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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 25

Topic:25. Introduction, Differential equation of the deflection curve

Instructor:

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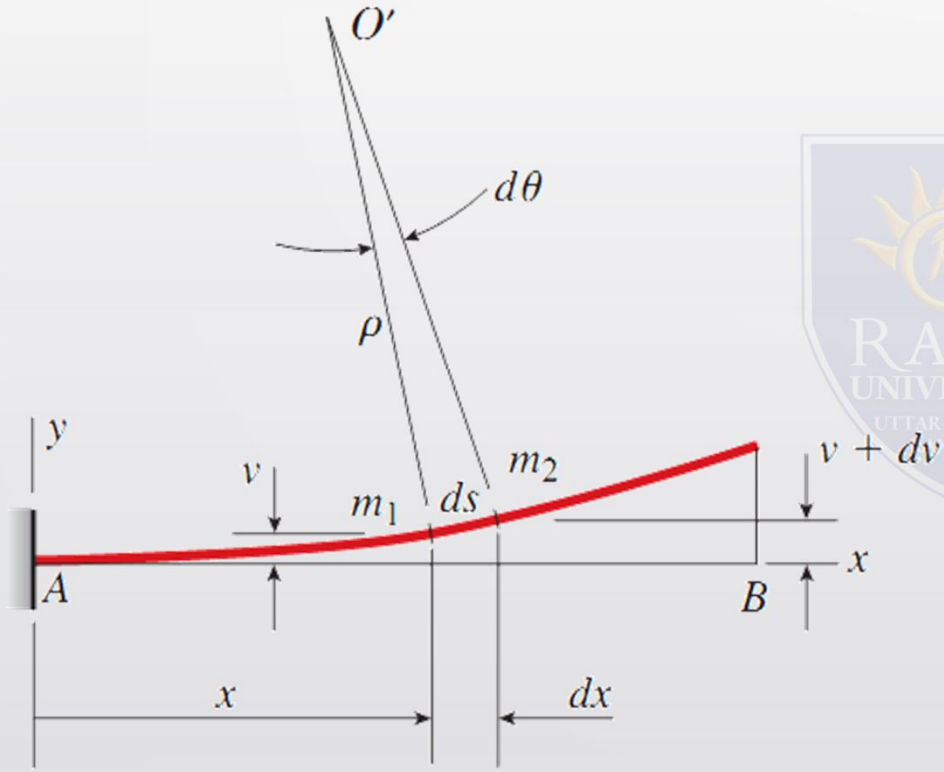
Introduction

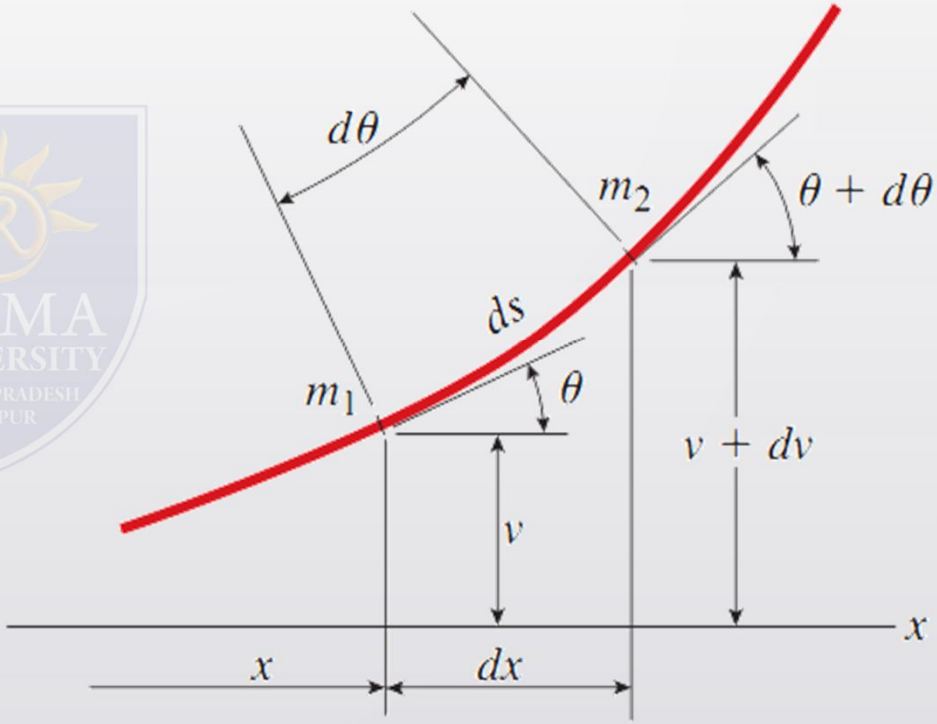
- ✓ When a beam with a straight longitudinal axis is loaded by lateral forces, the axis is deformed into a curve, called the deflection curve of the beam.
- ✓ Most procedures for finding beam deflections are based on the differential equations of the deflection curve and their associated relationships.
- ✓ The calculation of deflections is an important part of structural analysis and design. For example, finding deflections is an essential ingredient in the analysis of statically indeterminate structures.
- ✓ Deflections are also important in dynamic analyses, as when investigating the vibrations of aircraft or the response of buildings to earthquakes.
- ✓ Deflections are sometimes calculated in order to verify that they are within tolerable limits.

Differential Equation

- Consider a cantilever beam with a concentrated load acting upward at the free end.
- Due to this load, the axis of the beam deforms into a curve
- The reference axes have their origin at the fixed end of the beam
  - ***x axis*** directed to the right and the ***y axis*** directed upward.
  - ***z axis*** is directed outward from the figure (toward the viewer).
  - ***xy plane*** is a plane of symmetry of the beam
  - all loads act in this plane (the plane of bending)
- The deflection ***v*** is the displacement in the ***y direction*** of any point on the axis of the beam

Differential Equation





Differential Equation

- ✓ To obtain the equation of the deflection curve, we must express the deflection vas a function of the coordinate **x**.
- ✓ When the beam is bent, there is not only a deflection at each point along the axis but also a rotation
- ✓ The angle of rotation **θ** (angle of inclination or angle of slope) of the axis of the beam is the angle between the ***x axis*** and the tangent to the deflection curve. **ρ** is radius of curvature and **κ** is curvature. Sign convention of ***curvature is positive*** when the ***angle of rotation increases*** as we move along the beam in the positive ***x direction.*** Since $ds \approx dx$

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{ds} = \frac{d\theta}{dx}$$

- ✓ From above diagram on the right side we can see that. Also **θ** is very small so $\tan\theta \approx \theta$

$$\theta \approx \tan\theta = \frac{dv}{dx}$$

Differential Equation

- From above two equations we get

$$\frac{1}{\rho} = \frac{d\theta}{dx} = \frac{d^2v}{dx^2}$$

- This equation is valid for a beam of any material, provided the rotations are small quantities.
- If the material of a beam is linearly elastic and follows Hooke's law, the curvature then

$$\frac{1}{\rho} = \frac{M}{EI}$$

- Therefore, combining equations, we get **Differential equation of the deflection curve**

$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

- We know that $\frac{dM}{dx} = V$ and $\frac{dV}{dx} = -q$ so we can write above equation in other forms

Other Differential Equation

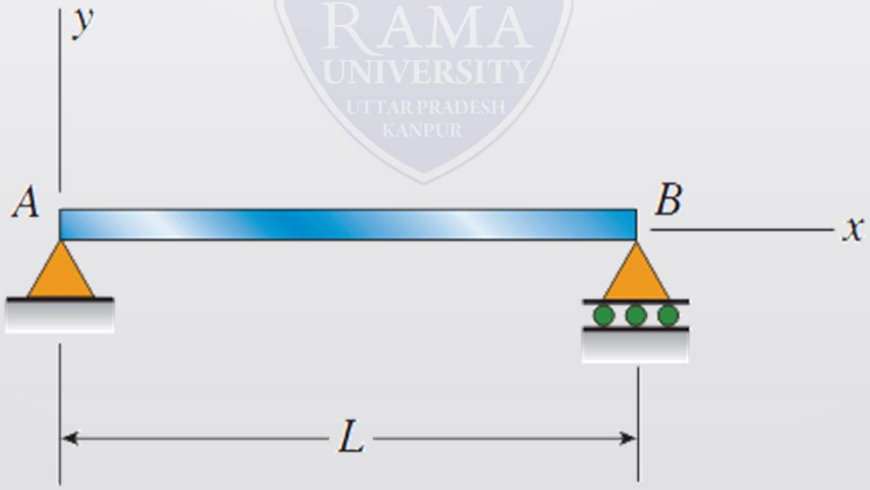
Bending –moment equation	Shear-force equation	Load equation
$EI\frac{d^2v}{dx^2} = M$ or $EIv'' = M$	$EI\frac{d^3v}{dx^3} = V$ or $EIv''' = V$	$EI\frac{d^4v}{dx^4} = -q$ or $EIv'''' = -q$

Question

9.2-1 The deflection curve for a simple beam AB (see figure) is given by the following equation:

$$v = -\frac{q_0 x}{360LEI}(7L^4 - 10L^2x^2 + 3x^4)$$

Describe the load acting on the beam.

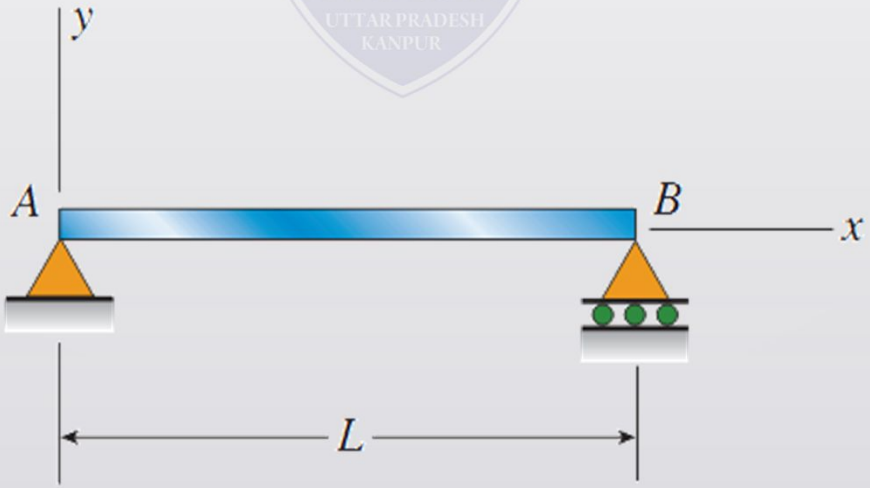


Question

9.2-2 The deflection curve for a simple beam AB (see figure) is given by the following equation:

$$v = -\frac{q_0 L^4}{\pi^4 EI} \sin \frac{\pi x}{L}$$

(a) Describe the load acting on the beam.
(b) Determine the reactions R_A and R_B at the supports.
(c) Determine the maximum bending moment M_{max}.

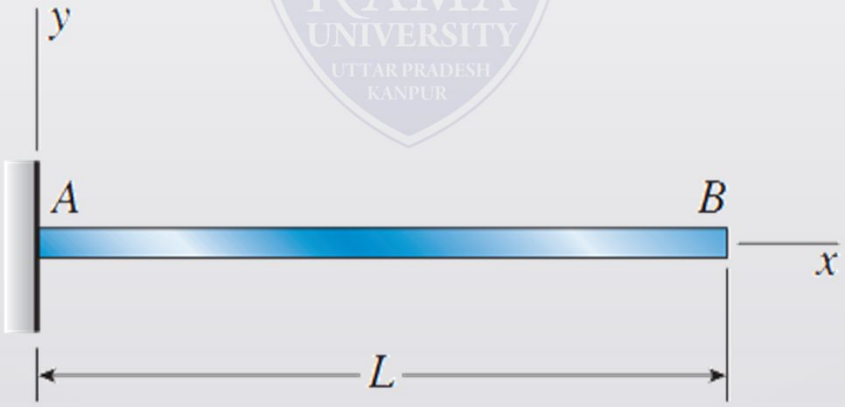


Question

9.2-3 The deflection curve for a cantilever beam AB (see figure) is given by the following equation:

$$v = -\frac{q_0 x^2}{120LEI}(10L^3 - 10L^2x + 5Lx^2 - x^3)$$

Describe the load acting on the beam.



Question

9.2-4 The deflection curve for a cantilever beam AB (see figure) is given by the following equation:

$$v = -\frac{q_0 x^2}{360L^2EI}(45L^4 - 40L^3x + 15L^2x^2 - x^4)$$

(a) Describe the load acting on the beam.

(b) Determine the reactions R_A and M_A at the support.