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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 27

Topic: Moment area method, First Moment-Area Theorem, Second Moment-Area Theorem

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First Moment-Area Theorem

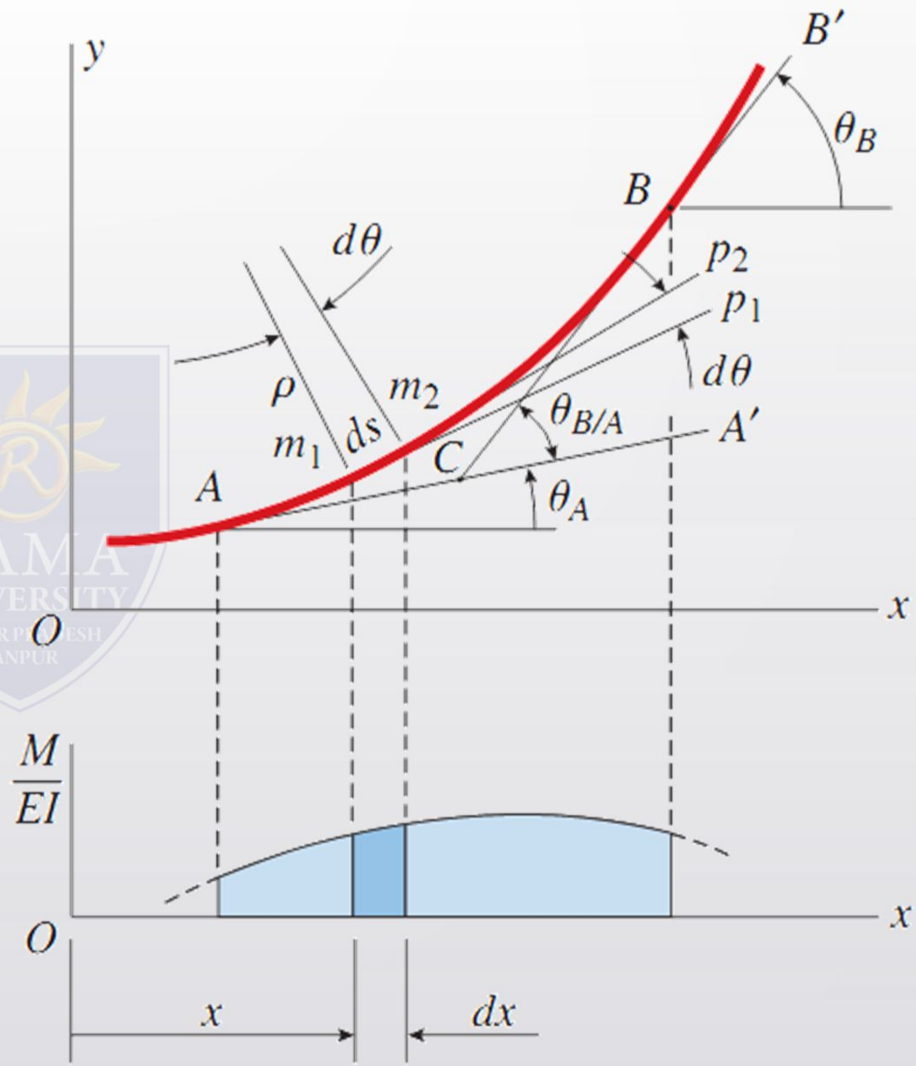
✓ We know that

$$\frac{d\theta}{dx} = \frac{M}{EI}$$

$$d\theta = \frac{M}{EI}dx$$

✓ The angle between the tangents, denoted $\theta_{B/A}$, is equal to the difference between θ_B and θ_A i.e.

$$\theta_{B/A} = \theta_B - \theta_A$$



First Moment-Area Theorem

- ✓ ***First moment-area theorem:*** *The angle* $\theta_{B/A}$ *between the tangents to the deflection curve at two points A and B is equal to the area of the M/EI diagram between those points.*

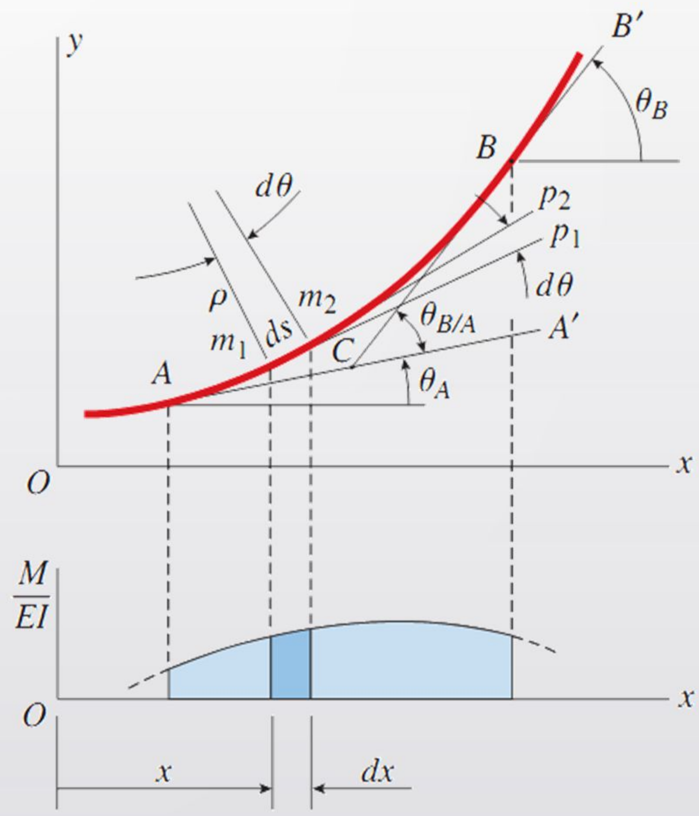
$$\int_A^B d\theta = \int_A^B \frac{M}{EI} dx$$

=Area of the M/EI diagram between point A and B

- ✓ Note that the angles θ_A and θ_B, which are the angles of rotation of the beam axis at points A and B, respectively, are also equal to the slopes at those points, because in reality the slopes and angles are very small quantities.

First Moment-Area Theorem

- ✓ The quantity Mdx/EI has a simple geometric interpretation. The M/EI diagram directly below the beam.
- ✓ At any point along the x axis, the height of this diagram is equal to the bending moment M at that point divided by the flexural rigidity EI at that point.
- ✓ Thus, the M/EI diagram has the same shape as the bending-moment diagram whenever EI is constant. The term Mdx/EI is the area of the shaded strip of width dx within the M/EI diagram. (Note that since the curvature of the deflection curve is positive, the bending moment M and the area of the M/EI diagram are also positive.)



First Moment-Area Theorem

✓ The sign conventions used in deriving the preceding theorem are as follows:

1. The angles θ_A and θ_B are positive when counterclockwise.

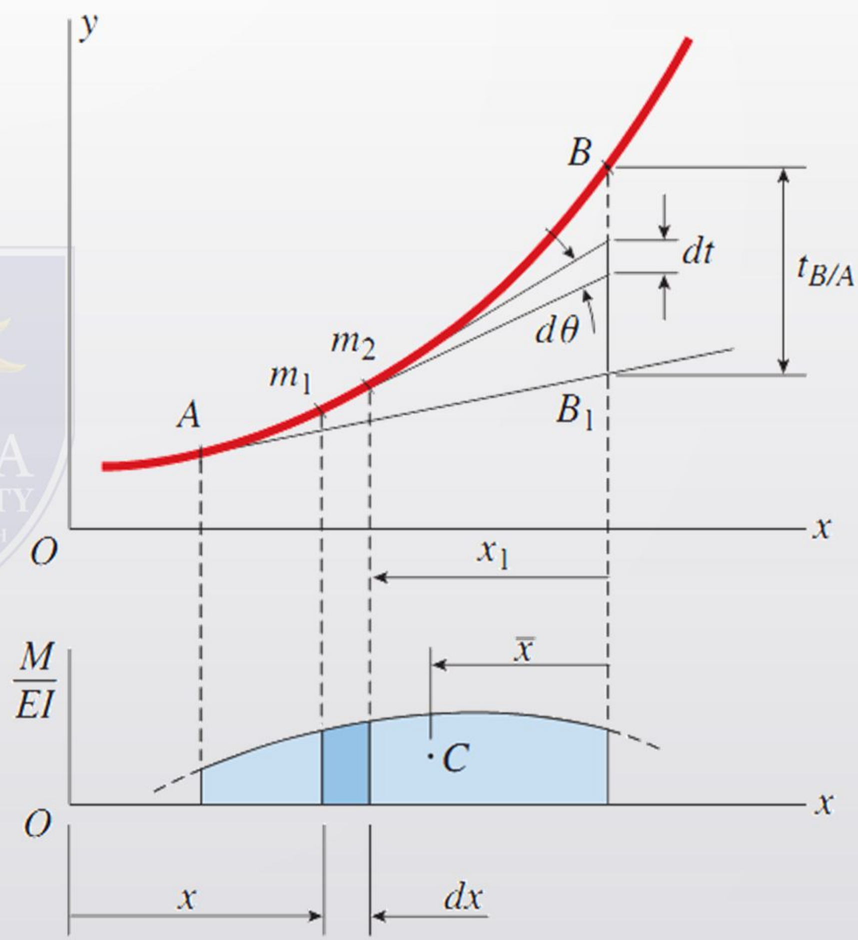
2. The angle between the tangents is positive when the angle θ_B is algebraically larger than the angle θ_A. Also, note that point B must be to the right of point A; that is, it must be further along the axis of the beam as we move in the x direction.

3. The bending moment M is positive according to our usual sign convention; that is, M is positive when it produces compression in the upper part of the beam.

4. The area of the M/EI diagram is given a positive or negative sign according to whether the bending moment is positive or negative. If part of the bending-moment diagram is positive and part is negative, then the corresponding parts of the M/EI diagram are given those same signs.

Second Moment-Area Theorem

- ✓ Second theorem is related primarily to deflections rather than to angles of rotation. Draw tangent at point ***A*** and draw a vertical line through point ***B*** they intersect at point B_1. The vertical distance B_1B is called tangential deviation of ***B*** with respect to ***A.***
- ✓ Tangential deviation is positive when point ***B*** is above ***A***



Second Moment-Area Theorem

- ✓ Under the assumption that angles are very small and deflection is small, from figure we can say that

$$dt = x_1 d\theta = x_1 \frac{M}{EI} dx$$

- ✓ ***Second moment-area theorem***: *The tangential deviation $t_{B/A}$ of point **B** from the tangent at point **A** is equal to the first moment of the area of the **M/EI** diagram between **A** and **B**, evaluated with respect to **B**.*

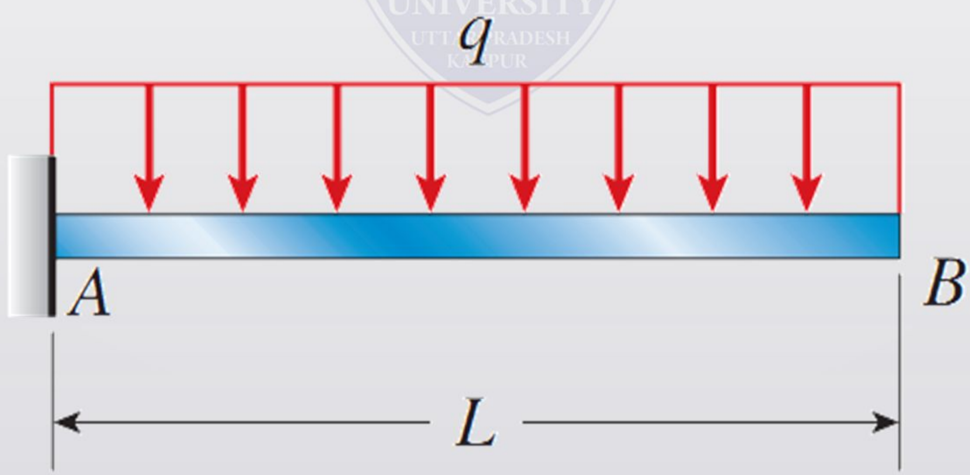
$$t_{\frac{B}{A}} = \int_A^B dt = \int_A^B x_1 \frac{M}{EI} dx$$

=1st moment of area of the M/EI diagram between point A and B, evaluated with respect to B

- ✓ The first moment of the area of the M/EI diagram can be obtained by taking the product of the area of the diagram and the distance $\bar{x}$ from point ***B*** to the centroid **C** of the area

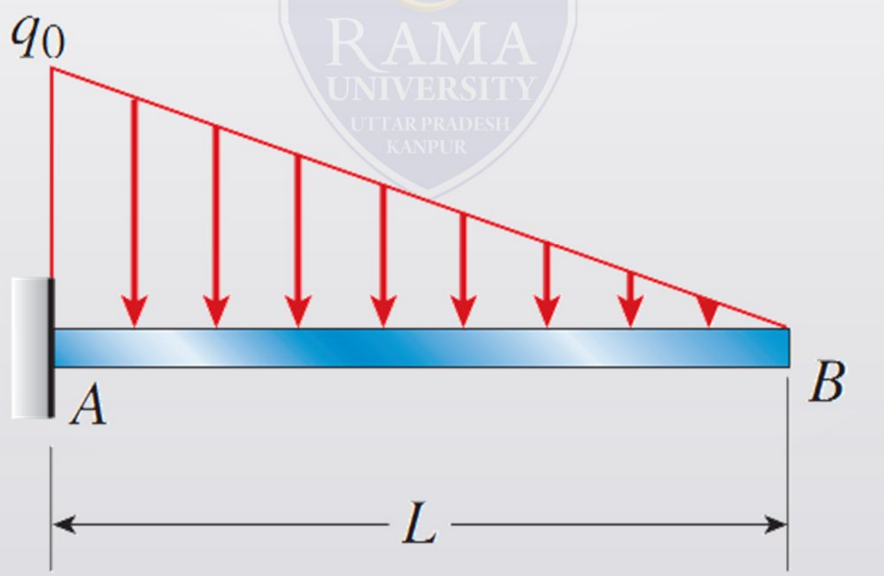
Question

9.6-1 A cantilever beam *AB* is subjected to a uniform load of intensity q acting throughout its length (see figure). Determine the angle of rotation θ_B and the deflection δ_B at the free end.



Question

9.6-2 The load on a cantilever beam *AB* has a triangular distribution with maximum intensity q_0 (see figure). Determine the angle of rotation θ_B and the deflection δ_B at the free end.



Question

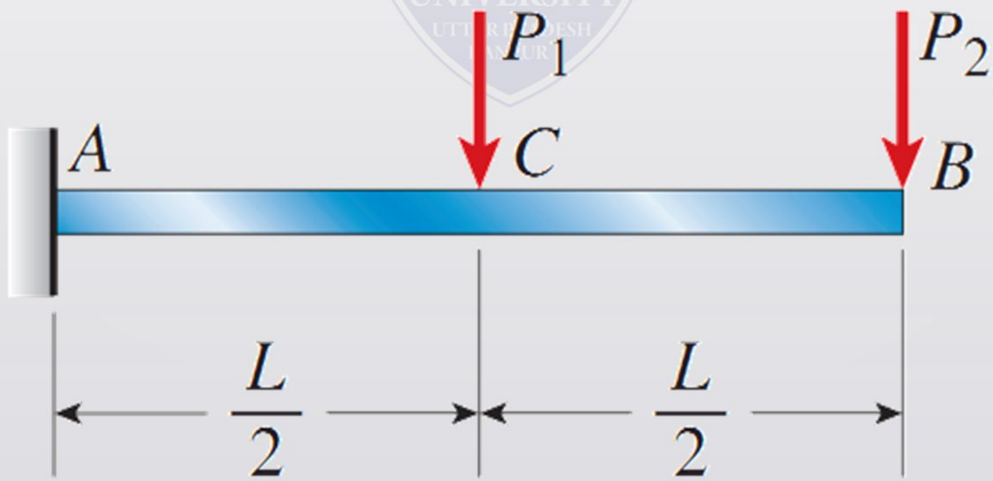
9.6-3 A cantilever beam *AB* is subjected to a concentrated load *P* and a couple M_0 acting at the free end (see figure).

Obtain formulas for the angle of rotation θ_B and the deflection δ_B at end *B*.

Question

9.6-6 A cantilever beam ACB supports two concentrated loads P_1 and P_2 as shown in the figure.

Calculate the deflections δ_B and δ_C at points B and C, respectively. Assume $P_1 = 10$ kN, $P_2 = 5$ kN, $L = 2.6$ m, $E = 200$ GPa, and $I = 20.1 \times 10^6$ mm^4.



Question

9.6-10 The simple beam *AB* shown in the figure supports two equal concentrated loads *P*, one acting downward and the other upward.

Determine the angle of rotation θ_A at the left-hand end, the deflection δ_1 under the downward load, and the deflection δ_2 at the midpoint of the beam.