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# FACULTY OF ENGINEERING AND TECHNOLOGY

## Department of Mechanical Engineering

# MEPS102:Strength of Material

# Lecture 23

# Topic:Shear stresses in beams

Instructor:

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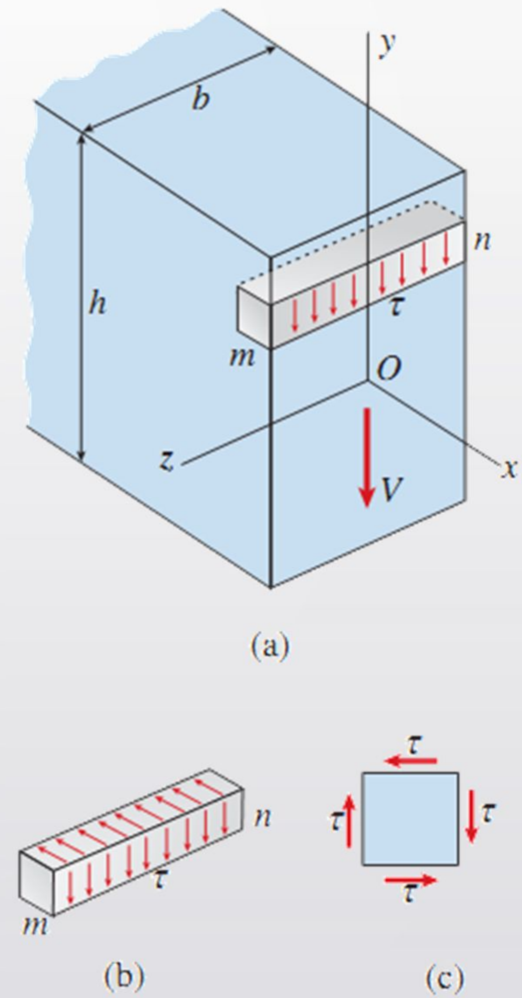
# Introduction

- ✓When a beam is in pure bending, the only stress resultants are the bending moments and the only stresses are the normal stresses acting on the cross sections.
- ✓Most beams are subjected to loads that produce both bending moments and shear forces (nonuniform bending). In these cases, both normal and shear stresses are developed in the beam.

# Vertical and Horizontal Shear Stresses

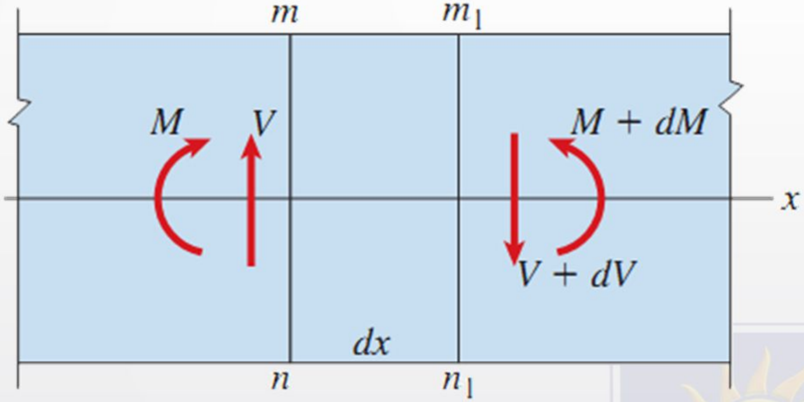
- ✓ Consider a beam of rectangular cross section (width b and height h) subjected to a positive shear force V
- ✓ Assumptions
  - ✓ The shear stresses τ acting on the cross section are parallel to the shear force
  - ✓ the shear stresses are uniformly distributed across the width of the beam, although they may vary over the height.
- ✓ For purposes of analysis, we isolate a small element mn of the beam by cutting between two adjacent cross sections and between two horizontal planes
  - ✓ We know that shear stresses acting on one side of an element are accompanied by shear stresses of equal magnitude acting on perpendicular faces of the element



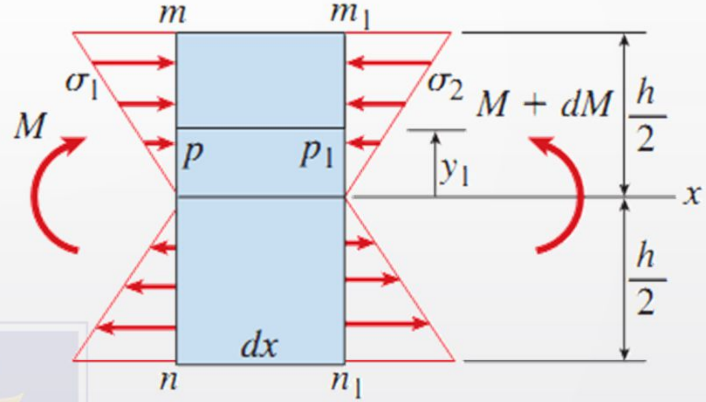
# Vertical and Horizontal Shear Stresses

- ✓ there are horizontal shear stresses acting between horizontal layers of the beam as well as vertical shear stresses acting on the cross sections.
- ✓ At any point in the beam, these complementary shear stresses are equal in magnitude.
- ✓ The equality of the horizontal and vertical shear stresses acting on an element leads to an important conclusion regarding the shear stresses at the top and bottom of the beam.
- ✓ If we imagine that the element mn is located at either the top or the bottom, we see that the horizontal shear stresses must vanish, because there are no stresses on the outer surfaces of the beam. It follows that the vertical shear stresses must also vanish at those locations; in other words, where $y = \pm\frac{h}{2}$

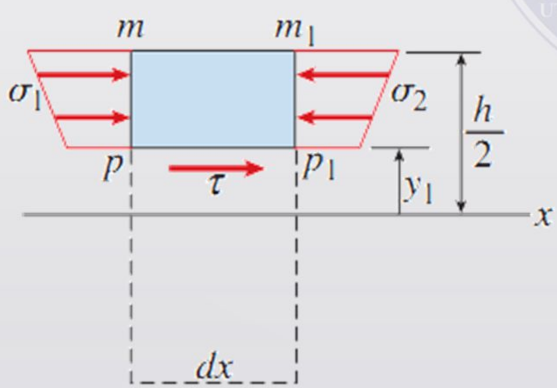
# Shear Formula



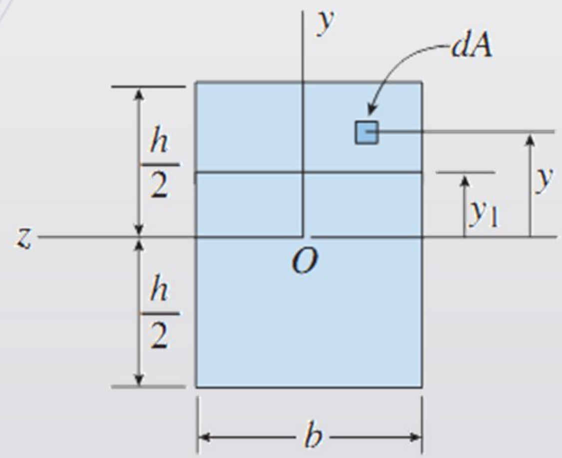

Side view of beam
(a)




Side view of element
(b)




Side view of subelement




Cross section of beam at subelement

# Shear Formula

- ✓ From last diagram we can write

$$F_3 = F_2 - F_1$$

- ✓ Now

$$F_1 = \int \sigma_1 \, \mathrm{d}A \; and \; F_2 = \int \sigma_2 \, \mathrm{d}A$$

$$F_1 = \int \frac{My}{I} \mathrm{d}A \; and \; F_2 = \int \frac{(M + \mathrm{d}M)y}{I} \mathrm{d}A$$

- ✓ If the shear stresses $\boldsymbol{\tau}$ are uniformly distributed across the width ***b*** of the beam, the force $\mathbf{F_3}$ is also equal to the following

$$F_3 = \tau b \, \mathrm{d}x = \int \frac{(M + \mathrm{d}M)y}{I} \mathrm{d}A - \int \frac{My}{I} \mathrm{d}A$$

$$\tau = \frac{\mathrm{d}M}{\mathrm{d}x} \left( \frac{1}{Ib} \right) \int y \, \mathrm{d}A$$

# Shear Formula

Now we know that $\frac{dM}{dx} = V$ and $\int y\,\mathrm{d}A = Q$ first moment of area

$$\tau = \frac{VQ}{Ib}$$

- ✓ This equation, known as the ***shear formula***, can be used to determine the shear stress $\boldsymbol{\tau}$ at any point in the cross section of a rectangular beam. Note that for a specific cross section, the shear force ***V***, moment of inertia ***I*** and width ***b*** are constants. However, the first moment ***Q*** (and hence the shear stress $\boldsymbol{\tau}$) varies with the distance $\boldsymbol{y_1}$ from the neutral axis.
- ✓ Shear stresses in a rectangular beam vary quadratically with the distance $(\bar{y})_1$ from the neutral axis.

# First Moment Q

✓ We usually use the area above the level $\boldsymbol{y_1}$ when the point where we are finding the shear stress is in the upper part of the beam, and we use the area below the level $\boldsymbol{y_1}$ when the point is in the lower part of the beam.

$$Q = A_1(\bar{y})_1$$

$$A_1 = \begin{Bmatrix} is\ area\ above\ the\ level\ y_1\ in\ upper\ part\ of\ beam \\ is\ area\ below\ the\ level\ y_1\ in\ lower\ part\ of\ beam \end{Bmatrix}$$

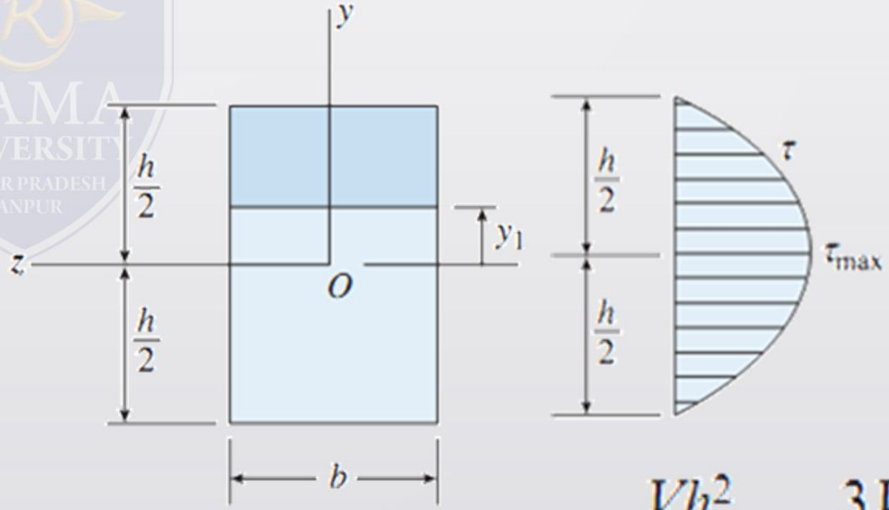
$$(\bar{y})_1 = \quad distance\ of\ centroid\ of\ area\ A_1\ from\ neutral\ axis$$

# Shear Stresses in Beams of Rectangular Cross Section

$$Q = b\left(\frac{h}{2} - y_1\right)\left(y_1 + \frac{h/2 - y_1}{2}\right) = \frac{b}{2}\left(\frac{h^2}{4} - y_1^2\right)$$

$$\tau = \frac{V}{2I}\left(\frac{h^2}{4} - y_1^2\right)$$

- ✓ Shear stresses in a rectangular beam vary quadratically with the distance $(\overline{y})_1$ from the neutral axis




$$\tau_{max} = \frac{Vh^2}{8I} = \frac{3V}{2A}$$

# Question

**5.8-1** The shear stresses $\tau$ in a rectangular beam are given by Eq. (5-43):

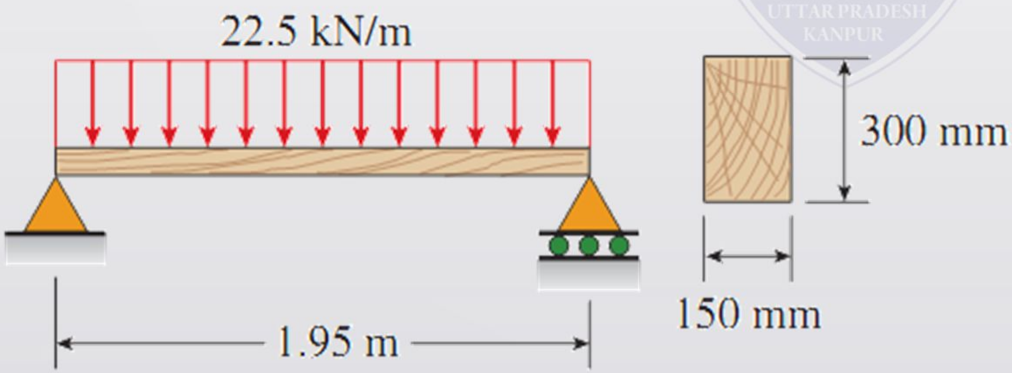
$$\tau = \frac{V}{2I}\left(\frac{h^2}{4} - y_1^2\right)$$

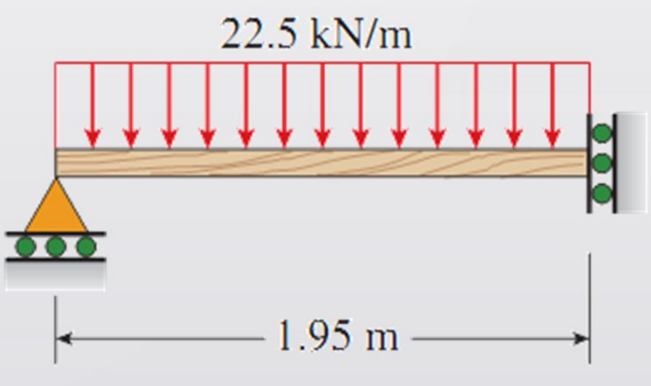
in which $V$ is the shear force, $I$ is the moment of inertia of the cross-sectional area, $h$ is the height of the beam, and $y_1$ is the distance from the neutral axis to the point where the shear stress is being determined (Fig. 5-30).

By integrating over the cross-sectional area, show that the resultant of the shear stresses is equal to the shear force $V$.

# Question

5.8-2 Calculate the maximum shear stress $\tau_{max}$ and the maximum bending stress $\sigma_{max}$ in a wood beam (see figure) carrying a uniform load of 22.5 kN/m (which includes the weight of the beam) if the length is 1.95 m and the cross section is rectangular with width 150 mm and height 300 mm, and the beam is either (a) simply supported as in the figure part a, or (b) has a sliding support at right as in the figure part b.



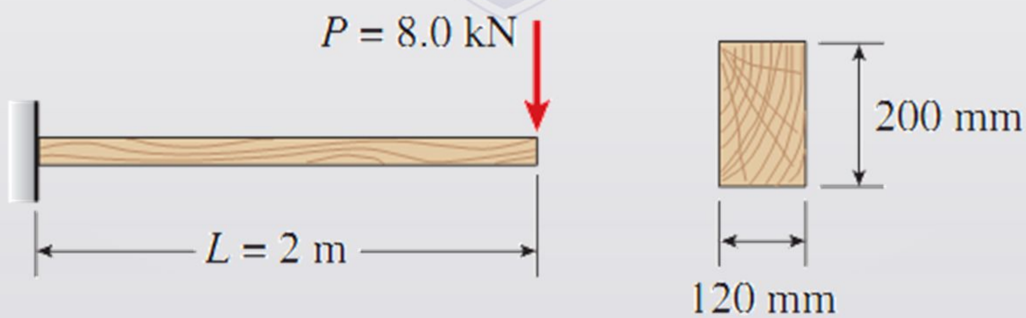



(b)

# Question

**5.8-4** A cantilever beam of length $L = 2$ m supports a load $P = 8.0$ kN (see figure). The beam is made of wood with cross-sectional dimensions 120 mm × 200 mm.

Calculate the shear stresses due to the load $P$ at points located 25 mm, 50 mm, 75 mm, and 100 mm from the top surface of the beam. From these results, plot a graph showing the distribution of shear stresses from top to bottom of the beam.

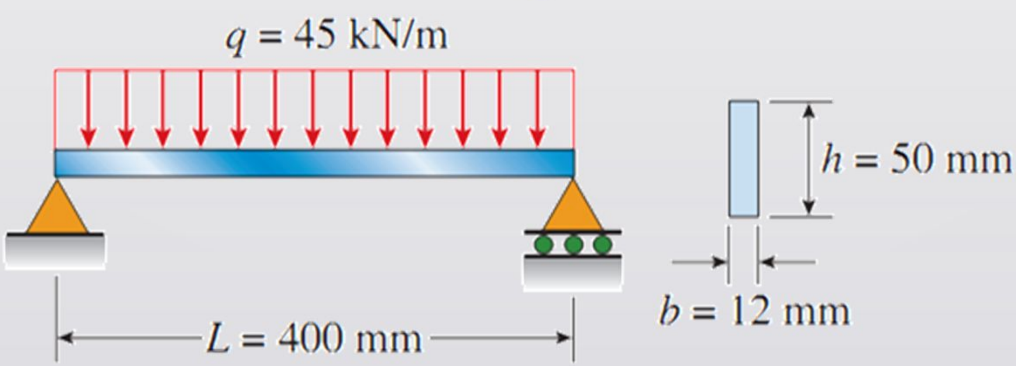


# Question

**5.8-5** A steel beam of length $L = 400$ mm and cross-sectional dimensions $b = 12$ mm and $h = 50$ mm (see figure) supports a uniform load of intensity $q = 45$ kN/m, which includes the weight of the beam.

Calculate the shear stresses in the beam (at the cross section of maximum shear force) at points located 6.25 mm, 12.5 mm, 18.75 mm, and 25 mm from the top surface of the beam. From these calculations, plot a graph showing the distribution of shear stresses from top to bottom of the beam.





# Question

5.8-10 A simply supported wood beam of rectangular cross section and span length 1.2 m carries a concentrated load $P$ at midspan in addition to its own weight (see figure). The cross section has width 140 mm and height 240 mm. The weight density of the wood is 5.4 kN/m$^3$.

Calculate the maximum permissible value of the load $P$ if (a) the allowable bending stress is 8.5 MPa, and (b) the allowable shear stress is 0.8 MPa.