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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 30

Topic: Columns with other support conditions

Instructor:

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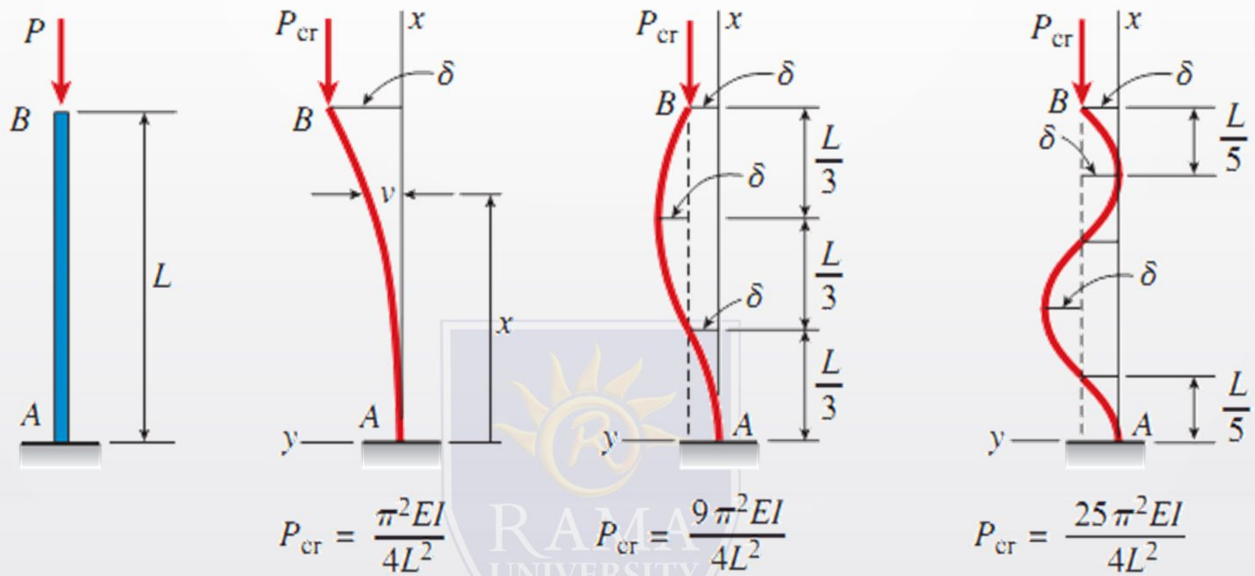
General Procedure

- ✓The critical loads for columns with various kinds of support conditions can be determined from the differential equation of the deflection curve by following the same procedure as above
  - ✓ **Step 1** With the column assumed to be in the buckled state, we obtain an ***expression for the bending moment*** in the column.
  - ✓ **Step 2** Set up the ***differential equation*** of the deflection curve, using the bending-moment equation
  - ✓ **Step 3 *Solve the equation*** and obtain its general solution, which contains two constants of integration plus any other unknown quantities.
  - ✓ **Step 4 *Apply boundary conditions*** pertaining to the deflection v and the slope v' and obtain a set of simultaneous equations.
  - ✓ **Step 5** Obtain the equation of deflection curve for buckled column
  - ✓ **Step 6** Solve those equations to obtain the ***critical load***

Fixed-Free column



$$M = -P(\delta - v)$$

$$EIv'' + P(\delta - v) = 0$$

$$\frac{d^2v}{dx^2} + k^2v = k^2\delta$$

$$v = C_1 \sin kx + C_2 \cos kx + \delta$$

Fixed-Free column

$$v(0) = 0 \quad \Rightarrow \; C_2 = -\delta$$

$$v'(0) = 0 \Rightarrow C_1 = 0$$

$$v = \delta(1 - \cos kx)$$

Applying third boundary condition

$$v(L) = \delta \Rightarrow \delta \cos kL = 0$$

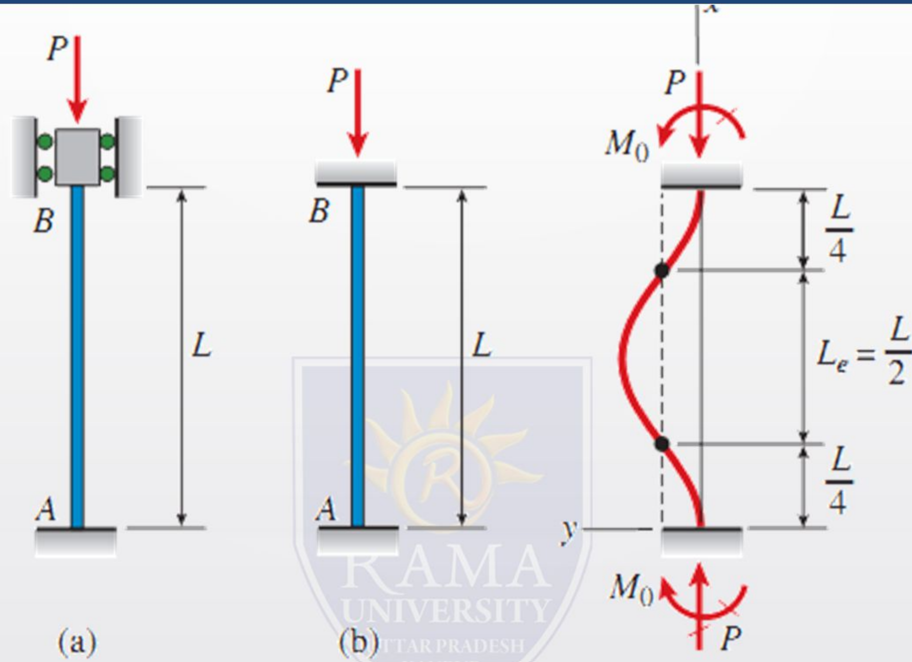
$$kL = \frac{n\pi}{2} \quad n = 1,3,5 \ldots.$$

$$P_{cr} = \frac{n^2\pi^2 EI}{4L^2}$$

Lowest critical load

$$P_{cr} = \frac{\pi^2 EI}{4L^2}$$

Fixed-Fixed Column



$$M = -(M_0 - Pv)$$

$$EIv'' + Pv - M_0 = 0$$

$$\frac{d^2v}{dx^2} + k^2 v = k^2 \frac{M_0}{P}$$

$$v = C_1 \sin kx + C_2 \cos kx + \frac{M_0}{P}$$

Fixed-Fixed Column

$$v(0) = 0 \;\Rightarrow\; C_2 = -\frac{M_0}{P}$$

$$v'(0) = 0 \;\Rightarrow\; C_1 = 0$$

$$v = -\frac{M_0}{P}\cos kx + \frac{M_0}{P}$$

Applying third boundary condition

$$v(L) = 0 \Rightarrow \cos kL = 1$$

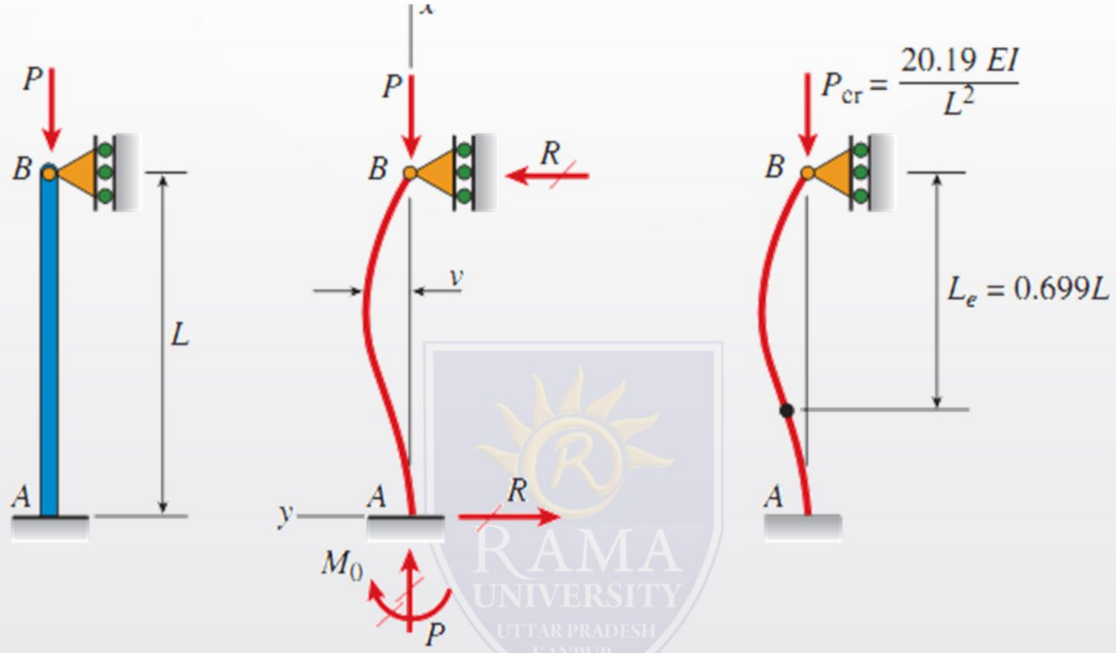
$$kL = \pi \quad n = 2{,}4{,}6 \ldots$$

$$P_{cr} = \frac{n^2\pi^2 EI}{L^2}$$

Lowest critical load

$$P_{cr} = \frac{4\pi^2 EI}{L^2}$$

Fixed-Pinned Column



$$M = -Pv + R(L - x)$$

$$EIv'' - Pv + R(L - x) = 0$$

$$\frac{d^2v}{dx^2} + k^2v = k^2\frac{R}{P}(L - x)$$

$$v = C_1 \sin kx + C_2 \cos kx + \frac{R}{P}(L - x)$$

Fixed-Pinned Column

$$v(0) = 0 \;\Rightarrow\; C_2 = -\frac{R}{P}L$$

$$v'(0) = 0 \;\Rightarrow C_1 = \frac{R}{kP}$$

$$v = \frac{R}{kP}\sin kx - \frac{R}{P}L\cos kx + \frac{R}{P}(L - x)$$

Applying third boundary condition

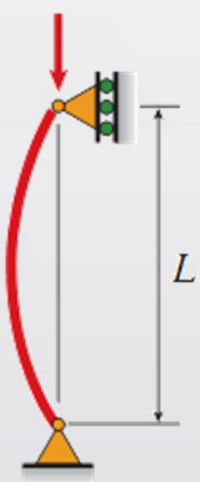
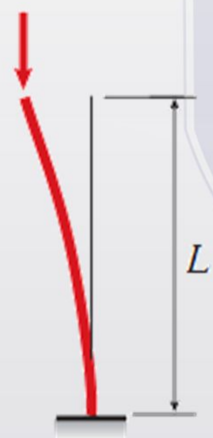
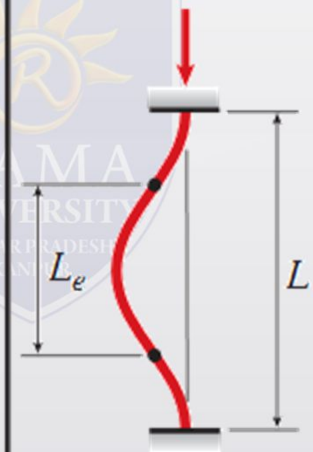
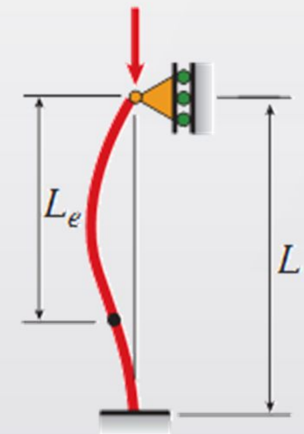
$$v(L) = 0 \Rightarrow \tan kL = kL$$

$$kL = 4.4934$$

Lowest critical load

$$P_{cr} = \frac{2.046\pi^2 EI}{L^2}$$

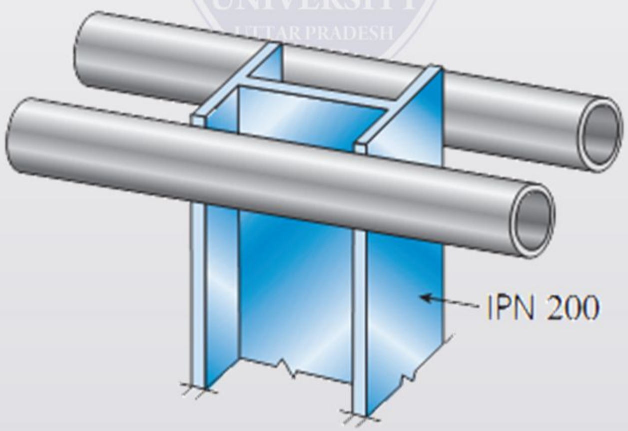
Summary

(a) Pinned-pinned column	(b) Fixed-free column	(c) Fixed-fixed column	(d) Fixed-pinned column
$P_{cr} = \dfrac{\pi^2 EI}{L^2}$	$P_{cr} = \dfrac{\pi^2 EI}{4L^2}$	$P_{cr} = \dfrac{4\pi^2 EI}{L^2}$	$P_{cr} = \dfrac{2.046\ \pi^2 EI}{L^2}$
			
$L_e = L$	$L_e = 2L$	$L_e = 0.5L$	$L_e = 0.699L$
$K = 1$	$K = 2$	$K = 0.5$	$K = 0.699$

Question

11.4-5 The upper end of an IPN 200 standard steel column (E = 200 GPa) is supported laterally between two pipes (see figure). The pipes are not attached to the column, and friction between the pipes and the column is unreliable. The base of the column provides a fixed support, and the column is 4 m long.

Determine the critical load for the column, considering Euler buckling in the plane of the web and also perpendicular to the plane of the web.

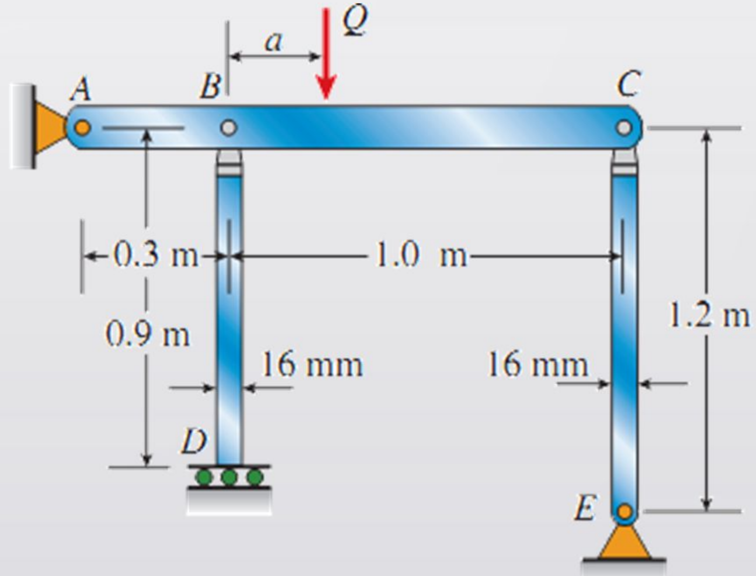


Question

11.4-7 The horizontal beam *ABC* shown in the figure is supported by columns *BD* and *CE*. The beam is prevented from moving horizontally by the pin support at end *A*. Each column is pinned at its upper end to the beam, but at the lower ends, support *D* is a guided support and support *E* is pinned. Both columns are solid steel bars (E = 200 GPa) of square cross section with width equal to 16 mm. A load Q acts at distance a from column *BD*.

(a) If the distance a = 0.5 m, what is the critical value Q_{cr} of the load?

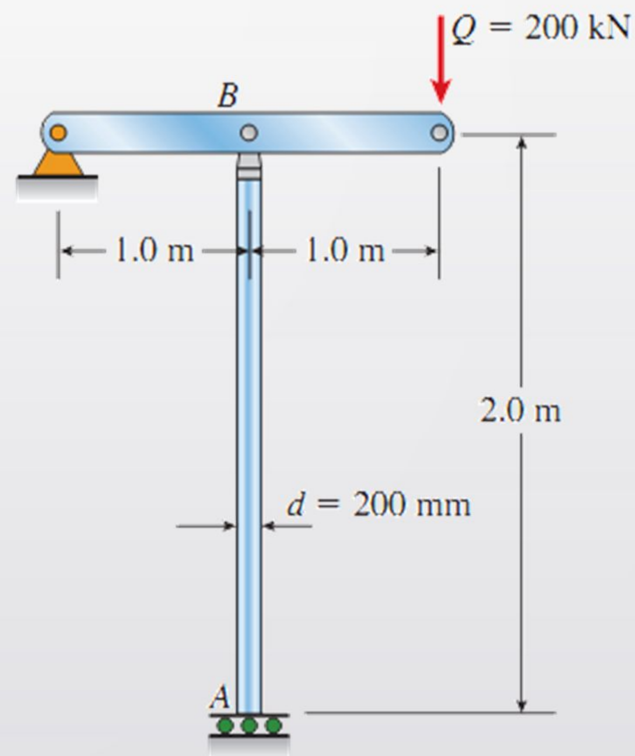
(b) If the distance a can be varied between 0 and 1.0 m, what is the maximum possible value of Q_{cr}? What is the corresponding value of the distance a?



Question

11.4-10 An aluminum tube AB of circular cross section has a guided support at the base and is pinned at the top to a horizontal beam supporting a load $Q = 200$ kN (see figure).

Determine the required thickness t of the tube if its outside diameter d is 200 mm and the desired factor of safety with respect to Euler buckling is $n = 3.0$. (Assume $E = 72$ GPa.)



Question

11.4-11 The frame ABC consists of two members AB and BC that are rigidly connected at joint B, as shown in part a of the figure. The frame has pin supports at A and C. A concentrated load P acts at joint B, thereby placing member AB in direct compression.

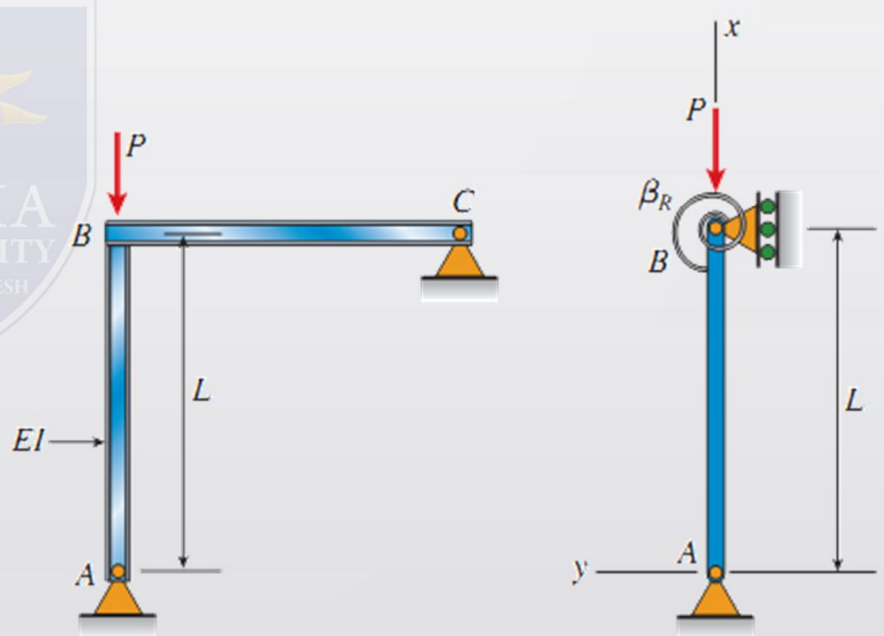
To assist in determining the buckling load for member AB, we represent it as a pinned-end column, as shown in part b of the figure. At the top of the column, a rotational spring of stiffness β_R represents the restraining action of the horizontal beam BC on the column (note that the horizontal beam provides resistance to rotation of joint B when the column buckles). Also, consider only bending effects in the analysis (i.e., disregard the effects of axial deformations).

(a) By solving the differential equation of the deflection curve, derive the following buckling equation for this column:

$$\frac{\beta_R L}{EI}(kL \cot kL - 1) - k^2L^2 = 0$$

in which L is the length of the column and EI is its flexural rigidity.

(b) For the particular case when member BC is identical to member AB, the rotational stiffness β_R equals $3EI/L$ (see Case 7, Table G-2, Appendix G). For this special case, determine the critical load P_{cr}.



Question

11.4-8 The roof beams of a warehouse are supported by pipe columns (see figure) having outer diameter d_2 = 100 mm and inner diameter d_1 = 90 mm. The columns have length L = 4.0 m, modulus E = 210 GPa, and fixed supports at the base.

Calculate the critical load P_{cr} of one of the columns using the following assumptions: (1) the upper end is pinned and the beam prevents horizontal displacement; (2) the upper end is fixed against rotation and the beam prevents horizontal displacement; (3) the upper end is pinned, but the beam is free to move horizontally; and (4) the upper end is fixed against rotation, but the beam is free to move horizontally.