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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 29

Topic:Differential Equation for Column Buckling, pinned-pinned column

Instructor:

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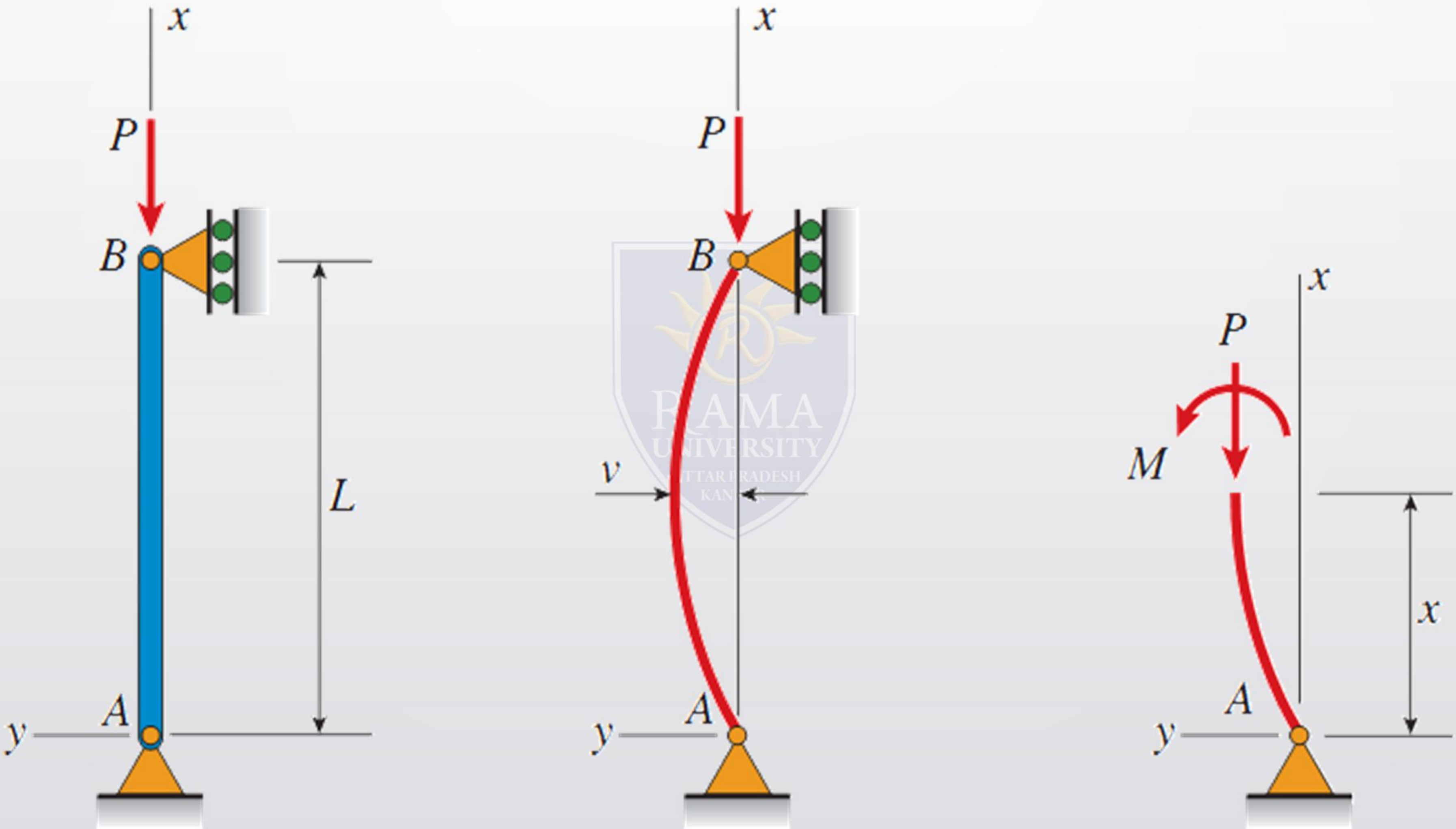
Column With Pinned Ends



Column With Pinned Ends: Assumptions

- ✓The column is loaded by a vertical force P that is applied through the centroid of the end cross section. The column itself is perfectly straight and is made of a linearly elastic material that follows Hooke's law. Since the column is assumed to have no imperfections, it is referred to as an ideal column.
- ✓ coordinate system with its origin at support A and with the x axis along the longitudinal axis of the column. The y axis is directed to the left in the figure, and the z axis (not shown) comes out of the plane of the figure toward the viewer. We assume that the xy plane is a plane of symmetry of the column and that any bending takes place in that plane

Column With Pinned Ends

- ✓When the axial load P has a small value, the column remains perfectly straight and undergoes direct axial compression.
- ✓The only stresses are the uniform compressive stresses obtained from the equation $\sigma = P/A$.
- ✓The column is in stable equilibrium, which means that it returns to the straight position after a disturbance.
  - ✓ For instance, if we apply a small lateral load and cause the column to bend, the deflection will disappear and the column will return to its original position when the lateral load is removed

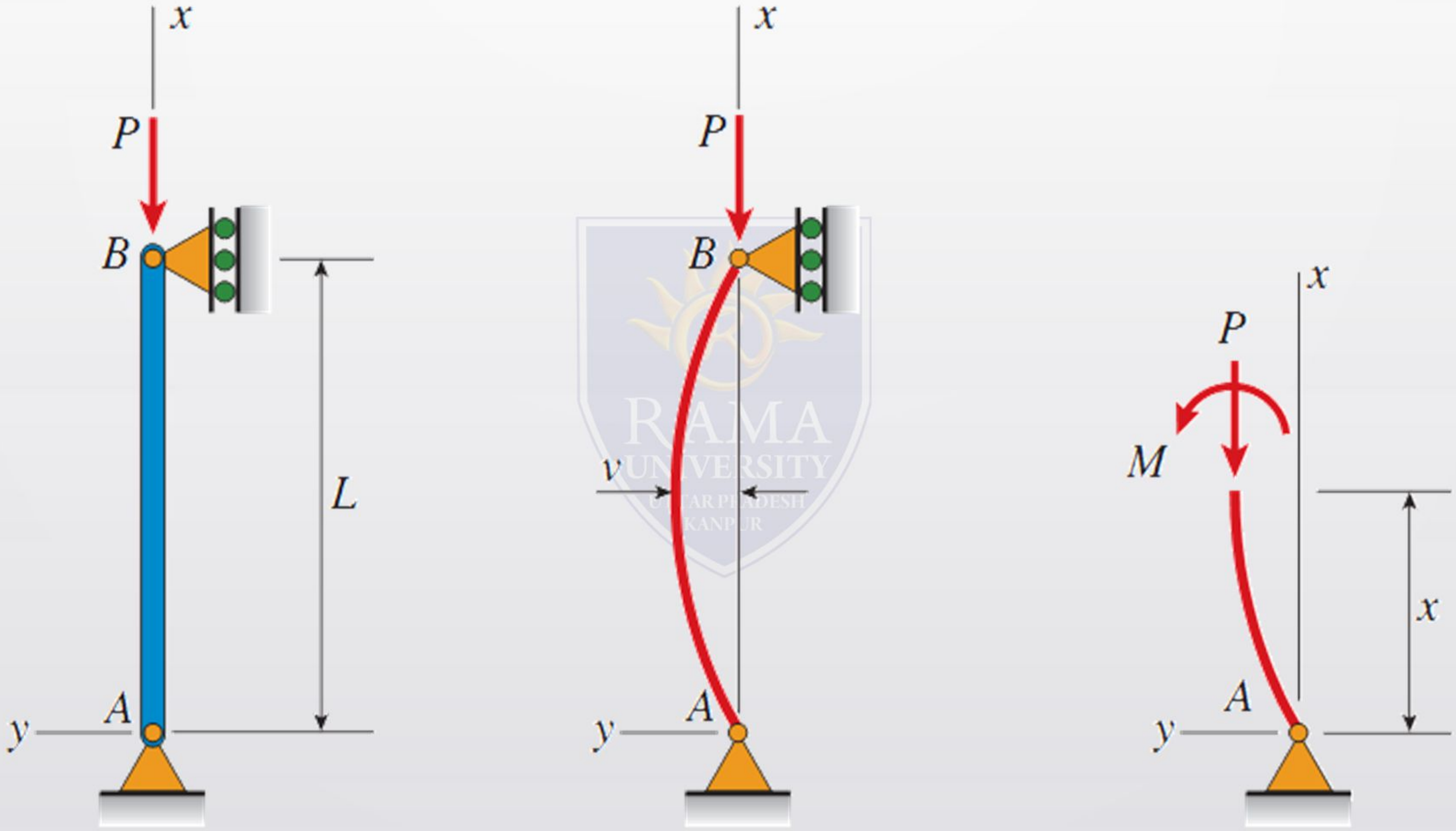
Column With Pinned Ends

- ✓As the axial load P is gradually increased, we reach a condition of neutral equilibrium in which the column may have a bent shape.
- ✓The corresponding value of the load is the critical load $\mathbf{P}_{cr}$. At this load the column may undergo small lateral deflections with no change in the axial force.
  - ✓ For instance, a small lateral load will produce a bent shape that does not disappear when the lateral load is removed.
- ✓The critical load can maintain the column in equilibrium either in the straight position or in a slightly bent position.

Column With Pinned Ends

- ✓At higher values of the load, the column is unstable and may collapse by buckling, that is, by excessive bending.
- ✓For the ideal case that we are discussing, the column will be in equilibrium in the straight position even when the axial force P is greater than the critical load.
- ✓However, since the equilibrium is unstable, the smallest imaginable disturbance will cause the column to deflect sideways.
- ✓Once that happens, the deflections will immediately increase and the column will fail by buckling. The behavior is similar to that described in the preceding section for the idealized buckling model

Differential Equation for Column



Differential Equation for Column

- ✓ We will use bending-moment equation to find the general solution

$$EIv'' = M$$

- ✓ Here **y-axis** is positive towards left and deflection **v** is positive towards left side
- ✓ From equilibrium of moments about point A, we obtain

$$M + Pv = 0 \;\; or\; M = -Pv$$

- ✓ From above two equations we will get

$$EIv'' + Pv = 0$$

$$v'' + \frac{P}{EI}v = 0 \;\;\; put\; k^2 = \frac{P}{EI}$$

- ✓ Therefore, final equation is

$$\frac{d^2v}{dx^2} + k^2v = 0$$

Solution to Differential Equation

- ✓ General solution

$$v = C_1 \sin kx + C_2 \cos kx$$

- ✓ Value of constants is found out from boundary condition which in this case is basically end conditions. Buckling of pinned-end column in the first mode is called the **fundamental case of column buckling.** The type of buckling described in this section is called **Euler buckling**, and the critical load for an ideal elastic column is often called the Euler load.

- ✓ Boundary condition

$$v(0) = 0 \quad \Rightarrow \quad C_2 = 0$$
$$v(L) = 0 \quad \Rightarrow \quad C_1 \sin kL = 0$$

- ✓ From second Boundary condition we will get two cases

- ✓ **Case 1** If $C_1 = 0$ then $v = 0$ therefore any value of the quantity kL will satisfy the equation. Consequently, the axial load ***P*** may also have any value.

Solution to Differential Equation

- **Case 2** The second possibility is given by the following equation, known as the buckling equation

$$\sin kL = 0$$

$$kL = n\pi$$

$$P_{cr} = \frac{n^2\pi^2 EI}{L^2} \quad n = 1,2,3 \ldots$$

- Values of P given by above equation are known as critical load of column. Only when P has one of the values given is it theoretically possible for the column to have a bent shape. For all other values of P, the column is in equilibrium only if it remains straight.
- Lowest critical load when value of n=1

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

The corresponding buckled shape (sometimes called a mode shape) is

$$v = C_1 \sin\frac{\pi x}{L}$$

Solution to Differential Equation

- ✓ Buckling of a pinned-end column in the first mode is called the fundamental case of column buckling
- ✓ The type of buckling described in this section is called Euler buckling, and the critical load for an ideal elastic column is often called the Euler load.
- ✓ After finding the critical load for a column, we can calculate the corresponding critical stress by dividing the load by the cross-sectional area

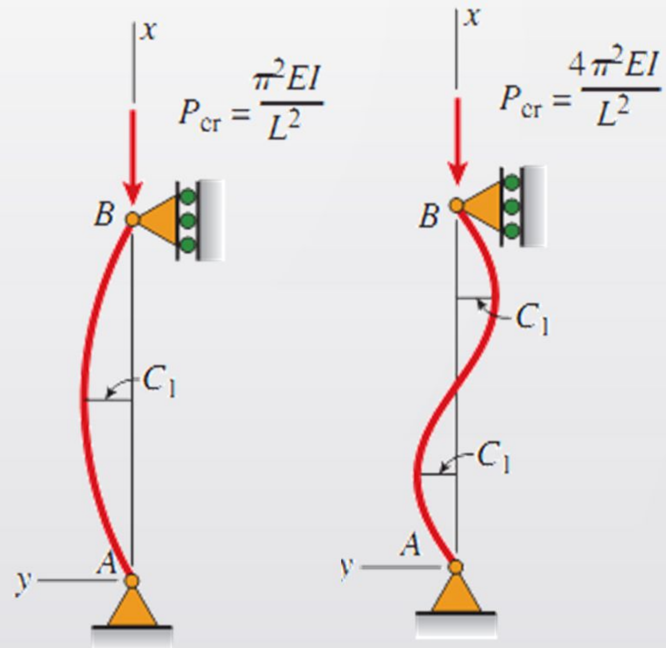
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{AL^2} = \frac{\pi^2 EAr^2}{AL^2} = \frac{\pi^2 E}{\left(L/r\right)^2}$$

$$Slenderness\ ratio = \frac{L}{r}$$

- ✓ $\boldsymbol{r}$ is least radius of gyration $= \sqrt{I_{min}/A}$ the buckling will takes place along this direction i.e. $\boldsymbol{r\ or\ I_{min}}$

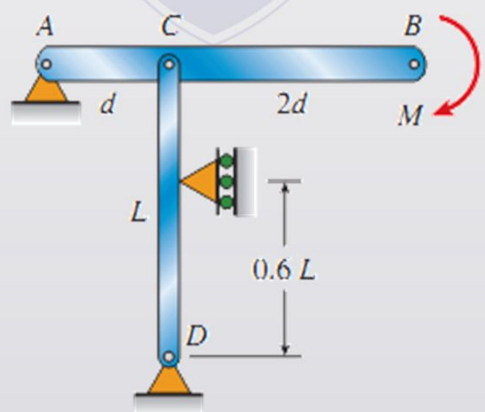
Solution to Differential Equation

- ✓ By taking higher values of the index n we obtain an infinite number of critical loads and corresponding mode shapes.
- ✓ The corresponding critical load is four times larger than the critical load for the fundamental case. The magnitudes of the critical loads are proportional to the square of n, and the number of half-waves in the buckled shape is equal to n.
- ✓ Buckled shapes for the higher modes are often of no practical interest because the column buckles when the axial load P reaches its lowest critical value. The only way to obtain modes of buckling higher than the first is to provide lateral support of the column at intermediate points,



Question

11.3-4 A horizontal beam AB is pin supported at end A and carries a clockwise moment M at joint B, as shown in the figure. The beam is also supported at C by a pinned-end column of length L; the column is restrained laterally at $0.6L$ from the base at D. Assume the column can only buckle in the plane of the frame. The column is a solid steel bar ($E = 200$ GPa) of square cross section having length $L = 2.4$ m and side dimensions $b = 70$ mm. Let dimensions $d = L/2$. Based upon the critical load of the column, determine the allowable moment M if the factor of safety with respect to buckling is $n = 2.0$.

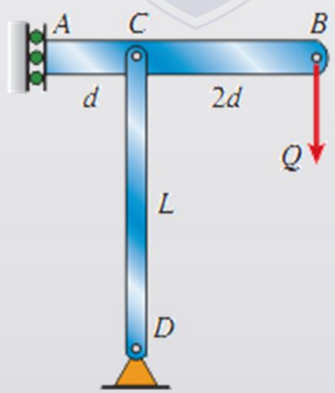


Question

11.3-6 A horizontal beam *AB* is supported at end A and carries a load Q at joint *B*, as shown in the figure part a. The beam is also supported at *C* by a pinned-end column of length L. The column has flexural rigidity EI.

(a) For the case of a guided support at *A* (figure part a), what is the critical load Q_{cr}? (In other words, at what load Q_{cr} does the system collapse because of Euler buckling of the column *DC*?)

(b) Repeat part (a) if the guided support at *A* is replaced by column *AF* with length $3L/2$ and flexural rigidity EI (see figure part b).



Question

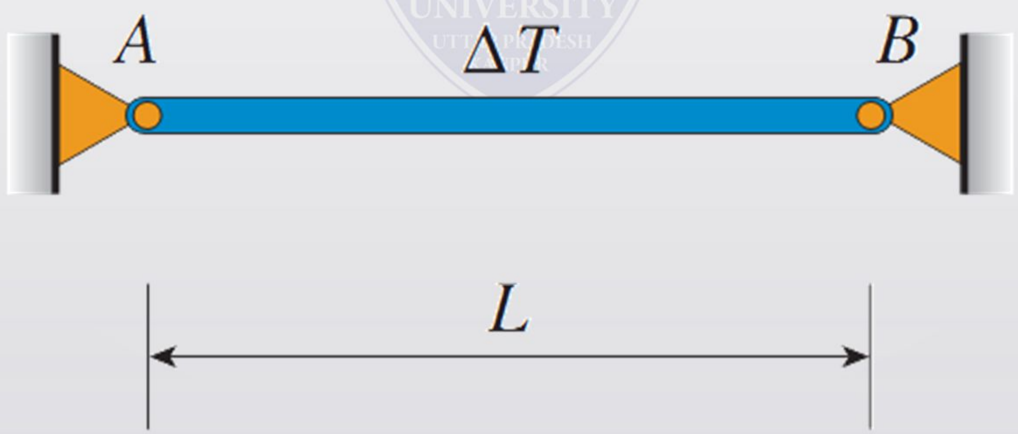
11.3-11 Three pinned-end columns of the same material have the same length and the same cross-sectional area (see figure). The columns are free to buckle in any direction. The columns have cross sections as follows: (1) a circle, (2) a square, and (3) an equilateral triangle.

Determine the ratios $P_1 : P_2 : P_3$ of the critical loads for these columns.

Question

11.3-8 A slender bar AB with pinned ends and length L is held between immovable supports (see figure).

What increase ΔT in the temperature of the bar will produce buckling at the Euler load?



Question

11.3-9 A rectangular column with cross-sectional dimensions b and h is pin supported at ends A and C (see figure). At midheight, the column is restrained in the plane of the figure, but is free to deflect perpendicularly to the plane of the figure.

Determine the ratio h/b such that the critical load is the same for buckling in the two principal planes of the column.