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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 31

Topic: Column with eccentric load

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Column with Eccentric Load

- Ideal columns in which the axial loads acted through the centroids of the cross sections. Under these conditions, the columns remain straight until the critical loads are reached, after which bending may occur.
- Now we will assume that a column is compressed by loads P that are applied with a small eccentricity e measured from the axis of the column
- Each eccentric axial load is equivalent to a centric load P and a couple of moment $\boldsymbol{M_0 = Pe}$
- This moment exists from the instant the load is first applied, and therefore the column begins to deflect at the onset of loading.
- Now follow the same Steps as mentioned before to get answer

$$M = M_0 - Pv = Pe - Pv$$

- From above two equations we will get

$$EIv'' + Pv = Pe$$

$$v'' + \frac{P}{EI}v = \frac{P}{EI}e \quad put\ k^2 = \frac{P}{EI}$$

Column with Eccentric Load

- Therefore, final equation is

$$\frac{d^2v}{dx^2} + k^2v = k^2e$$

- General solution

$$v = C_1 \sin kx + C_2 \cos kx + e$$

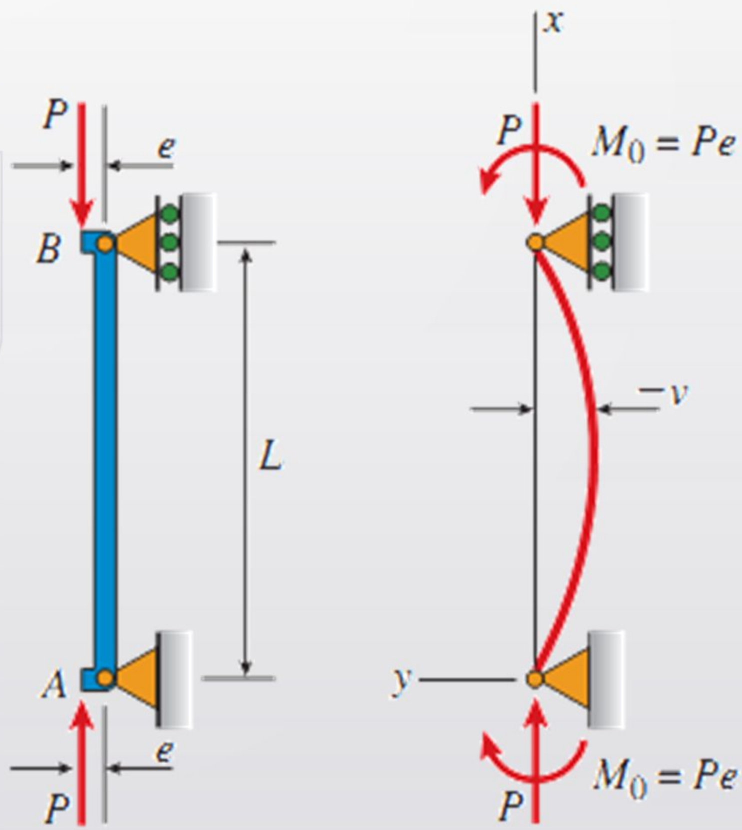
- Boundary conditions

$$v(0) = 0 \quad \& \quad v(L) = 0$$

$$C_1 = -e \quad and \quad C_2 = -e \tan\frac{kL}{2}$$

- Equation of the deflection curve

$$v = -e(\tan\frac{kL}{2}\sin kx + \cos kx - 1)$$



Column with Eccentric Load

- ✓ Maximum Deflection

$$\delta = v\Big|_{\frac{L}{2}} = -e\left(\tan\frac{kL}{2}\sin\frac{kL}{2} + \cos\frac{kL}{2} - 1\right)$$

$$\delta = e\left(\sec\frac{kL}{2} - 1\right) \qquad k = \frac{\pi}{L}\sqrt{\frac{P}{P_{cr}}}$$

$$\delta_{max} = e\left(\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right)$$

Column with Eccentric Load

Maximum Bending Moment

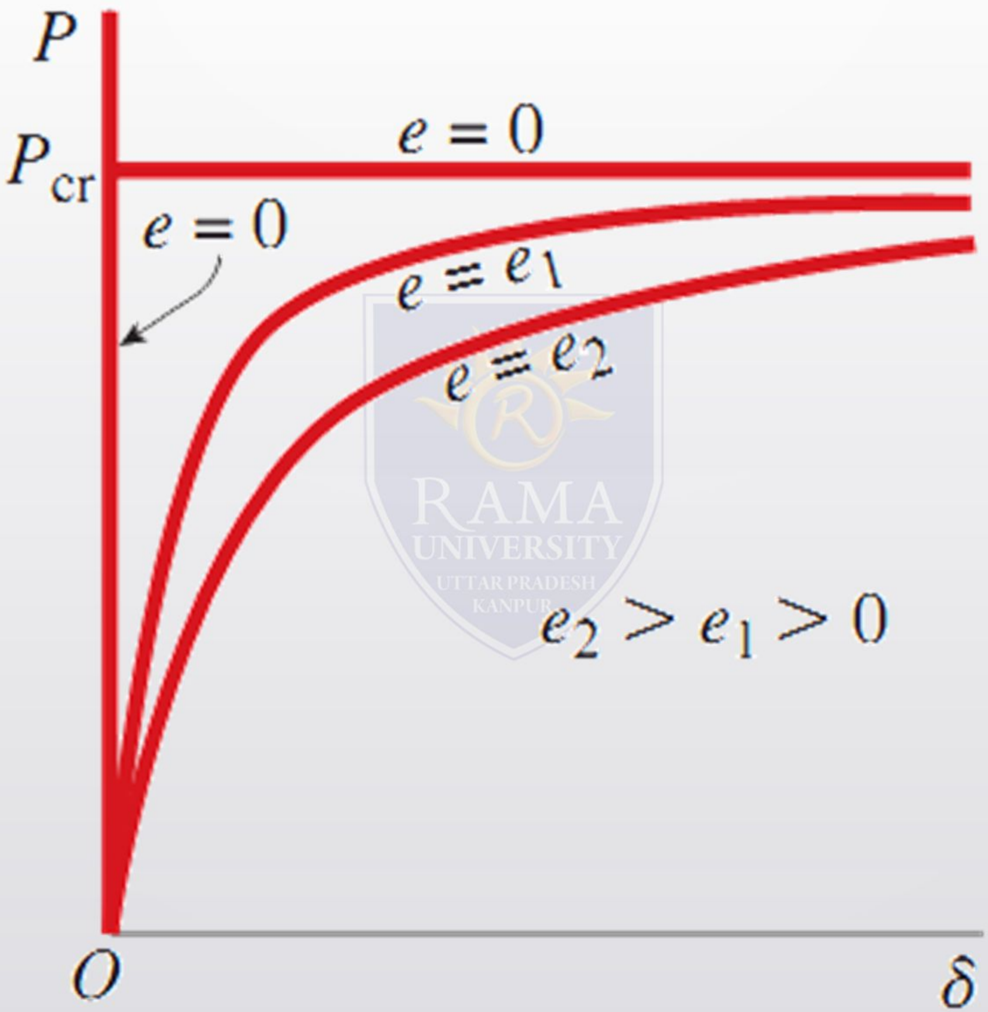
$$M_{max} = P(e+\delta) = Pe\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$

- ✓ Secant Formula
  - ✓ Now we will find out the maximum stress in column

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}c}{I}$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right], \qquad \frac{ec}{r^2} = Eccentricity\ ratio$$

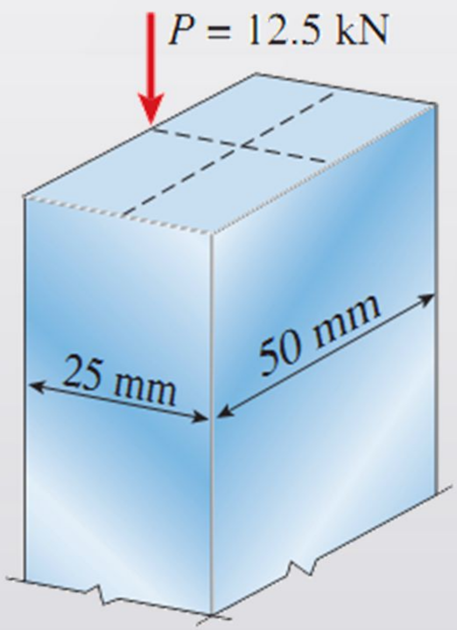
Column with Eccentric Load



Question

11.5-1 An aluminum bar having a rectangular cross section (50 mm × 25 mm) and length $L = 1.0$ m is compressed by axial loads that have a resultant $P = 12.5$ kN acting at the midpoint of the long side of the cross section (see figure).

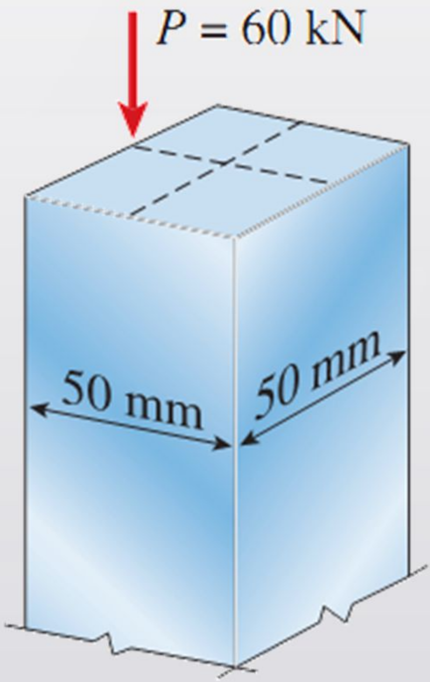
Assuming that the modulus of elasticity E is equal to 70 GPa and that the ends of the bar are pinned, calculate the maximum deflection δ and the maximum bending moment M_{max}.



Question

11.5-2 A steel bar having a square cross section (50 mm × 50 mm) and length $L = 2.0$ m is compressed by axial loads that have a resultant $P = 60$ kN acting at the midpoint of one side of the cross section (see figure).

Assuming that the modulus of elasticity E is equal to 210 GPa and that the ends of the bar are pinned, calculate the maximum deflection δ and the maximum bending moment M_{max}.

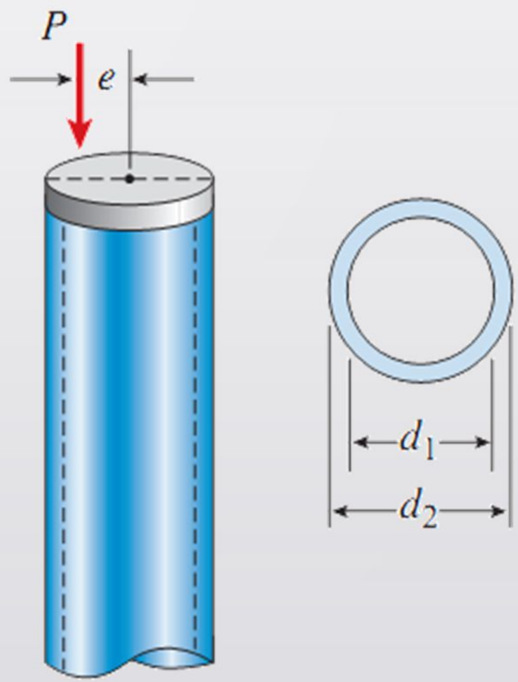


Question

11.6-4 A pinned-end column of length $L = 2.1$ m is constructed of steel pipe ($E = 210$ GPa) having inside diameter $d_1 = 60$ mm and outside diameter $d_2 = 68$ mm (see figure). A compressive load $P = 10$ kN acts with eccentricity $e = 30$ mm.

(a) What is the maximum compressive stress σ_{max} in the column?

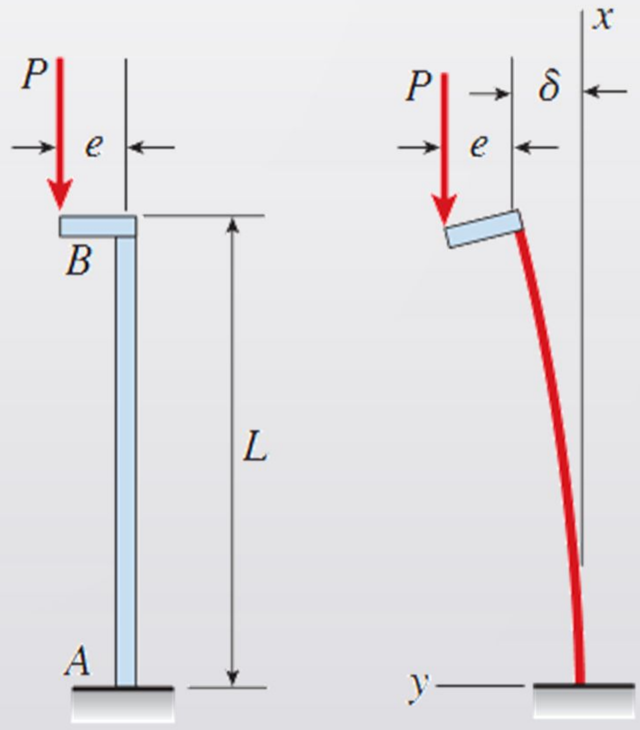
(b) If the allowable stress in the steel is 50 MPa, what is the maximum permissible length L_{max} of the column?



Question

11.5-9 The column shown in the figure is fixed at the base and free at the upper end. A compressive load P acts at the top of the column with an eccentricity e from the axis of the column.

Beginning with the differential equation of the deflection curve, derive formulas for the maximum deflection δ of the column and the maximum bending moment M_{max} in the column.



Question

11.6-1 A steel bar has a square cross section of width $b = 50$ mm (see figure). The bar has pinned supports at the ends and is 1 m long. The axial forces acting at the end of the bar have a resultant $P = 80$ kN located at distance $e = 20$ mm from the center of the cross section. Also, the modulus of elasticity of the steel is 200 GPa.

(a) Determine the maximum compressive stress σ_{max} in the bar.

(b) If the allowable stress in the steel is 125 MPa, what is the maximum permissible length L_{max} of the bar?