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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 21

Topic: Pure bending, Strain curvature relation

Instructor:

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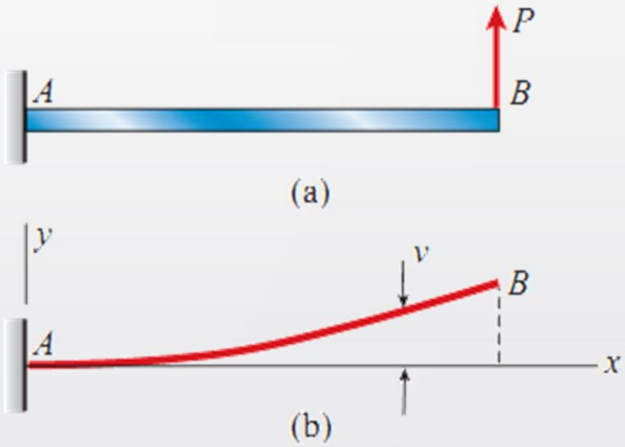
Deflection Curve

- he loads acting on a beam cause the beam to bend (or flex), thereby deforming its axis into a curve. As an example, consider a cantilever beam AB subjected to a load P at the free end. The initially straight axis is bent into a curve, called the **deflection curve of the beam**.
- The beams considered are assumed to be symmetric about the xy plane, which means that the y axis is an axis of symmetry of the cross section.

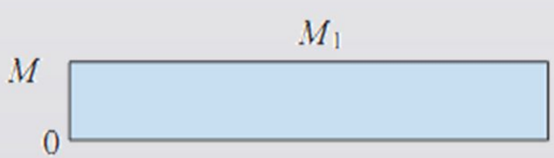


- All loads must act in the xy plane. Hence the bending deflections occur in this same plane, known as the plane of bending
- The deflection of the beam at any point along its axis is the displacement of that point from its original position, measured in the y direction. We denote the deflection by the letter **v** to distinguish it from the coordinate y itself

Pure Bending and Nonuniform Bending

- ✓ **Pure bending** refers to flexure of a beam under a constant bending moment. Therefore, pure bending occurs only in regions of a beam where the shear force is zero
- ✓ Nonuniform bending refers to flexure in the presence of shear forces, which means that the bending moment changes as we move along the axis of the beam.

(a)



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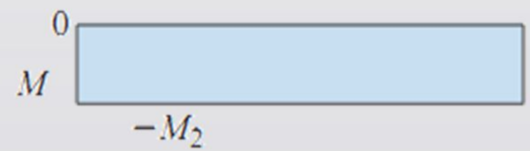
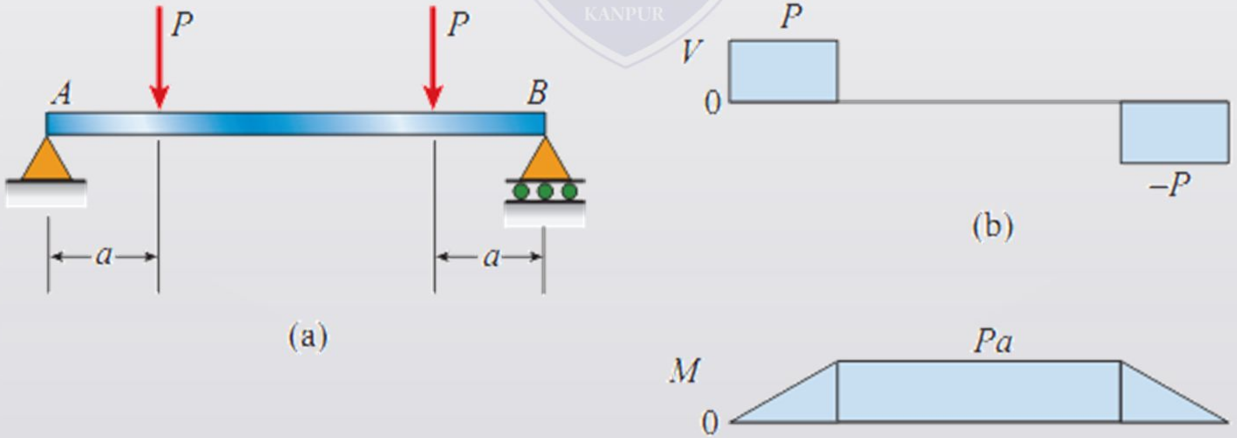


Pure Bending and Nonuniform Bending

✓ The symmetrically loaded simple beam is an example of a beam that is partly in pure bending and partly in nonuniform bending, as seen from the shear-force and bending-moment diagrams. The central region of the beam is in pure bending because the shear force is zero and the bending moment is constant. The parts of the beam near the ends are in nonuniform bending because shear forces are present and the bending moments vary.



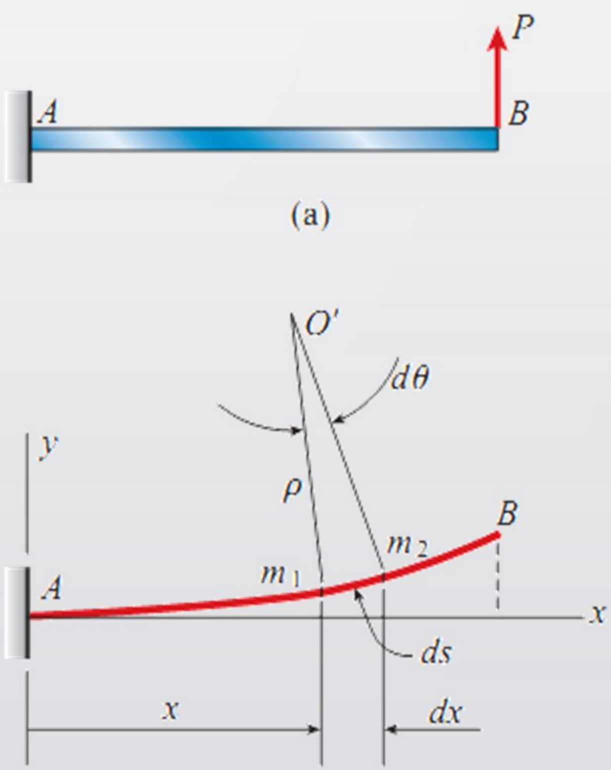
Curvature of a Beam

- ✓ When loads are applied to a beam, its longitudinal axis is deformed into a curve. The resulting strains and stresses in the beam are directly related to the curvature of the deflection curve.

The distance m1O’ from the curve to the center of curvature is called the radius of curvature ρ , and the curvature κ (Greek letter kappa) is defined as $\boldsymbol{\kappa} = {}^{1}/_{\rho}$

- Curvature is a measure of how sharply a beam is bent
- If the load on a beam is small, the beam will be nearly straight, the radius of curvature will be very large, and the curvature will be very small and vice versa.
- Under these special conditions of small deflections, the equation for the curvature becomes

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{ds} = \frac{d\theta}{dx}$$

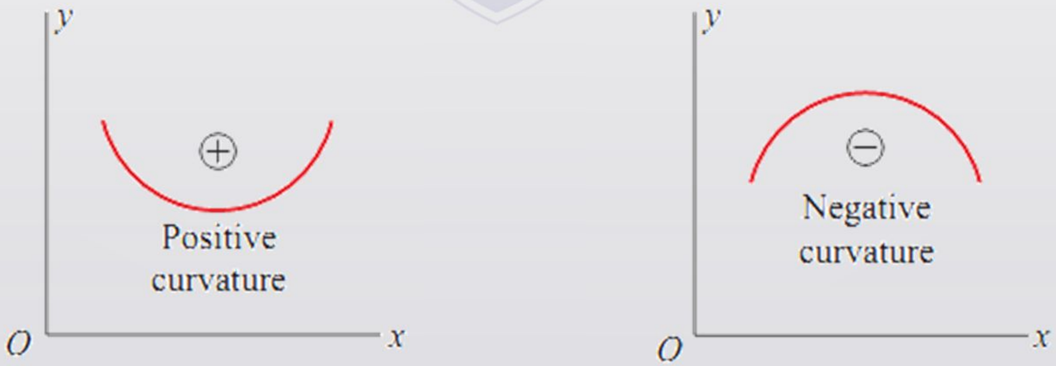


Curvature of a Beam

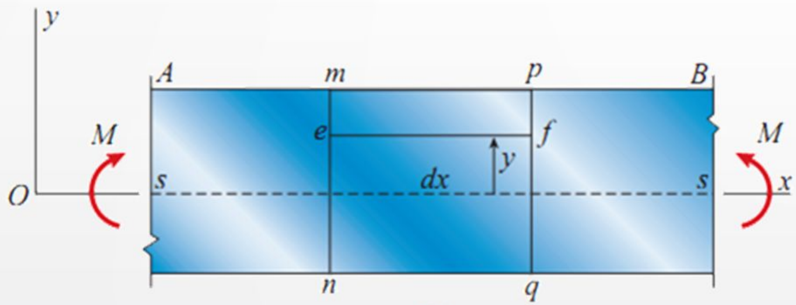
- ✓ Both the curvature and the radius of curvature are functions of the distance x measured along the x axis. It follows that the position O' of the center of curvature also depends upon the distance x.
- ✓ The curvature at a particular point on the axis of a beam depends upon the bending moment at that point and upon the properties of the beam itself (shape of cross section and type of material).
- ✓ If the beam is prismatic and the material is homogeneous, the curvature will vary only with the bending moment.
- ✓ a beam in pure bending will have constant curvature and a beam in nonuniform bending will have varying curvature.

Curvature of a Beam

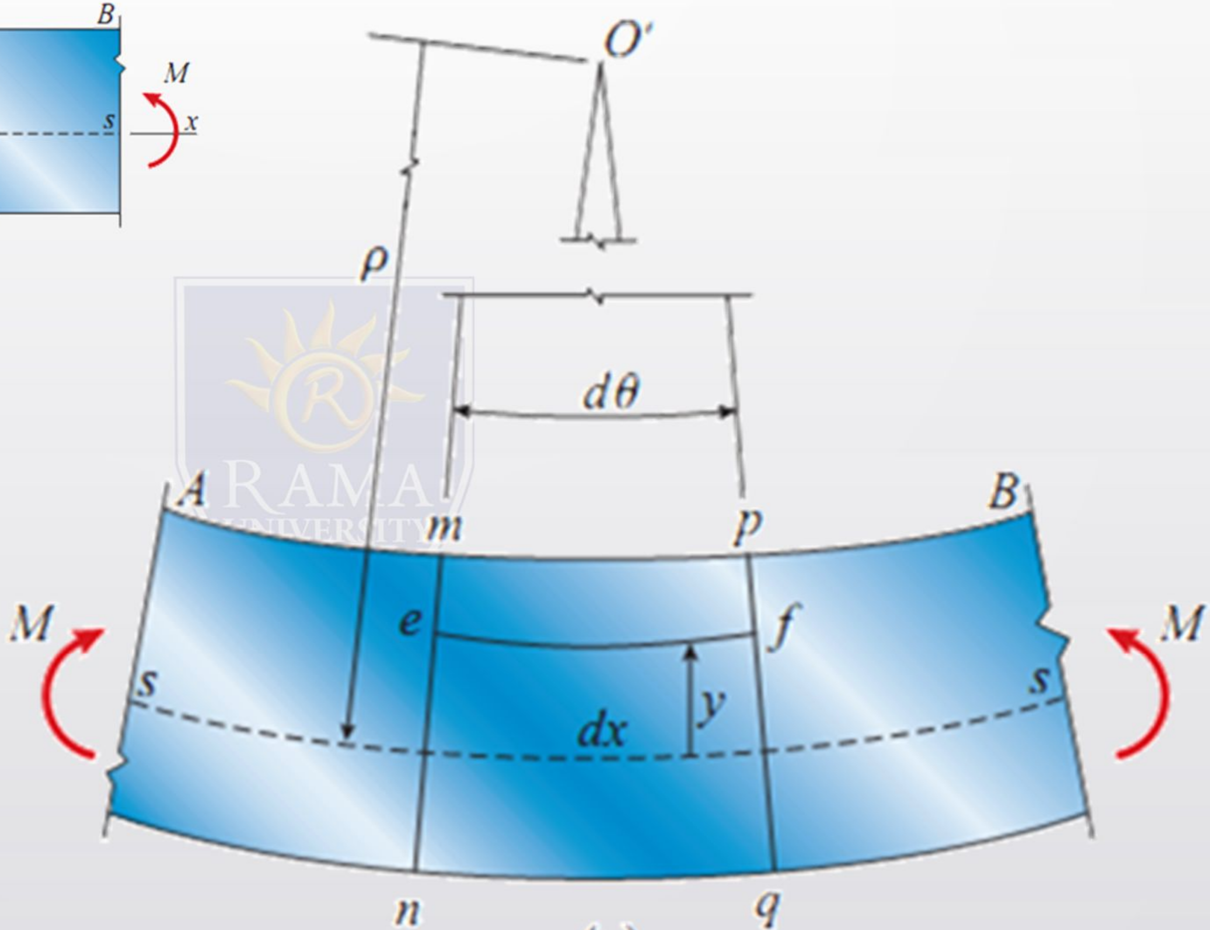
- ✓The sign convention for curvature depends upon the orientation of the coordinate axes. If the x axis is positive to the right and the y axis is positive upward
- ✓The curvature is positive when the beam is bent concave upward and the center of curvature is above the beam.
- ✓Conversely, the curvature is negative when the beam is bent concave downward and the center of curvature is below the beam.



Longitudinal Strains in Beams



(a)



(c)

Longitudinal Strains in Beams

- ✓ Under the action of the bending moments, the beam deflects in the ***xy*** plane (the plane of bending) and its longitudinal axis is bent into a circular curve
- ✓ Cross sections of the beam, such as sections ***mn*** and ***pq*** in, ***remain plane*** and normal to the longitudinal axis
- ✓ The basic point is that the symmetry of the beam and its loading means that all elements of the beam (such as element ***mpqn***) must deform in an identical manner, which is possible only if ***cross sections remain plane*** during bending. This conclusion is valid for beams of any material, whether the material is elastic or inelastic, linear or nonlinear

Longitudinal Strains in Beams

- ✓ Because of the bending deformations shown, cross sections ***mn*** and ***pq*** rotate with respect to each other about axes perpendicular to the ***xy-plane***. Longitudinal lines on the ***lower part*** of the beam are ***elongated***, whereas those on the ***upper part are shortened***. Thus, the ***lower part*** of the beam is in ***tension*** and the ***upper part is in compression***.
- ✓ Somewhere between the top and bottom of the beam is a surface in which ***longitudinal lines do not change in length***. This surface, indicated by the dashed line, is called the ***neutral surface of the beam***. Its intersection with any cross-sectional plane is called the ***neutral axis*** of the cross section

Strain curvature relation

- ✓ Let the length of line ***ef*** be **dx** and ***y*** is distance from neutral surface
- ✓ Now length of line ***ef*** after deformation $\mathbf{L_1}$ therefore change in length will be $\mathbf{L_1}$**-dx**

$$L_1 = (\rho - y)\mathrm{d}\theta = \mathrm{d}x - \frac{y}{\rho}\mathrm{d}x \qquad \because \mathrm{d}x = \rho \mathrm{d}\theta$$

$$\epsilon_x = -\frac{y}{\rho} = -\kappa y$$

- ✓ Longitudinal strains in the beam are proportional to the curvature and vary linearly with the distance ***y*** from the neutral surface regardless of the shape of the stress-strain curve of the material i.e. properties of material. When the point under consideration is above the neutral surface, the distance ***y*** is positive.

Strain curvature relation

- ✓ The normal strain in a beam was derived solely from the geometry of the deformed beam—the properties of the material did not enter into the discussion.
- ✓ The longitudinal strains in a beam are ***accompanied by transverse strains*** (that is, normal strains in the ***y*** and ***z*** directions) because of the effects of Poisson's ratio. However, there are ***no*** accompanying ***transverse stresses*** because beams are ***free to deform*** laterally. This stress condition is analogous to that of a prismatic bar in tension or compression, and therefore longitudinal elements in a beam in pure bending are in a state of ***uniaxial stress***.

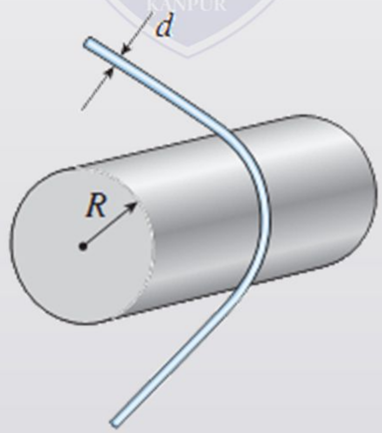
Question

5.4-1 A steel wire with a diameter of $d = 1.6$ mm is bent around a cylindrical drum with a radius of $R = 0.9$ m (see figure).

(a) Determine the maximum normal strain ε_{max}.

(b) What is the minimum acceptable radius of the drum if the maximum normal strain must remain below yield? Assume $E = 210$ GPa and $\sigma_Y = 690$ MPa.

(c) If $R = 0.9$ m, what is the maximum acceptable diameter of the wire if the maximum normal strain must remain below yield?

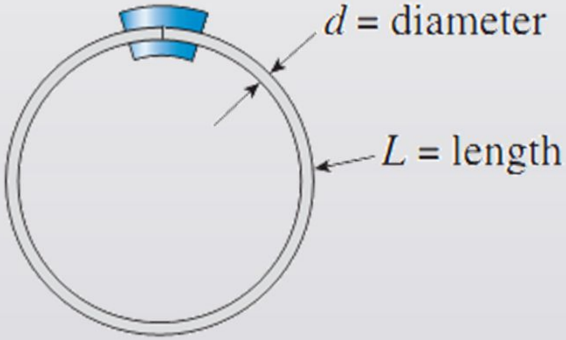


Question

5.4-2 A copper wire having a diameter of $d = 4$ mm is bent into a circle and held with the ends just touching (see figure).

(a) If the maximum permissible strain in the copper is $\varepsilon_{max} = 0.0024$, what is the shortest length L of wire that can be used?

(b) If $L = 5.5$ m, what is the maximum acceptable diameter of the wire if the maximum normal strain must remain below yield? Assume $E = 120$ GPa and $\sigma_Y = 300$ MPa.



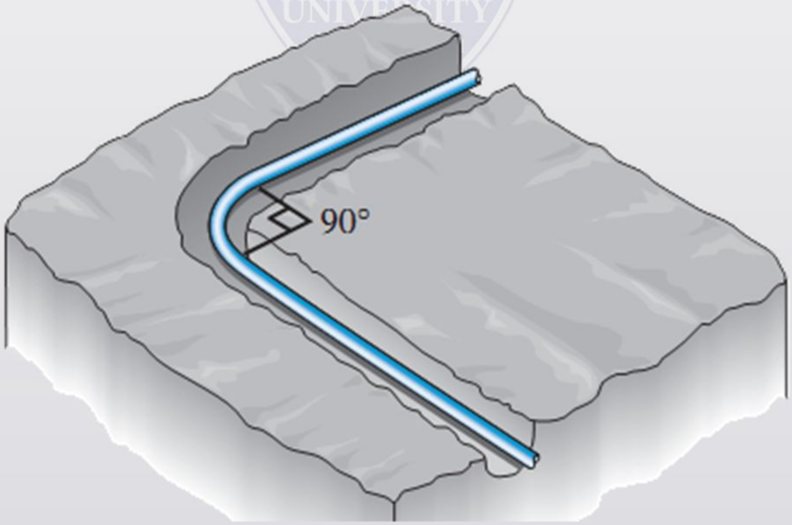
Question

5.4-3 A 120 mm outside diameter polyethylene pipe designed to carry chemical wastes is placed in a trench and bent around a quarter-circular 90° bend (see figure). The bent section of the pipe is 16 m long.

(a) Determine the maximum compressive strain ε_{max} in the pipe.

(b) If the normal strain cannot exceed 6.1×10^{-3}, what is the maximum diameter of the pipe?

(c) If $d = 120$ mm, what is the minimum acceptable length of the bent section of the pipe?



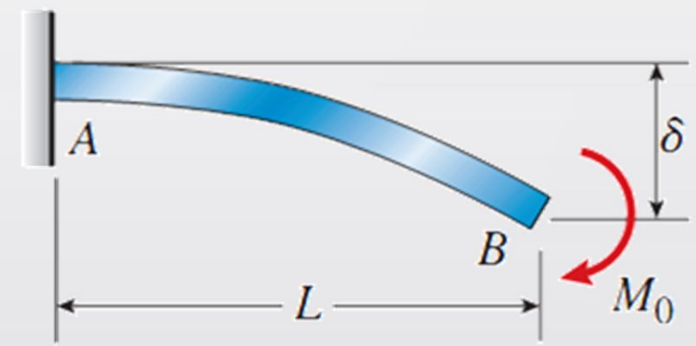
Question

5.4-4 A cantilever beam AB is loaded by a couple M_0 at its free end (see figure). The length of the beam is $L = 2.0$ m, and the longitudinal normal strain at the top surface is $\varepsilon = 0.0010$. The distance from the top surface of the beam to the neutral surface is $c = 85$ mm.

(a) Calculate the radius of curvature ρ, the curvature κ, and the vertical deflection δ at the end of the beam.

(b) If allowable strain $\varepsilon_a = 0.0008$, what is the maximum acceptable depth of the beam? [Assume that the curvature is unchanged from part(a)].

(c) If allowable strain $\varepsilon_a = 0.0008$, $c = 85$ mm, and $L = 4$ m, what is deflection δ?



Question

5.4-5 A thin strip of steel with a length of $L = 0.5$ m and thickness of $t = 7$ mm is bent by couples M_0 (see figure). The deflection at the midpoint of the strip (measured from a line joining its end points) is found to be 7.5 mm.

(a) Determine the longitudinal normal strain ε at the top surface of the strip.

(b) If allowable strain $\varepsilon_a = 0.0008$, what is the maximum acceptable thickness of the strip?

(c) If allowable strain $\varepsilon_a = 0.0008$, $t = 7$ mm, and $L = 0.8$ m, what is deflection δ?

(d) If allowable strain $\varepsilon_a = 0.0008$, $t = 7$ mm, and the deflection cannot exceed 25 mm, what is the maximum permissible length of the strip?