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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 26

Topic:Deflections by integration of the bending-moment equation

Instructor:

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Bending-Moment equation

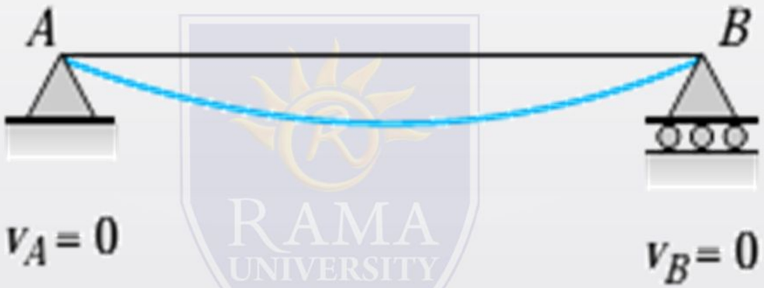
- ✓ Solve the differential equations of the deflection curve and obtain deflections of beams.
- ✓ Since this equation is of second order, two integrations are required.
- ✓ The first integration produces the slope $v' = \frac{dv}{dx}$, and the second produces the deflection v.
- ✓ In some cases a single bending-moment expression holds for the entire length of the beam
- ✓ In other cases the bending moment changes abruptly at one or more points along the axis of the beam.
- ✓ Then we must write separate bending-moment expressions for each region of the beam between points where changes occur

Bending-Moment equation

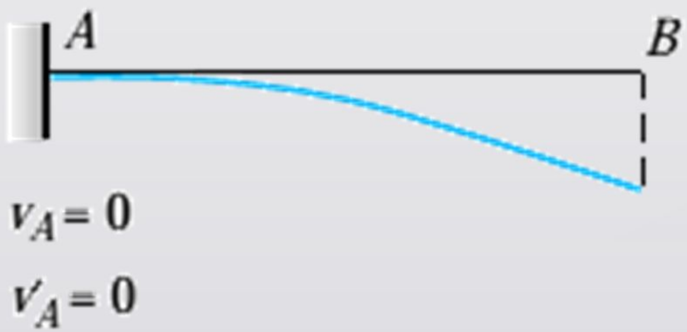
- ✓ Regardless of the number of bending-moment expressions, the general procedure for solving the differential equations
  - ✓ For each region of the beam, we substitute the expression for M found out from equation of equilibrium into the differential equation and integrate to obtain the slope v'. Each such integration produces ***one constant*** of integration.
  - ✓ Next integrate each slope equation to obtain the corresponding deflection ***v***. Each integration produces a new constant
  - ✓ Thus, there are ***two constants*** of integration for ***each region*** of the beam**.** So if there are ***two region*** of beam (two equation of M for same beam) there will be ***four constants.***
  - ✓ These constants are evaluated from known conditions pertaining to the slopes and deflections. The conditions fall into three categories: (1) boundary conditions, (2) continuity conditions, and (3) symmetry conditions

Boundary conditions

- ✓ **Boundary conditions** pertain to the deflections and slopes at the sup-ports of a beam
- ✓ Deflection is zero at the supports



- ✓ Deflection and Slope is zero at the fixed beam

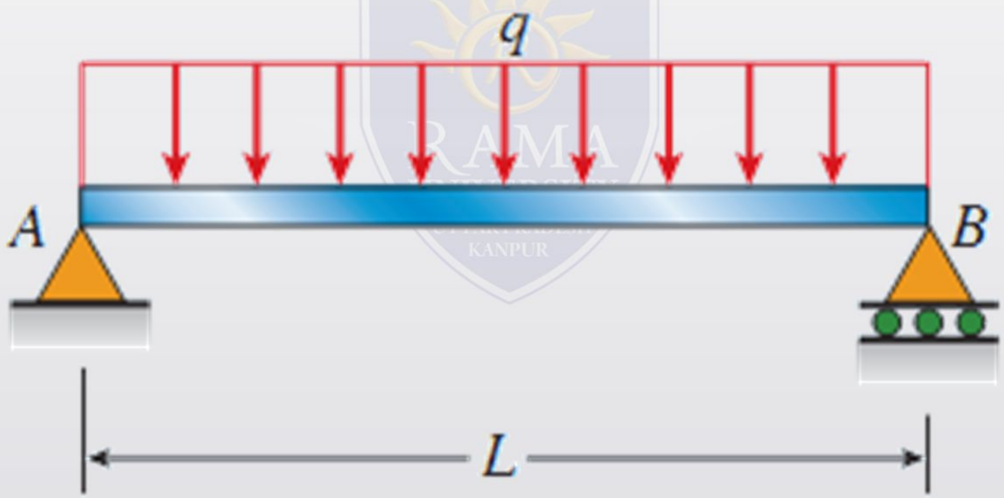


Continuity conditions

- ✓ Points where the regions of integration meet
- ✓ The deflection curve of this beam is physically continuous at point C. So deflection and slope the slopes found for each part of the beam must be equal at point C.

Symmetry conditions

✓ If a simple beam supports a uniform load throughout its length, we know in advance that the slope of the deflection curve at the midpoint must be zero.



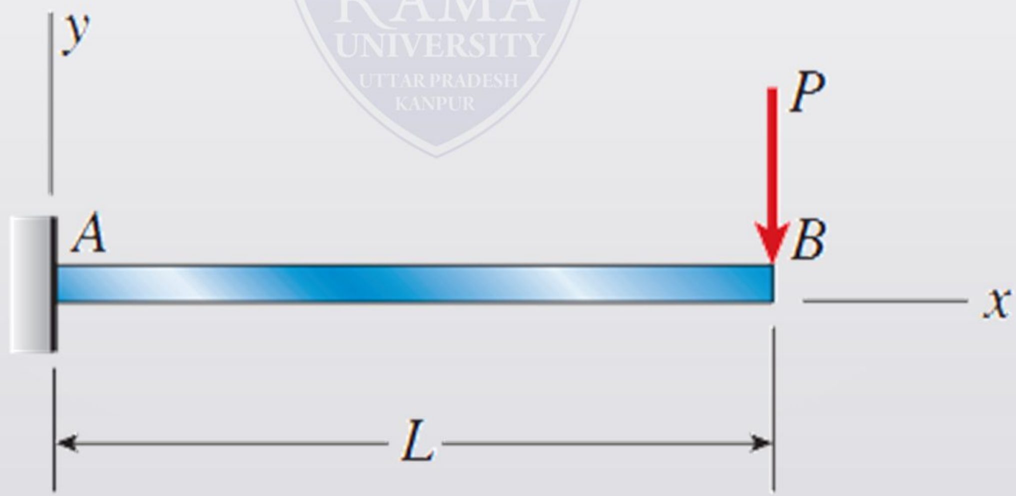
Bending-Moment equation

- The preceding method for finding deflections is sometimes called the ***method of successive integrations.***
- *Point of inflection. At point D the curvature of the deflection curve is zero because the bending moment is zero. A point such as D where the curvature and bending moment change signs is called a point of inflection (or point of contraflexure).*
- The equations of the deflection curve in terms of the shear force **V** and the load **q** may also be integrated to obtain slopes and deflections.

$$EIv''' = V \;\&\; EIv'''' = -q$$

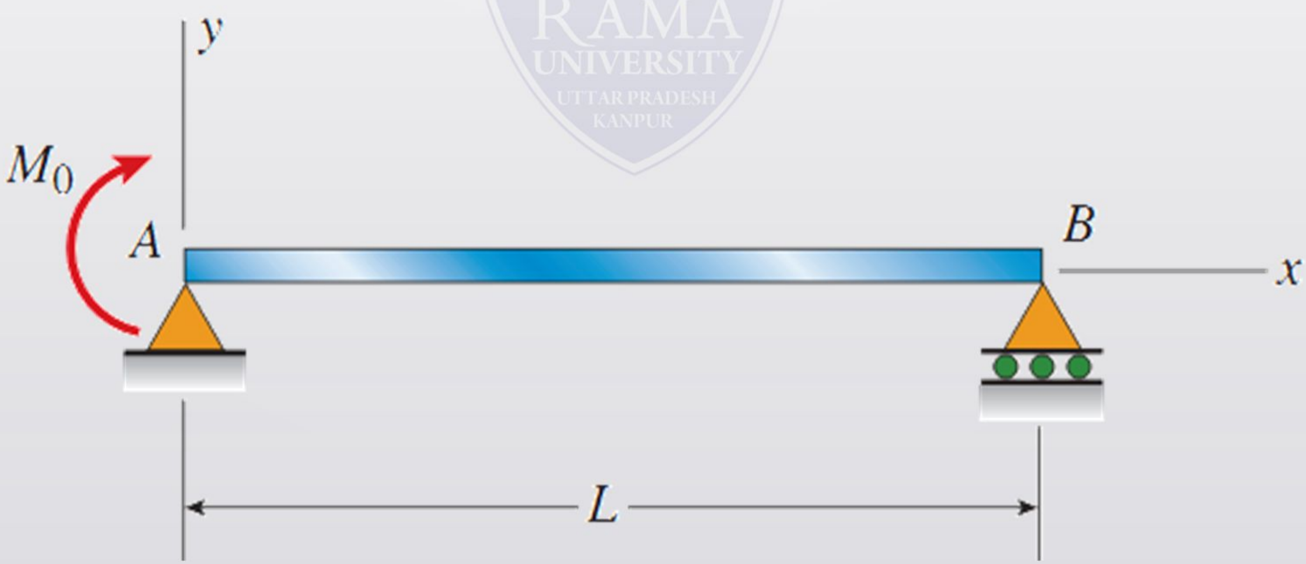
Question

9.3-8 Derive the equation of the deflection curve for a cantilever beam *AB* supporting a load *P* at the free end (see figure). Also, determine the deflection δ_B and angle of rotation θ_B at the free end. (*Note:* Use the second-order differential equation of the deflection curve.)



Question

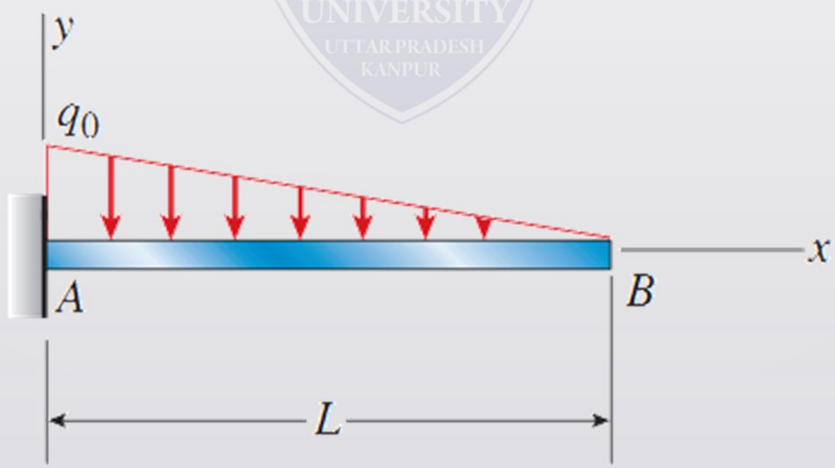
9.3-9 Derive the equation of the deflection curve for a simple beam AB loaded by a couple M_0 at the left-hand support (see figure). Also, determine the maximum deflection δ_{max}. (*Note:* Use the second-order differential equation of the deflection curve.)



Question

9.3-10 A cantilever beam AB supporting a triangularly distributed load of maximum intensity q_0 is shown in the figure.

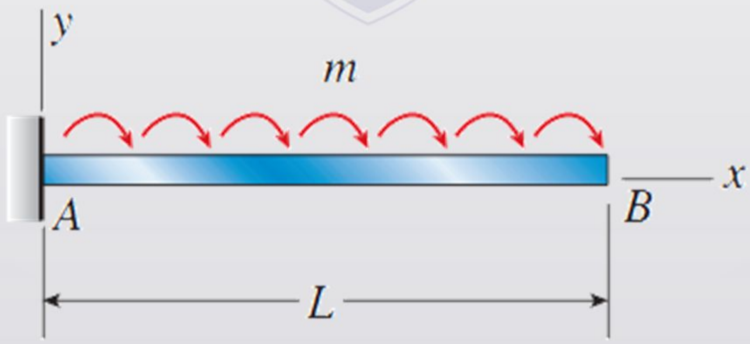
Derive the equation of the deflection curve and then obtain formulas for the deflection δ_B and angle of rotation θ_B at the free end. (*Note:* Use the second-order differential equation of the deflection curve.)



Question

9.3-11 A cantilever beam AB is acted upon by a uniformly distributed moment (bending moment, not torque) of intensity m per unit distance along the axis of the beam (see figure).

Derive the equation of the deflection curve and then obtain formulas for the deflection δ_B and angle of rotation θ_B at the free end. (*Note:* Use the second-order differential equation of the deflection curve.)



Question

9.4-2 A simple beam AB is subjected to a distributed load of intensity $q = q_0 \sin \pi x/L$, where q_0 is the maximum intensity of the load (see figure).

Derive the equation of the deflection curve, and then determine the deflection δ_{max} at the midpoint of the beam. Use the fourth-order differential equation of the deflection curve (the load equation).