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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 3

Topic: **Hooke's Law, Elasticity, plasticity, creep ,Relaxation,**

Instructor:

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Hooke's Law Uniaxial Case (1 D)

- ✓ The linear relationship between stress and strain for a bar in simple tension or compression is expressed by the equation

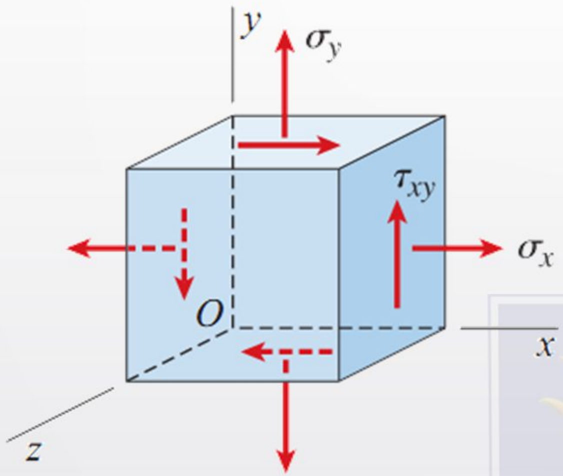
$$\sigma = E\epsilon$$

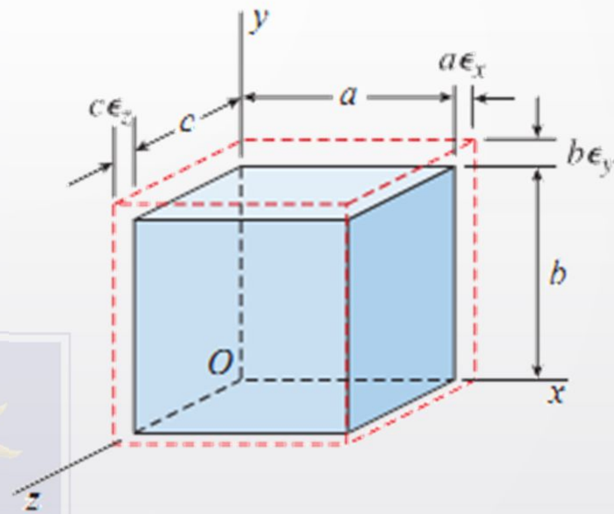
Where

$$\sigma \text{ is axial stress}$$
$$\epsilon \text{ is axial strain}$$
$$E \text{ is modulus of elasticity}$$

- ✓ The above equation is know as Hooke's Law
- ✓ It related only longitudinal stresses and strains developed in simple tension or compression of a bar (uniaxial stress).
- ✓ Modulus of elasticity is often called Young's modulus
    - ✓ For most materials, the value of E in compression is nearly the same as in tension.

Hooke's Law Biaxial Case (2 D)

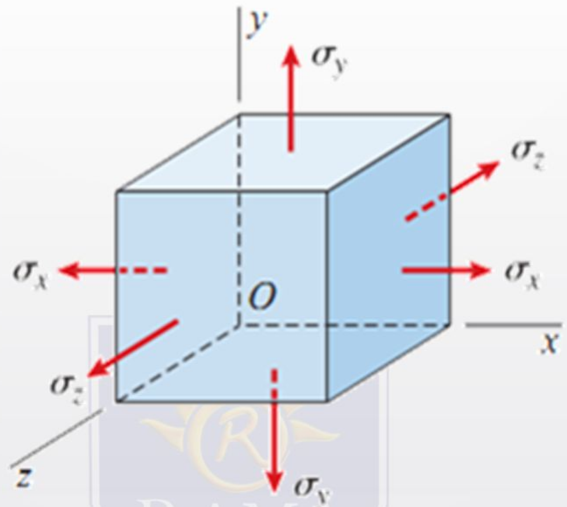




Strain ϵ_x x direction due to σ_x is $\frac{\sigma_x}{E}$ and due to σ_y is $-\nu\frac{\sigma_x}{E}$

	Strain	Stress
x-direction	$\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$	$\sigma_x = \frac{E}{1-\nu^2}(\epsilon_x + \nu\epsilon_y)$
y-direction	$\epsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$	$\sigma_y = \frac{E}{1-\nu^2}(\epsilon_y + \nu\epsilon_x)$
z-direction	$\epsilon_z = -\frac{1}{E}(\sigma_x + \sigma_y)$	$\sigma_z = 0$

Hooke's Law Triaxial Case (3 D)

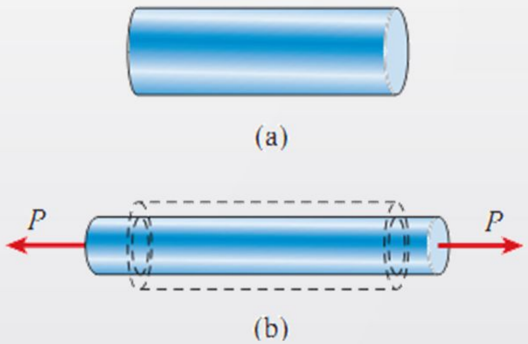


	Strain	Stress
x-direction	$\epsilon_x = \frac{1}{E}\{\sigma_x - \nu(\sigma_y + \sigma_z)\}$	$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\{(1-\nu)\epsilon_x + \nu(\epsilon_y + \epsilon_z)\}$
y-direction	$\epsilon_y = \frac{1}{E}\{\sigma_y - \nu(\sigma_x + \sigma_z)\}$	$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\{(1-\nu)\epsilon_y + \nu(\epsilon_x + \epsilon_z)\}$
z-direction	$\epsilon_z = \frac{1}{E}\{\sigma_z - \nu(\sigma_x + \sigma_y)\}$	$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}\{(1-\nu)\epsilon_z + \nu(\epsilon_x + \epsilon_y)\}$

Poisson's Ratio

✓ When a prismatic bar is loaded in tension, the axial elongation is accompanied by lateral contraction (that is, contraction normal to the direction of the applied load).

- The lateral strain ϵ' at any point in a bar is proportional to the axial strain ϵ at that same point if the material is linearly elastic.
- The ratio of these strains is a property of the material known as **Poisson's ratio**.
- This dimensionless ratio, usually denoted by the Greek letter ν (nu)
- Poisson's ratio remains constant only in the linearly elastic range.
- In nonlinear case, the ratio of lateral strain to axial strain is often called the ***contraction ratio***

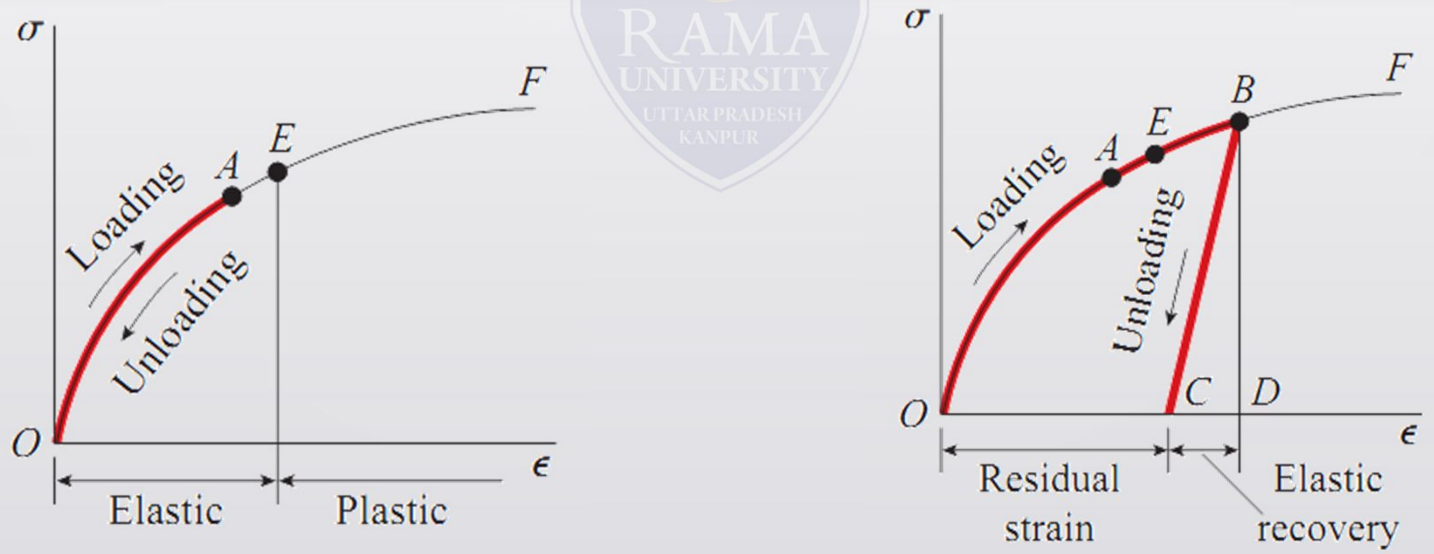


$$\nu = -\frac{Lateral\ Strain}{Axial\ Strain} = -\frac{\epsilon'}{\epsilon}$$

✍ *The minus sign is inserted in the equation to compensate for the fact that the lateral and axial strains normally have opposite signs.*

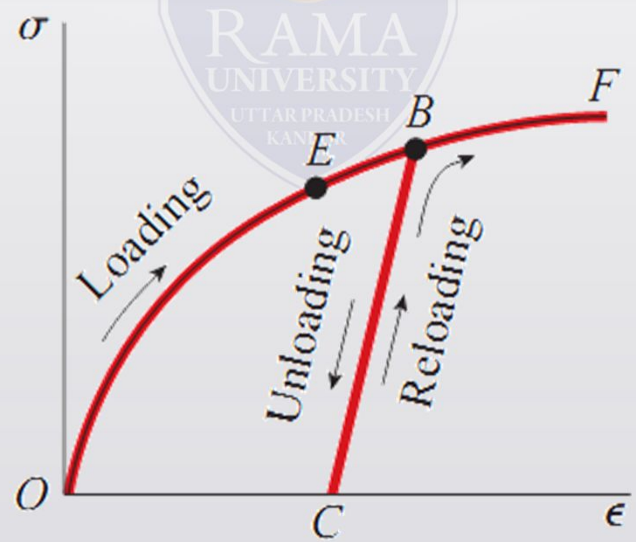
Elasticity and Plasticity

- ✓ **Elasticity:** Property of a material, by which it returns to its original dimension during unloading
- ✓ **Partially elastic:** During unloading the bar returns partially to its original shape
  - ✓ Residual elongation of the bar is called the ***permanent set*** and the strain associated with that is known as ***residual strain***
- ✓ **Plasticity:** characteristic of a material by which it undergoes ***inelastic*** strains beyond the strain at the elastic limit



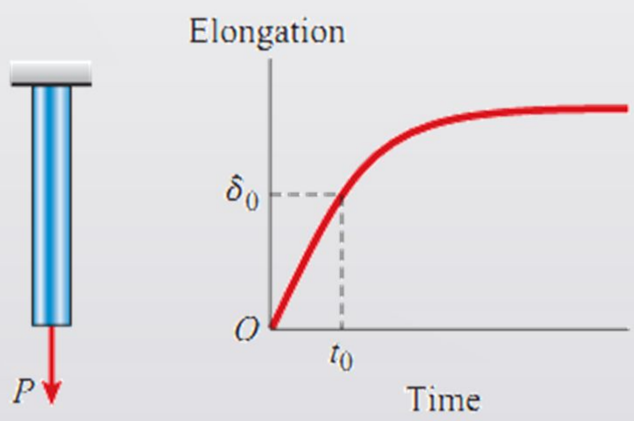
Reloading of Material

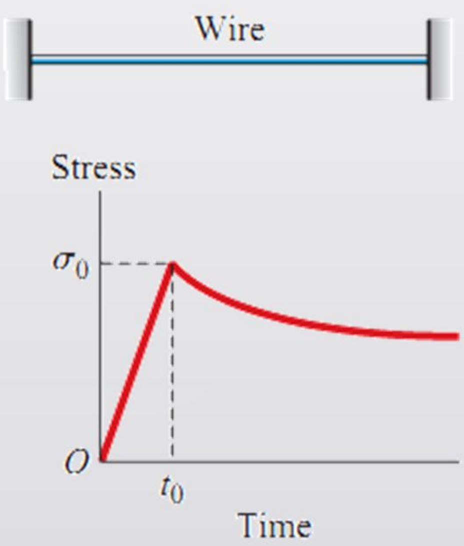
- ✓ If the material remains within the elastic range, it can be loaded, unloaded, and loaded again without significantly changing the behavior
- ✓ By stretching a material such as steel or aluminum into the inelastic or plastic range, the properties of the material are changed
    - ✓ the linearly elastic region is increased,
    - ✓ the proportional limit is raised
    - ✓ elastic limit is raised
    - ✓ ductility is reduced



Creep and Relaxation

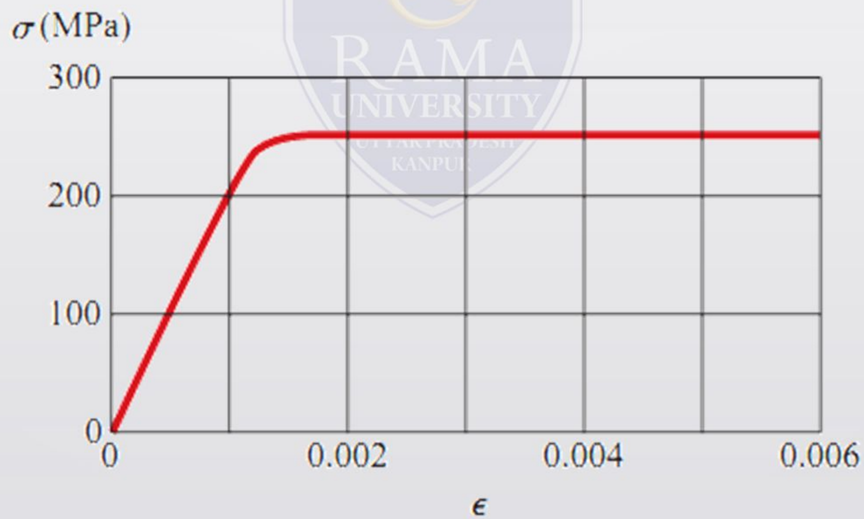
- ✓ **Creep**
  - ✓ When loaded for long periods of time, some materials develop additional strains and are said to creep
- ✓ **Relaxation**
  - ✓ a wire that is stretched between two immovable supports so that it has an initial tensile stress
  - ✓ With the elapse of time, the stress in the wire gradually diminishes, eventually reaching a constant value, even though the supports at the ends of the wire do not move.





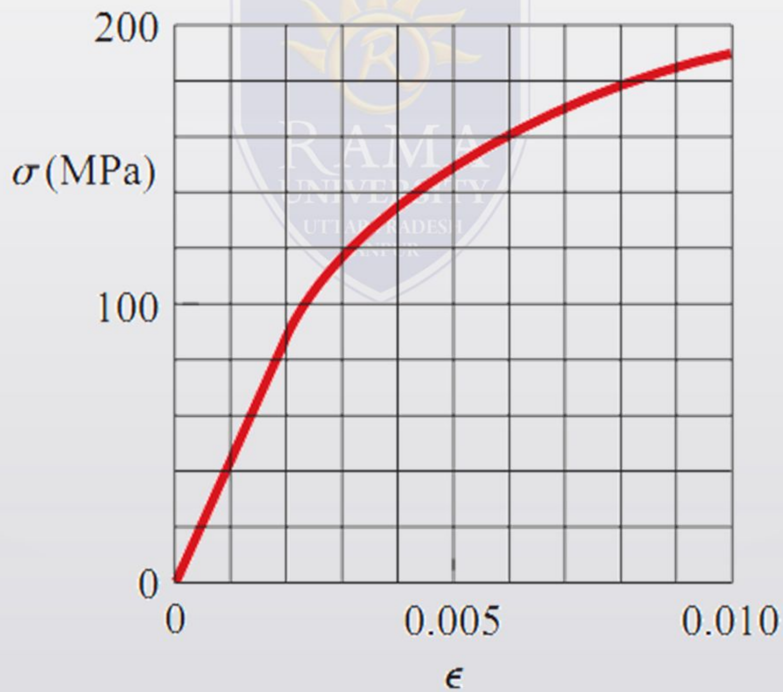
Review Questions

Q1 A bar of length 2.0 m is made of a structural steel having the stress-strain diagram shown in the figure. The yield stress of the steel is 250 MPa and the slope of the initial linear part of the stress-strain curve (modulus of elasticity) is 200 GPa. The bar is loaded axially until it elongates 6.5 mm, and then the load is removed. How does the final length of the bar compare with its original length?



Review Questions

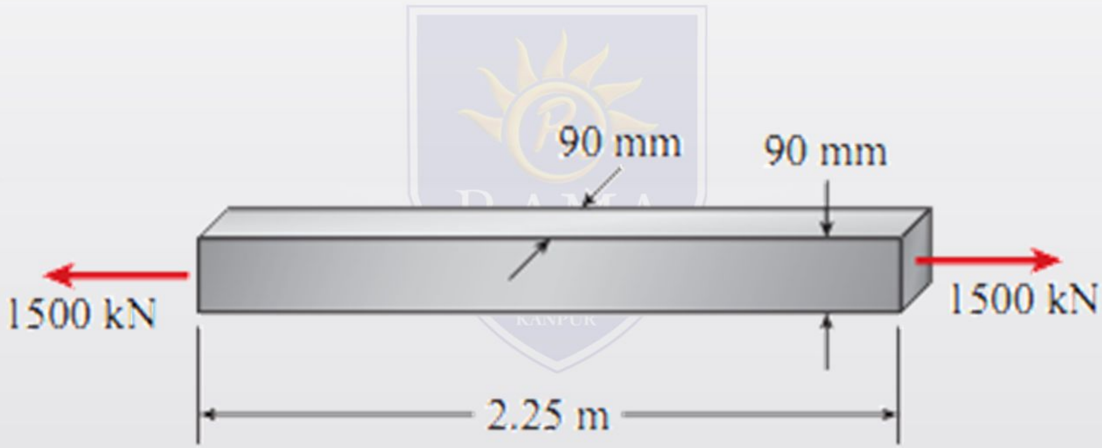
Q2 A circular bar of magnesium alloy is 750 mm long. The stress-strain diagram for the material is shown in the figure. The bar is loaded in tension to an elongation of 6.0 mm, and then the load is removed.
(a) What is the permanent set of the bar?
(b) If the bar is reloaded, what is the proportional limit?



Review Questions

A brass bar of length 2.25 m with a square cross section of 90 mm on each side is subjected to an axial tensile force of 1500 kN (see figure). Assume that E = 110 GPa and $\nu = 0.34$.

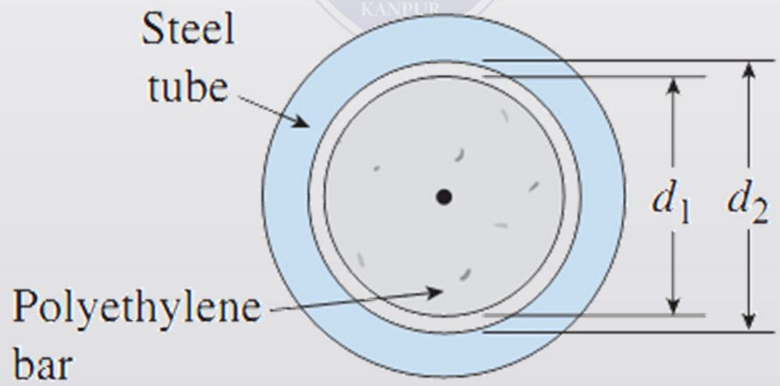
Determine the increase in volume of the bar.



Review Questions

1.6-3 A polyethylene bar having diameter $d_1 = 70$ mm is placed inside a steel tube having inner diameter $d_2 = 70.2$ mm (see figure). The polyethylene bar is then compressed by an axial force P.

At what value of the force P will the space between the polyethylene bar and the steel tube be closed? (For polyethylene, assume $E = 1.4$ GPa and $\upsilon = 0.4$.)



Review Questions

1.6-7 A hollow, brass circular pipe ABC (see figure) supports a load $P_1 = 118$ kN acting at the top. A second load $P_2 = 98$ kN is uniformly distributed around the cap plate at B. The diameters and thicknesses of the upper and lower parts of the pipe are $d_{AB} = 31$ mm, $t_{AB} = 12$ mm, $d_{BC} = 57$ mm, and $t_{BC} = 9$ mm, respectively. The modulus of elasticity is 96 GPa. When both loads are fully applied, the wall thickness of pipe BC increases by 5×10^{-3} mm.

(a) Find the increase in the inner diameter of pipe segment BC.

(b) Find Poisson's ratio for the brass.

(c) Find the increase in the wall thickness of pipe segment AB and the increase in the inner diameter of AB.