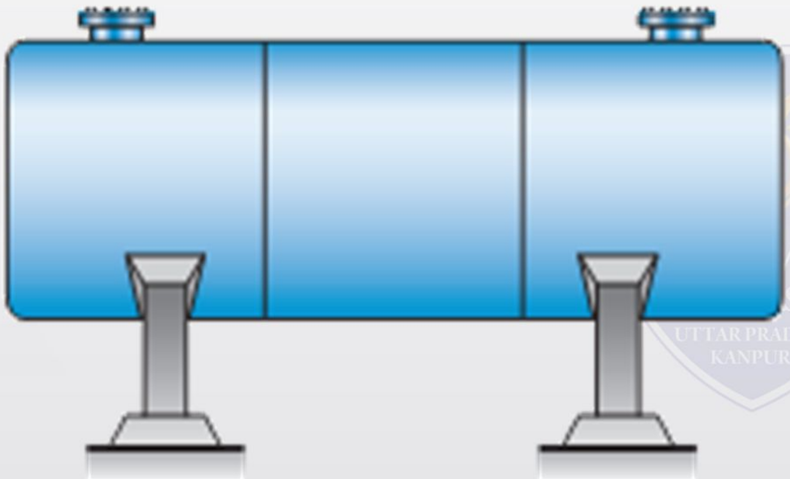
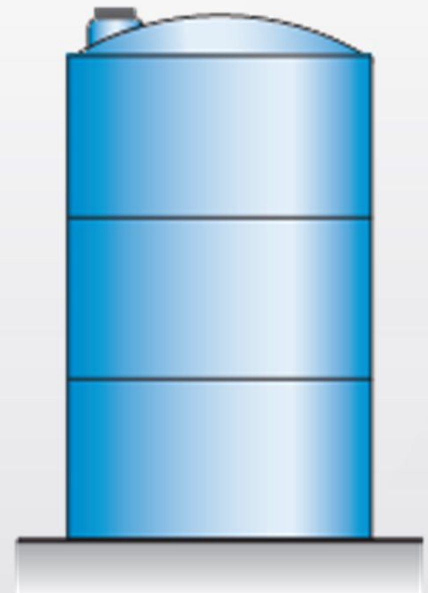
Pressure Vessels: Recap

- ✓ Pressure vessels are closed structures containing liquids or gases under pressure. These are special case of **plane stress** i.e. $\sigma_{z\ or\ 3} = 0$
- ✓ When pressure vessels have ***walls*** that are ***thin*** in comparison to their overall dimensions, they are included within a more general category known as ***shell structures***.
- ✓ Pressure vessels are considered to be thin-walled when the ratio of ***radius r*** to wall ***thickness t*** is greater than 10

$$\frac{d}{t} > 20 \quad or \quad \frac{t}{d} < \frac{1}{20}$$

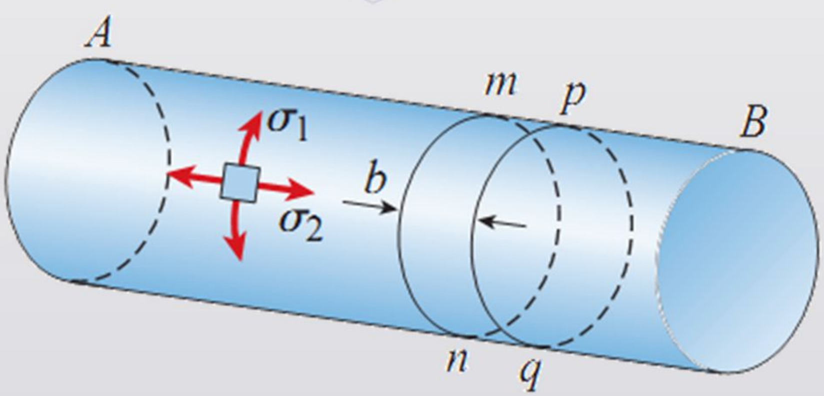
Cylindrical Pressure Vessels





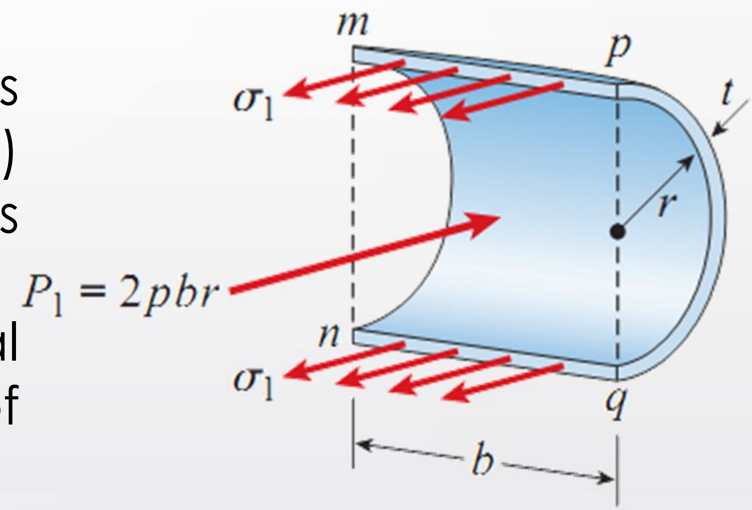
Cylindrical Pressure Vessels

- A stress element with its faces parallel and perpendicular to the axis of the tank is shown on the wall of the tank. The normal stresses σ_1 *and* σ_2 acting on the side faces of this element are the membrane stresses in the wall.
- No shear stresses act on these faces because of the symmetry of the vessel and its loading. Therefore, the stresses σ_1 *and* σ_2 are principal stresses
- Because of their directions, the stress σ_1 is called the circumferential stress or the hoop stress, and the stress σ_2 is called the longitudinal stress or the axial stress.
- Each of these stresses can be calculated from equilibrium by using appropriate free-body diagrams.



Cylindrical Pressure Vessels: Circumferential Stress

- ✓ To determine the circumferential stress we make two cuts (mn and pq) perpendicular to the longitudinal axis and distance b apart.
- ✓ Then we make a third cut in a vertical plane through the longitudinal axis of the tank, resulting in the free body



- ✓ This free body consists not only of the half-circular piece of the tank but also of the fluid contained within the cuts. Acting on the longitudinal cut (plane mpqn) are the circumferential stresses σ_1 and the internal pressure p.

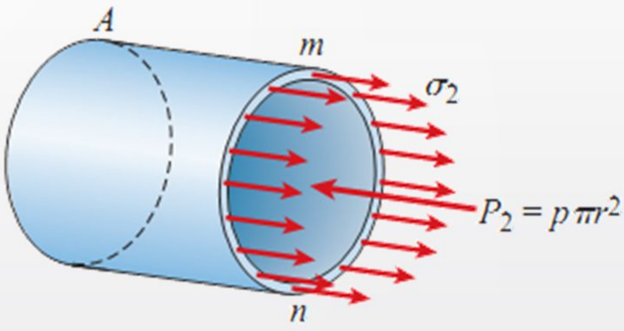
Force due to fluid pressure = Force due to Hoop stress

$$p \times (2rb) = \sigma_1 \times (2bt)$$

$$\sigma_1 = \frac{pr}{t}$$

Cylindrical Pressure Vessels: Longitudinal Stress

- ✓ The longitudinal stress is obtained from the equilibrium of a free body of the part of the vessel to the left of cross section mn The resultant of the tensile stresses σ in the wall is a horizontal force equal to the stress σ times the area over which it acts



Force due to fluid pressure = Force due to longitudinal stress

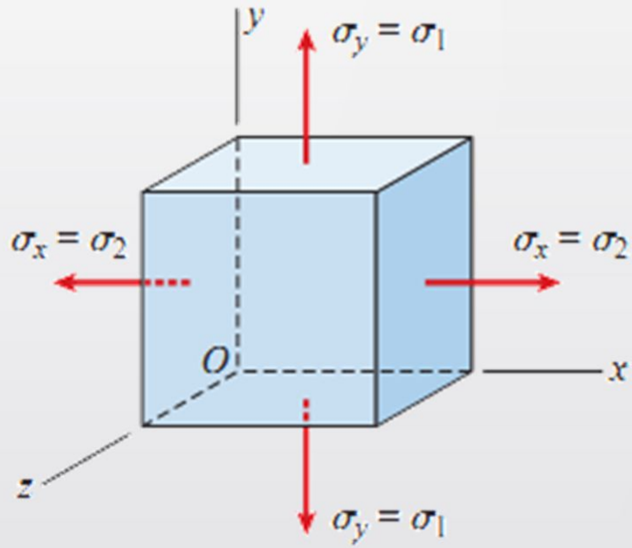
$$p \times \pi r^2 = \sigma_2 \times (2\pi r t)$$

$$\sigma_2 = \frac{pr}{2t}$$

Cylindrical Pressure Vessels: Stresses at the Outer Surface

- ✓ The outer surface of a Cylindrical pressure vessel is usually free of any loads. Therefore, the element shown below is in biaxial stress.
- ✓ The x and y axes are tangential to the surface of the sphere, and the z axis is perpendicular to the surface.

$$\sigma_x = \sigma_2$$
$$\sigma_y = \sigma_1$$



- ✓ If we analyse the element of by using the transformation equations for maximum in-plane shear stress

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{pr}{4t}$$

- ✓ Maximum out-of-plane shear stress

$$\tau_{max} = \frac{\sigma_1}{2} = \frac{pr}{2t}$$

Cylindrical Pressure Vessels: Stresses at the Inner Surface

$$\sigma_1 = \frac{pr}{t}$$

$$\sigma_2 = \frac{pr}{2t}$$

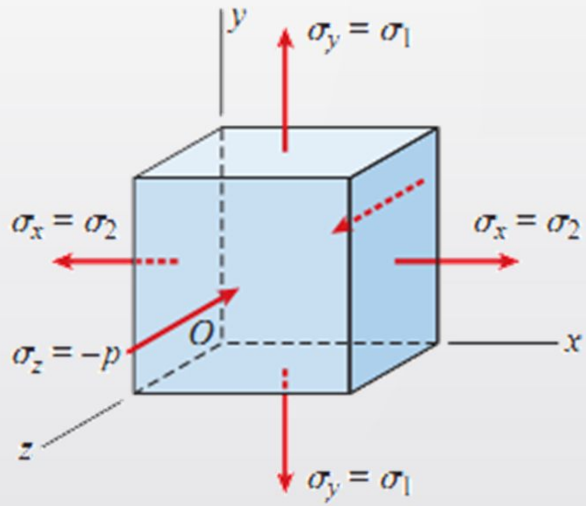
$$\sigma_3 = -p$$

- Maximum shear stress

$$(\tau_{max})_x = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma + p}{2} = \frac{pr}{2t} + \frac{\mathrm{p}}{2}$$

$$(\tau_{max})_y = \frac{\sigma_2 - \sigma_3}{2} = \frac{\sigma + p}{2} = \frac{pr}{4t} + \frac{\mathrm{p}}{2}$$

$$(\tau_{max})_x = \frac{\sigma_1 - \sigma_2}{2} = \frac{pr}{4t}$$

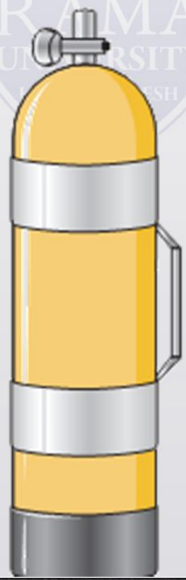


Questions

8.3-1 A scuba tank (see figure) is being designed for an internal pressure of 12 MPa with a factor of safety of 2.0 with respect to yielding. The yield stress of the steel is 300 MPa in tension and 140 MPa in shear.

(a) If the diameter of the tank is 150 mm, what is the minimum required wall thickness?

(b) If the wall thickness is 6 mm, what is the maximum acceptable internal pressure?

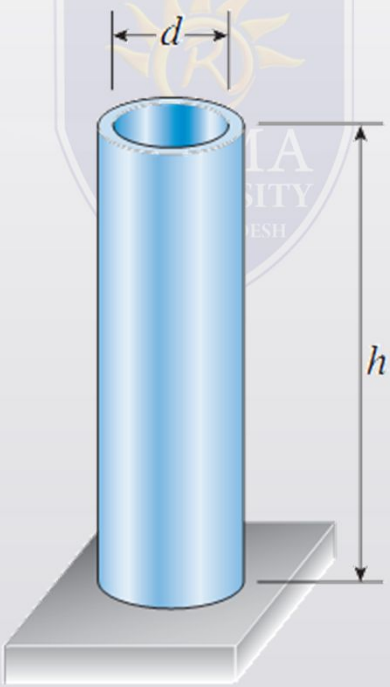


Questions

8.3-2 A tall standpipe with an open top (see figure) has diameter $d = 2.2$ m and wall thickness $t = 20$ mm.

(a) What height h of water will produce a circumferential stress of 12 MPa in the wall of the standpipe?

(b) What is the axial stress in the wall of the tank due to the water pressure?



Questions

8.3-5 A strain gage is installed in the longitudinal direction on the surface of an aluminum beverage can (see figure). The radius-to-thickness ratio of the can is 200. When the lid of the can is popped open, the strain changes by $\varepsilon_0 = 170 \times 10^{-6}$.

(a) What was the internal pressure p in the can? (Assume $E = 70$ GPa and $\nu = 0.33$.)

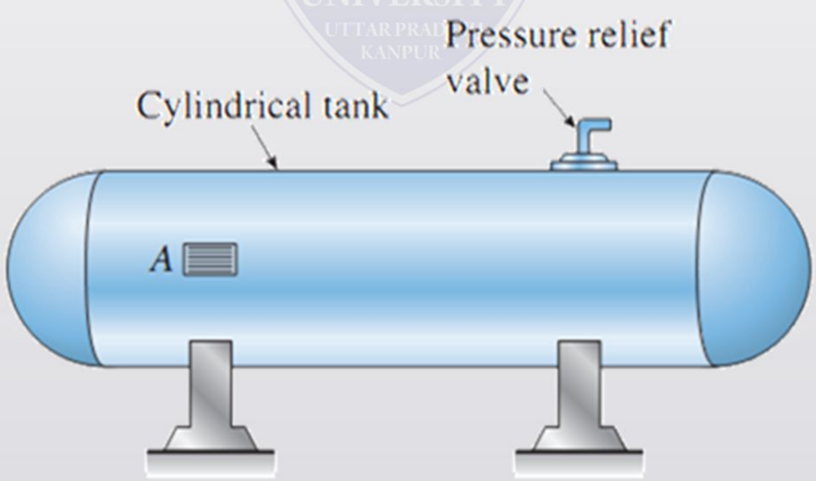
(b) What is the change in strain in the radial direction when the lid is opened?

Questions

8.3-6 A circular cylindrical steel tank (see figure) contains a volatile fuel under pressure. A strain gage at point A records the longitudinal strain in the tank and transmits this information to a control room. The ultimate shear stress in the wall of the tank is 98 MPa, and a factor of safety of 2.8 is required.

(a) At what value of the strain should the operators take action to reduce the pressure in the tank? (Data for the steel are as follows: modulus of elasticity $E = 210$ GPa and Poisson's ratio $\nu = 0.30$.)

(b) What is the associated strain in the radial direction?



Questions

8.3-12 A pressurized steel tank is constructed with a helical weld that makes an angle $\alpha = 55°$ with the longitudinal axis (see figure). The tank has radius $r = 0.6$ m, wall thickness $t = 18$ mm, and internal pressure $p = 2.8$ MPa. Also, the steel has modulus of elasticity $E = 200$ GPa and Poisson's ratio $\nu = 0.30$.

Determine the following quantities for the cylindrical part of the tank.

(a) The circumferential and longitudinal stresses.

(b) The maximum in-plane and out-of-plane shear stresses.

(c) The circumferential and longitudinal strains.

(d) The normal and shear stresses acting on planes parallel and perpendicular to the weld (show these stresses on a properly oriented stress element).