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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 34

Topic: Strain Energy in other loading conditions

Instructor:

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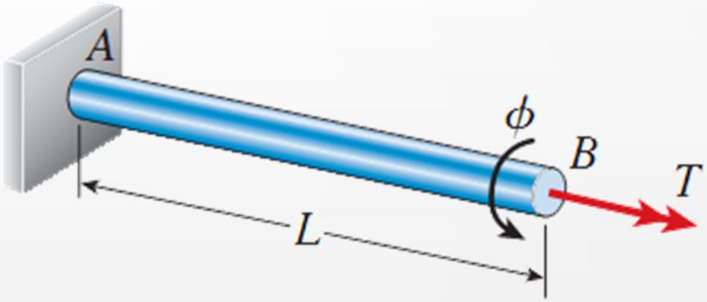
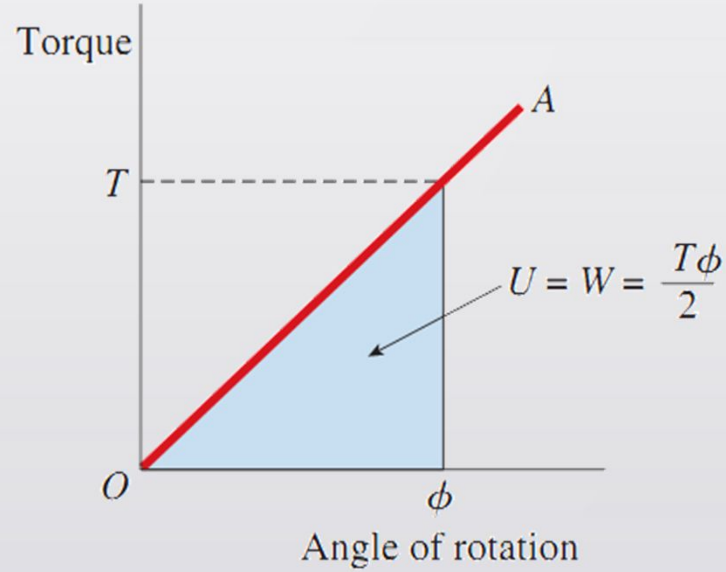
Strain Energy in Torsion and Pure Shear

- ✓Consider a prismatic bar AB in pure torsion under the action of a torque T.
- ✓When the load is applied statically, the bar twists and the free end rotates through an angle φ.
- ✓Assuming that the material of the bar is linearly elastic and follows Hooke's law, then the relationship between the applied torque and the angle of twist will also be linear.





Strain Energy in Torsion and Pure Shear

$$U = W = \frac{T\phi}{2}$$

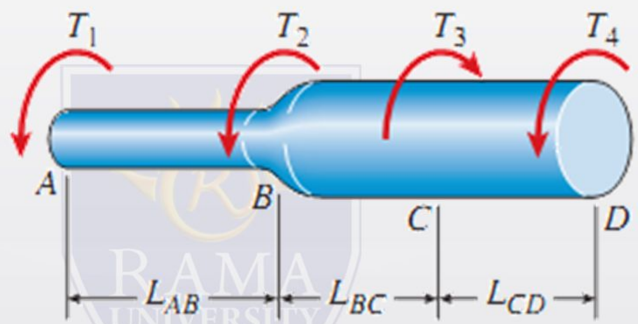
And we know that

$$\phi = \frac{TL}{GI_P}$$

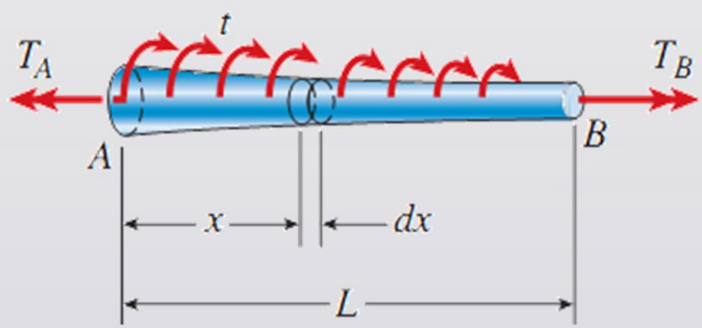
Therefore

$$\frac{T^2L}{2GI_P} \; and \; \frac{GI_P\phi^2}{2L}$$

For nonuniform case



$$U = \sum_{i=1}^{n} \frac{T_i^2 L_i}{2G_i(I_P)_i}$$



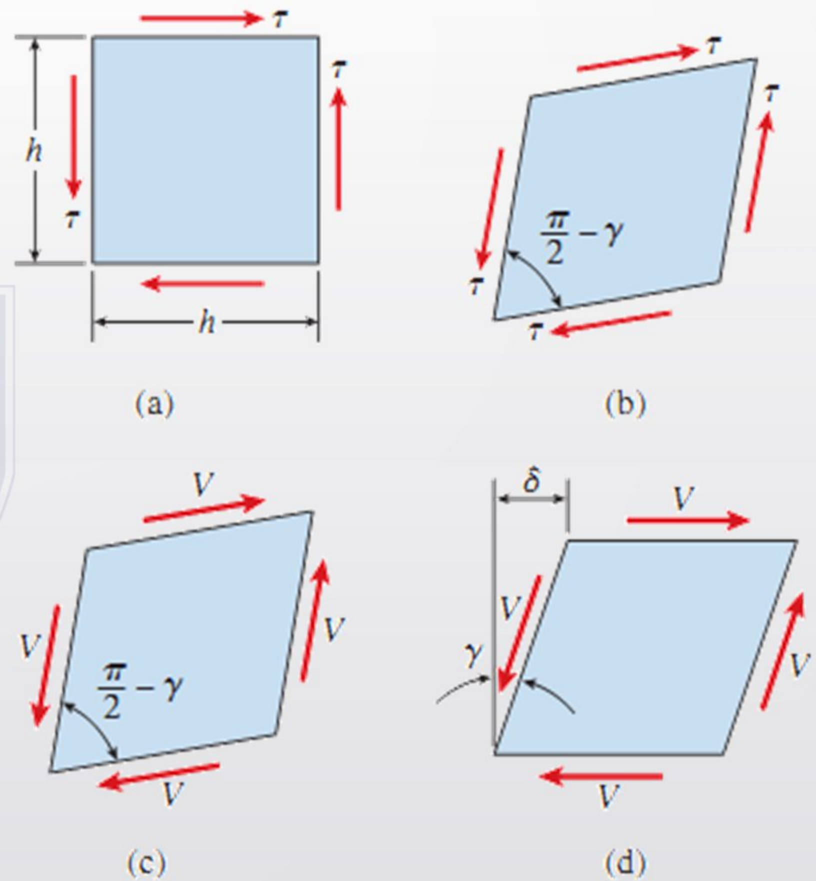
$$U = \int \frac{T^2(x)}{2ET_P(x)} dx$$

Strain-Energy Density in Pure Shear

- ✓ Assume that the front face of the element is square, with each side having length h. Although the figure shows only a two-dimensional view of the element, we recognize that the element is actually three dimensional with thickness t perpendicular to the plane of the figure

- ✓ The shear forces V acting on the side faces of the element are found by multiplying the stresses by the areas ht over which they ac

$$V = \tau h t$$



Strain-Energy Density in Pure Shear

- ✓ The top face of the element is displaced horizontally through a distance δ (relative to the bottom face) as the shear force is gradually increased from zero to its final value V. The displacement δ is equal to the product of the shear strain γ (which is a small angle) and the vertical dimension of the element

$$\delta = \gamma h$$

$$U = W = \frac{V\delta}{2}$$

Therefore

$$u = \frac{1}{2}\tau\gamma = \frac{\tau^2}{2G} = \frac{G\gamma^2}{2}$$

Strain Energy of Bending

- Simple beam AB in pure bending under the action of two couples, each having a moment M.

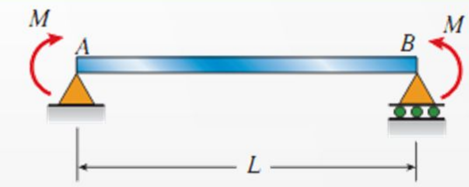
$$\theta = \frac{L}{\rho} = \kappa L = \frac{ML}{EI}$$

$$U = W = \frac{M\theta}{2}$$

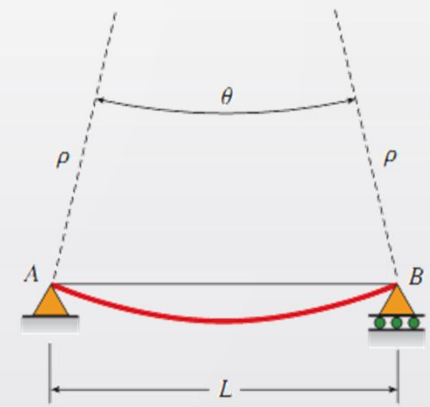
$$\frac{M^2 L}{2EI} \; and \; \frac{EI}{2L}(\theta)^2$$

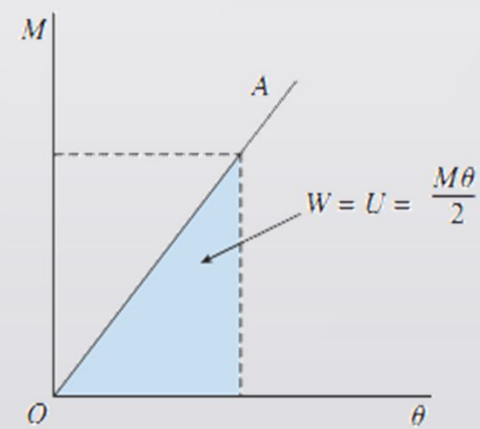
- If the bending moment in a beam varies along its length (nonuniform bending), then

$$\int \frac{M^2}{2EI} dx \;\; and \;\; \int \frac{EI}{2}\left(\frac{d^2 v}{dx^2}\right)^2 dx$$



(a)



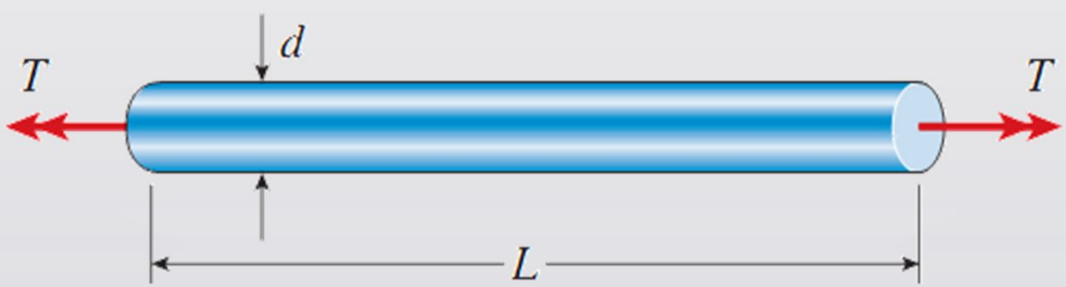


Question

3.9-1 A solid circular bar of steel (G = 80 GPa) with length L = 1.5 m and diameter d = 75 mm is subjected to pure torsion by torques T acting at the ends (see figure).

(a) Calculate the amount of strain energy U stored in the bar when the maximum shear stress is 45 MPa.

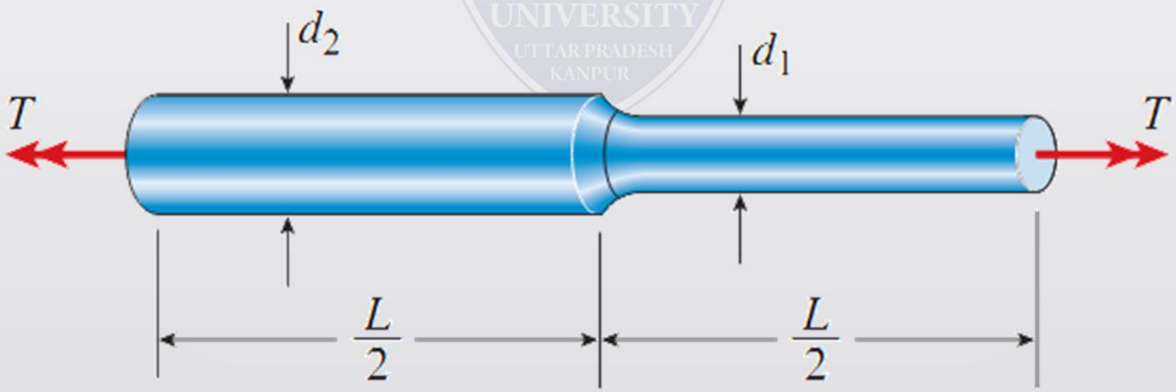
(b) From the strain energy, calculate the angle of twist ϕ (in degrees).



Question

3.9-3 A stepped shaft of solid circular cross sections (see figure) has length $L = 2.6$ m, diameter $d_2 = 50$ mm, and diameter $d_1 = 40$ mm. The material is brass with $G = 81$ GPa.

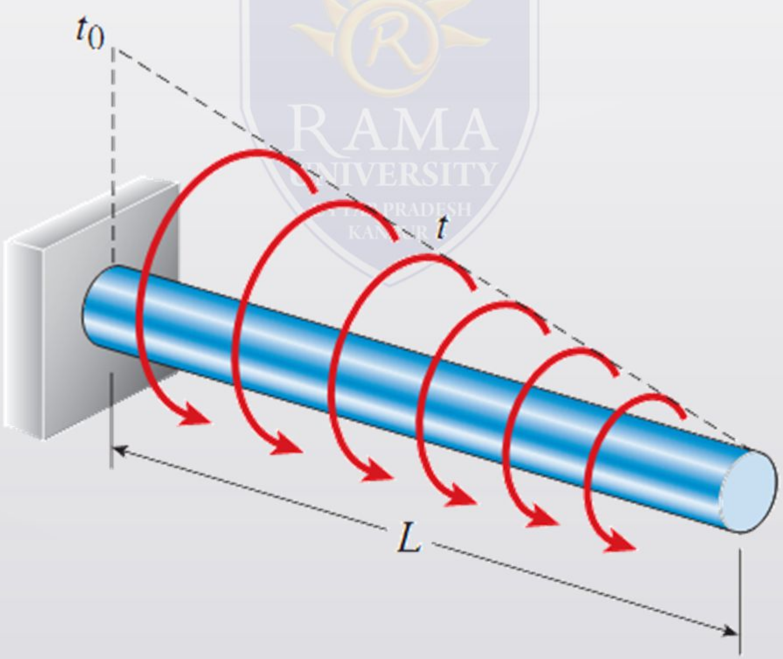
Determine the strain energy U of the shaft if the angle of twist is 3.0°.



Question

3.9-8 Derive a formula for the strain energy U of the cantilever bar shown in the figure.

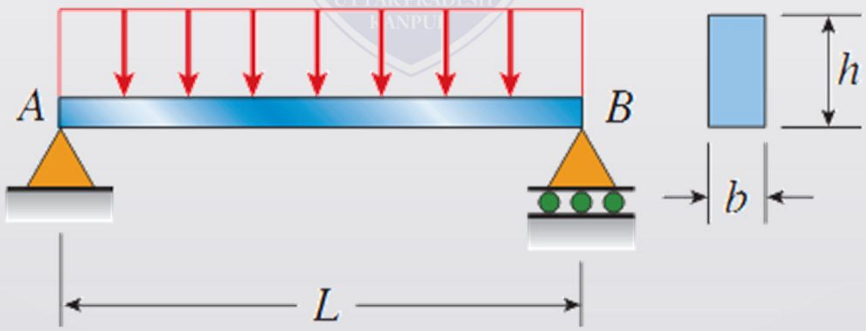
The bar has circular cross sections and length L. It is subjected to a distributed torque of intensity t per unit distance. The intensity varies linearly from $t = 0$ at the free end to a maximum value $t = t_0$ at the support.



Question

9.8-1 A uniformly loaded simple beam AB (see figure) of span length L and rectangular cross section (b = width, h = height) has a maximum bending stress σ_{max} due to the uniform load.

Determine the strain energy U stored in the beam.



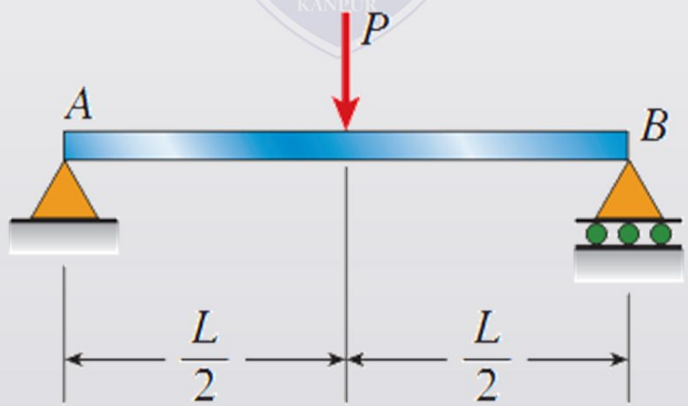
Question

9.8-2 A simple beam AB of length L supports a concentrated load P at the midpoint (see figure).

(a) Evaluate the strain energy of the beam from the bending moment in the beam.

(b) Evaluate the strain energy of the beam from the equation of the deflection curve.

(c) From the strain energy, determine the deflection δ under the load P.



Question

9.8-4 A simple beam AB of length L is subjected to loads that produce a symmetric deflection curve with maximum deflection δ at the midpoint of the span (see figure).

How much strain energy U is stored in the beam if the deflection curve is (a) a parabola, and (b) a half wave of a sine curve?