



# **FACULTY OF ENGINEERING AND TECHNOLOGY**

**Department of Mechanical Engineering**



# MEPS102:Strength of Material

## Lecture 18

**Topic:Relationships between loads, shear forces, and bending moments**

Instructor:

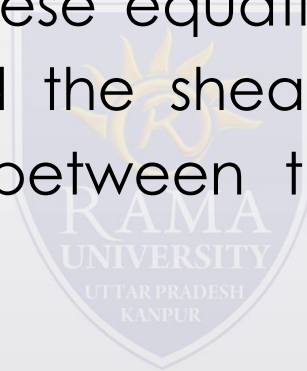
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# Load Sign Convention

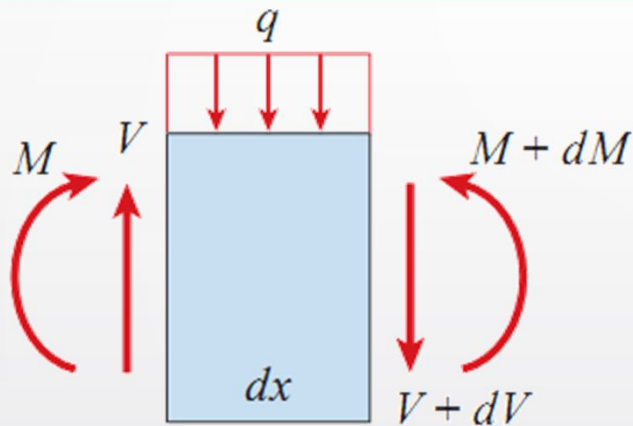
- ✓ Distributed loads and concentrated loads are positive when they act downward on the beam and negative when they act upward.
- ✓ A couple acting as a load on a beam is positive when it is counterclockwise and negative when it is clockwise.
- ✓ In general, the shear forces and bending moments vary along the axis of the beam. Therefore, their values on the right-hand face of the element may be different from their values on the left-hand face.

# Relationships between loads, shear forces, and bending moments

- ✓ For each type of loading we can write two equations of equilibrium for the element—one equation for equilibrium of forces in the vertical direction and one for equilibrium of moments. The first of these equations gives the relationship between the load and the shear force, and the second gives the relationship between the shear force and the bending moment



# Distributed Loads: Shear Force



$$\sum F_{\text{vert}} = 0 \quad V - q \, dx - (V + dV) = 0$$

- ✓ Shear Force. Equilibrium of forces in the vertical direction (upward forces are positive) gives

$$\frac{dV}{dx} = -q$$

- ✓ If the sign convention for the distributed load is reversed, so that  $q$  is positive upward instead of downward, then the minus sign is omitted in the preceding equation.

# Distributed Loads: Shear Force

- ✓ If there is no distributed load on a segment of the beam shear stress is either zero or the shear force is constant in that part of the beam
- ✓ If the distributed load is uniform along part of the beam the shear force varies linearly in that part of the beam

$$V_B - V_A = \int_A^B dV = - \int_A^B q dx$$

*= -(area of loading diagram between A and B)*

# Distributed Loads: Bending Moment

- ✓ Summing moments about an axis at the left-hand side of the element (the axis is perpendicular to the plane of the figure), and taking counterclockwise moments as positive, we obtain

$$\sum M = 0 \quad -M - q dx \left( \frac{dx}{2} \right) - (V + dV) dx + M + dM = 0$$

- ✓ Discarding products of differentials (because they are negligible compared to the other terms), we obtain the following relationship

$$\frac{dM}{dx} = V$$

- ✓ if the shear force is zero in a region of the beam, then the bending moment is constant in that same region.

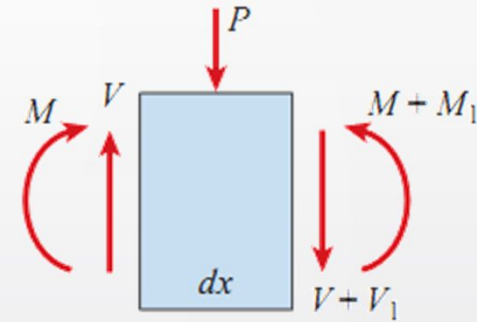
$$\begin{aligned} M_B - M_A &= \int_A^B dM = \int_A^B V dx \\ &= (\text{area of shear force diagram between A and B}) \end{aligned}$$

# Concentrated Loads

- ✓ From equilibrium of forces in the vertical direction, we get

$$V - P - (V + V_1) = 0 \text{ or } V_1 = -P$$

- ✓ This result means that an abrupt change in the shear force occurs at any point where a concentrated load acts



- ✓ From equilibrium of moments about the left-hand face of the element

$$-M - P\left(\frac{dx}{2}\right) - (V + V_1)dx + M + M_1 = 0$$

$$M_1 = P\left(\frac{dx}{2}\right) + Vdx + V_1dx$$

1. The bending moment does not change as we pass through the point of application of a concentrated load.
2. At the point of application of a concentrated load  $P$ , the rate of change  $dM/dx$  of the bending moment decreases abruptly by an amount equal to  $P$ .



# Loads in the Form of Couples

- ✓ From equilibrium of the element in the vertical direction we obtain  $V_1 = 0$ , which shows that the shear force does not change at the point of application of a couple.
- ✓ Equilibrium of moments about the left-hand side of the element gives

$$-M + M_0 - (V + V_1)dx + M + M_1 = 0$$

$$M_1 = -M_0$$

*The bending moment changes abruptly at the point of application of a couple.*

