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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 22

Topic:22. Normal Stresses in Beams, Neutral axis, Moment-Curvature Relationship, Flexure Formula

Instructor:

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Normal Stresses in Beams

- Since longitudinal elements of a beam are subjected only to tension or compression, we can use the stress-strain curve for the material to determine the stresses from the strains.
- For linearly elastic material we can use Hooke's Law for uniaxial stress ($\sigma = E\epsilon$)
- Now combining Strain curvature relation and Hooke's Law we can get the ***Normal Stresses in Beams***

$$\sigma_x = E\epsilon_x = -\frac{Ey}{\rho} = -E\kappa y$$

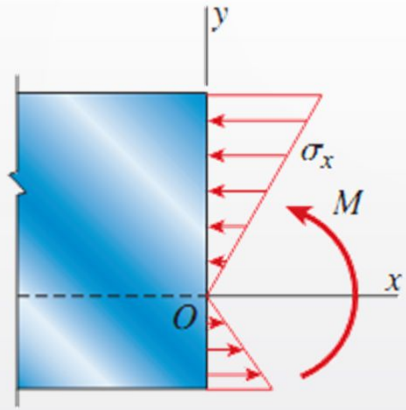
- This equation shows that the normal stresses acting on the cross section vary linearly with the distance y from the neutral surface
- When the curvature is positive, the stresses $\boldsymbol{\sigma_x}$ are negative (compression) above the neutral surface and vice-versa.

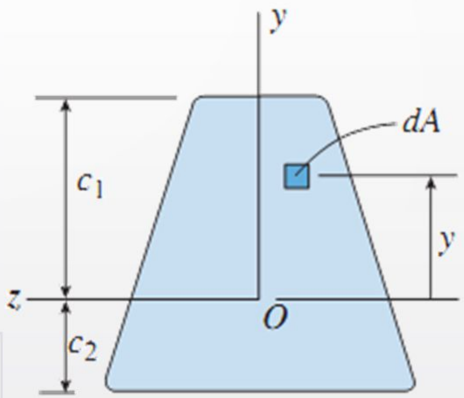
Normal Stresses in Beams

$$\sigma_x = E\epsilon_x = -\frac{Ey}{\rho} = -E\kappa y$$

- ✓ In order for above equation to be of practical value, we must locate the origin of coordinates so that we can determine the distance y.
- ✓ In other words, we must locate the neutral axis of the cross section.
- ✓ We also need to obtain a relationship between the curvature and the bending moment—so that we can substitute into above equation and obtain an equation relating the stresses to the bending moment.
- ✓ The above two relationship can be found out by determining the resultant (or net effect) of $\boldsymbol{\sigma_x}$ on the cross section area, which are as follows
  - ✓ A Resultant force acting in the x-direction whose value is zero
  - ✓ A moment acting about z-axis (in xy plane) whose values will be equal to the bending moment acting on that cross section

Location of neutral axis





$$\sum F_x = \int_A \sigma_x dA = -\int_A E\kappa y dA = -\int_A \frac{Ey}{\rho} dA = 0$$

- ✓ Now $\frac{\boldsymbol{E}}{\boldsymbol{\rho}}$ can be taken out as it is not a function of area i.e. they do not vary over the area $\int_A y dA = 0$ This equation states that the first moment of the area of the cross section, evaluated with respect to the z-axis, is zero. In other words, the z-axis must pass through the centroid of the cross section.
- ✓ *The neutral axis passes through the centroid of the cross-sectional area when the material follows Hooke's law and there is no axial force acting on the cross section*

Moment-Curvature Relationship

- ✓ The fact that the moment resultant of the normal stresses acting over the cross section is equal to the bending moment M
- ✓ Elemental Moment dM is

$$dM = -\sigma_x y dA$$

$$\sum M_z = Net\ Bending\ moment$$

$$\sum M_z = M = \int_A dM = -\int_A \sigma_x y dA = \int_A \frac{Ey^2}{\rho} dA$$

- ✓ Now $\frac{E}{\rho}$ can be taken out as they not a function of area i.e. they do not vary over the area

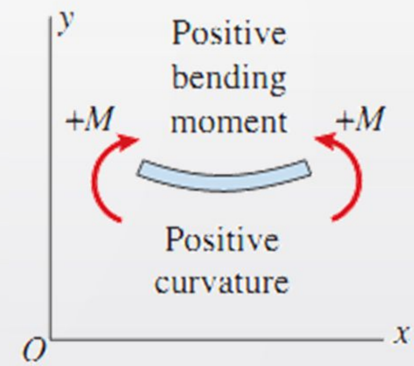
$$M = \frac{E}{\rho}\int_A y^2 dA = \frac{EI}{\rho} \quad \because I = \int_A y^2 dA$$

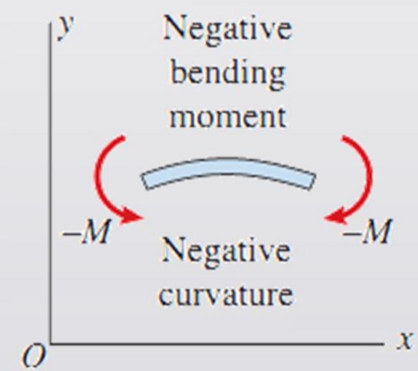
Moment-Curvature Relationship

- This integral $\boldsymbol{I = \int_A y^2 dA}$ is known as moment of inertia of cross section with respect ***to z axis (i.e. w.r.t to neutral axis***). The above equation can be rearranged into ***Moment-Curvature equation,*** where ***EI*** is known as ***flexural rigidity*** of beam.

$$\frac{1}{\rho} = \frac{M}{EI}$$

- Flexural rigidity is a measure of the resistance of a beam to bending, that is, the larger the flexural rigidity, the smaller the curvature for a given bending moment.
- A positive bending moment produces positive curvature and a negative bending moment produces negative curvature





Moment-Curvature Relationship

$$\sigma_x = E\epsilon_x = -\frac{Ey}{\rho} = -E\kappa y$$

$$\frac{1}{\rho} = \frac{M}{EI}$$

- ✓ Now combining Moment-Curvature equation and Normal Stresses in Beams we will get

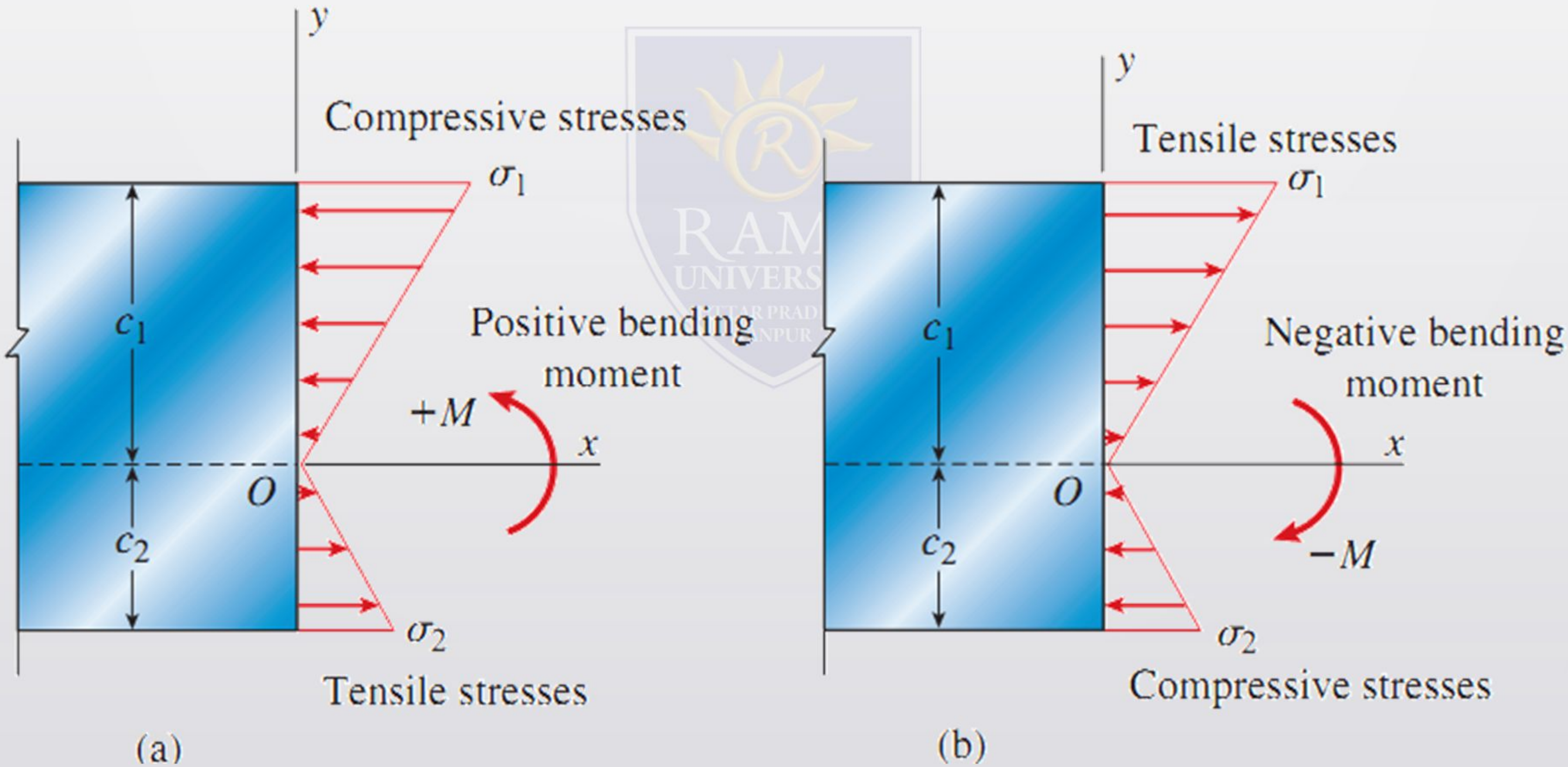
$$\sigma_x = -\frac{My}{I}$$

- ✓ Stresses calculated from the flexure formula are called ***bending stresses or flexural stresses***.

- ✓ If the bending moment in the beam is positive, the bending stresses will be positive (tension) over the part of the cross section where ***y*** is negative, that is, over the lower part of the beam. The stresses in the upper part of the beam will be negative (compression). If the bending moment is negative, the stresses will be reversed.

Moment-Curvature Relationship

- ✓ The maximum tensile and compressive bending stresses acting at any given cross section occur at points located farthest from the neutral axis



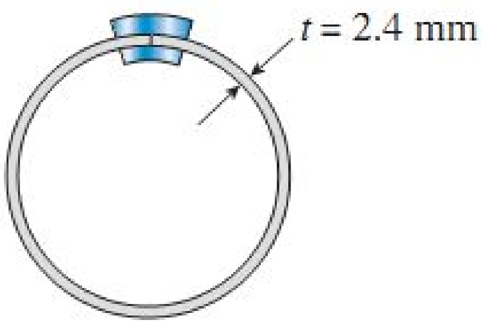
Question

5.5-1 A thin strip of hard copper ($E = 110$ GPa) having length $L = 2.3$ m and thickness $t = 2.4$ mm is bent into a circle and held with the ends just touching (see figure).

(a) Calculate the maximum bending stress σ_{max} in the strip.

(b) By what percent does the stress increase or decrease if the thickness of the strip is increased by 0.8 mm?

(c) Find the new length of the strip so that the stress in part (b) ($t = 3.2$ mm and $L = 2.3$ m) is equal to that in part (a) ($t = 2.4$ mm and $L = 2.3$ m).



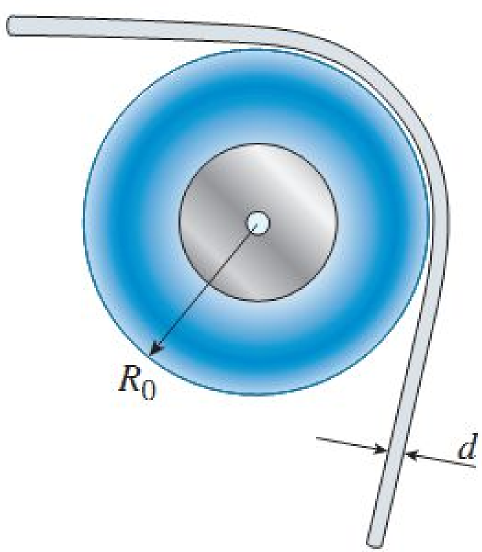
Question

5.5-2 A steel wire ($E = 200$ GPa) of diameter $d = 1.25$ mm is bent around a pulley of radius $R_0 = 500$ mm (see figure).

(a) What is the maximum stress σ_{max} in the wire?

(b) By what percent does the stress increase or decrease if the radius of the pulley is increased by 25%?

(c) By what percent does the stress increase or decrease if the diameter of the wire is increased by 25% while the pulley radius remains at $R_0 = 500$ mm?

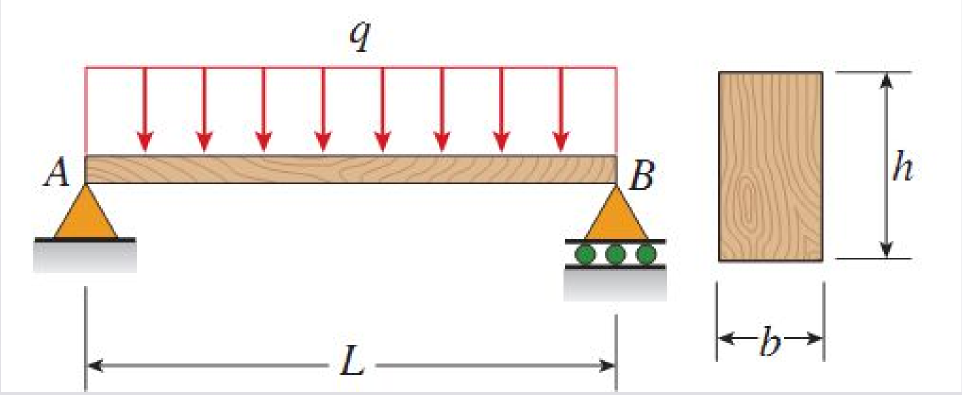


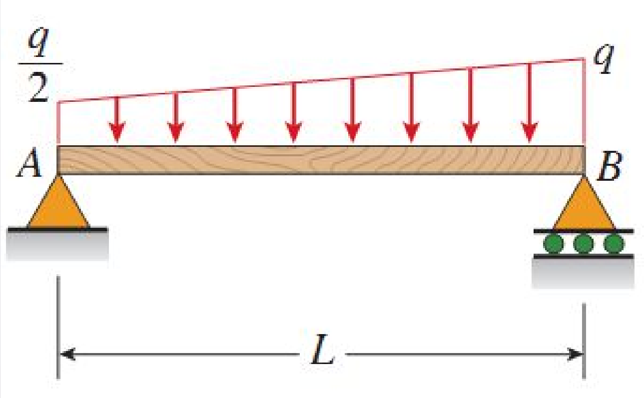
Question

5.5-4 A simply supported wood beam AB with span length $L = 4$ m carries a uniform load of intensity $q = 5.8$ kN/m (see figure).

(a) Calculate the maximum bending stress σ_{max} due to the load q if the beam has a rectangular cross section with width $b = 140$ mm and height $h = 240$ mm.

(b) Repeat part (a) but use the trapezoidal distributed load shown in the figure part b.

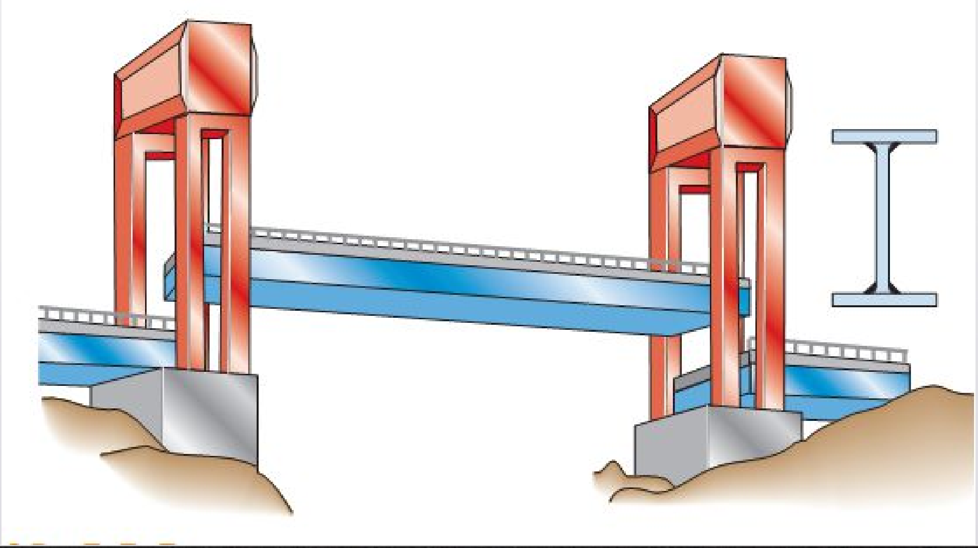




Question

5.5-5 Each girder of the lift bridge (see figure) is 50 m long and simply supported at the ends. The design load for each girder is a uniform load of intensity 18 kN/m. The girders are fabricated by welding three steel plates so as to form an I-shaped cross section (see figure) having section modulus $S = 46{,}000 \text{ cm}^3$.

What is the maximum bending stress σ_{max} in a girder due to the uniform load?



Question

5.5-20 A frame *ABC* travels horizontally with an acceleration a_0 (see figure). Obtain a formula for the maximum stress σ_{max} in the vertical arm *AB*, which has length *L*, thickness *t*, and mass density ρ.