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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 35

Topic:Castigliano's Theorem

Instructor:

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Castigliano's Theorem

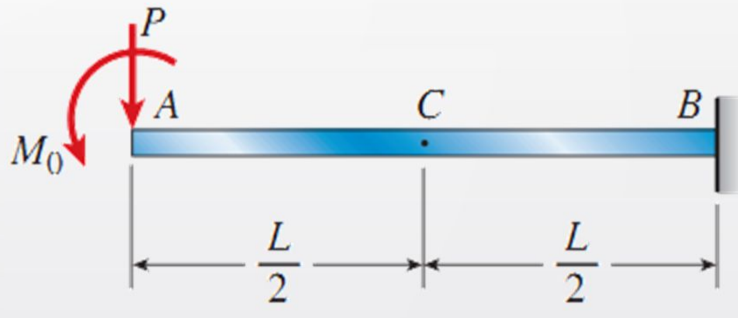
- ✓ It provides a means for finding the deflections of a structure from the strain energy of the structure
- ✓ *The partial derivative of the strain energy of a structure with respect to any load is equal to the displacement corresponding to that load*

$$\delta_i = \frac{\partial U}{\partial P_i}$$

- ✓ The theorem can be applied to any kind of loading. The important requirements are that the structure be linearly elastic and that the principle of superposition be applicable. Also, note that the strain energy must be expressed as a function of the loads (and not as a function of the displacements), a condition which is implied in the theorem itself, since the partial derivative is taken with respect to a load.

Application of Castigliano’s Theorem

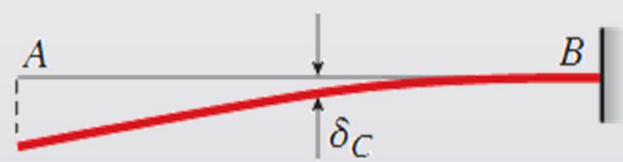
$$M = -Px - M_0$$



(a)

$$U = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\int_0^L (-Px - M_0)^2 dx$$

$$= \frac{P^2L^3}{6EI} + \frac{PM_0L^2}{2EI} + \frac{M_0^2L}{2EI}$$



$$\delta_A = \frac{\partial U}{\partial P} = \frac{PL^3}{3EI} + \frac{M_0L^2}{2EI}$$

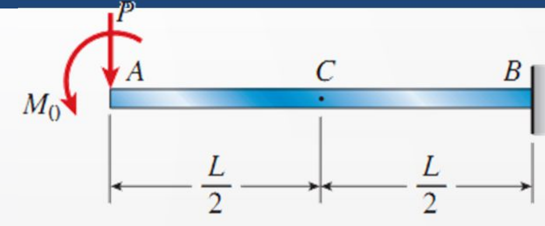
$$\theta_A = \frac{\partial U}{\partial M_0} = \frac{PL^2}{2EI} + \frac{M_0L}{EI}$$

Use of a Fictitious Load

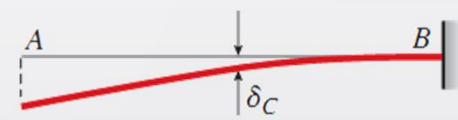
- ✓ The only displacements that can be found from Castigliano's theorem are those that correspond to loads acting on the structure.
- ✓ If we wish to calculate a displacement at a point on a structure where there is no load, then a **fictitious load** corresponding to the desired displacement must be applied to the structure.
- ✓ We can then determine the displacement by evaluating the strain energy and taking the partial derivative with respect to the fictitious load.
- ✓ The result is the displacement produced by the actual loads and the fictitious load acting simultaneously.
- ✓ By setting the fictitious load equal to zero, we obtain the displacement produced only by the actual loads.

Use of a Fictitious Load

✓ We wish to find out deflection at C



(a)



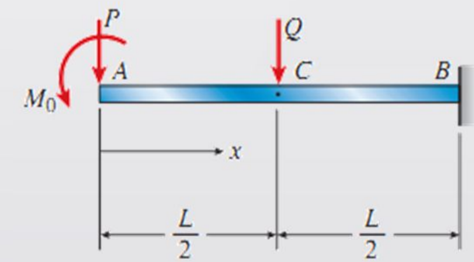
$$M = -Px - M_0 \qquad \left(0 \leq x \leq \frac{L}{2}\right)$$

$$M = -Px - M_0 - Q\left(x - \frac{L}{2}\right) \quad \left(\frac{L}{2} \leq x \leq L\right)$$

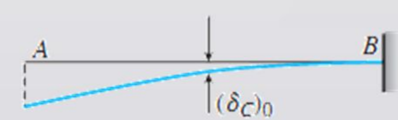
$$U_{AC} = \int_0^{L/2} \frac{M^2\,dx}{2EI} = \frac{1}{2EI}\int_0^{L/2}(-Px - M_0)^2\,dx$$

$$= \frac{P^2L^3}{48EI} + \frac{PM_0L^2}{8EI} + \frac{M_0^2L}{4EI}$$

Beam with a fictitious load Q



(a)



Use of a Fictitious Load

$$U_{CB} = \int_{L/2}^{L} \frac{M^2\,dx}{2EI} = \frac{1}{2EI}\int_{L/2}^{L}\left[-Px - M_0 - Q\left(x - \frac{L}{2}\right)\right]^2 dx$$

$$= \frac{7P^2L^3}{48EI} + \frac{3PM_0L^2}{8EI} + \frac{5PQL^3}{48EI} + \frac{M_0^2L}{4EI} + \frac{M_0Q^2}{8EI} + \frac{Q^2L^3}{48EI}$$

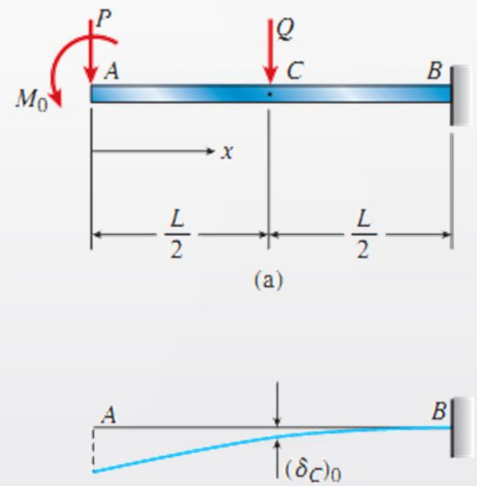
$$U = U_{AC} + U_{CB}$$

$$= \frac{P^2L^3}{6EI} + \frac{PM_0L^2}{2EI} + \frac{5PQL^3}{48EI} + \frac{M_0^2L}{2EI} + \frac{M_0QL^2}{8EI} + \frac{Q^2L^3}{48EI}$$

$$(\delta_C)_0 = \frac{\partial U}{\partial Q} = \frac{5PL^3}{48EI} + \frac{M_0L^2}{8EI} + \frac{QL^3}{24EI}$$

$$\delta_C = \frac{5PL^3}{48EI} + \frac{M_0L^2}{8EI}$$

Beam with a fictitious load Q



This method is sometimes called the dummy-load method, because of the introduction of a fictitious, or dummy, load.

Modified Castigliano's theorem (Unit Load Method)

- ✓ The use of Castigliano's theorem for determining beam deflections may lead to lengthy integrations, if the bending moment expression has three terms, its square may have as many as six terms, each of which must be integrated.

- ✓ we can bypass the step of finding the strain energy by differentiating before integrating.

$$\delta_i = \frac{\partial U}{\partial P_i} = \frac{\partial}{\partial P_i}\int \frac{M^2}{2EI}dx = \int \frac{M}{EI}\left(\frac{\partial M}{\partial P_i}\right)dx$$

Question

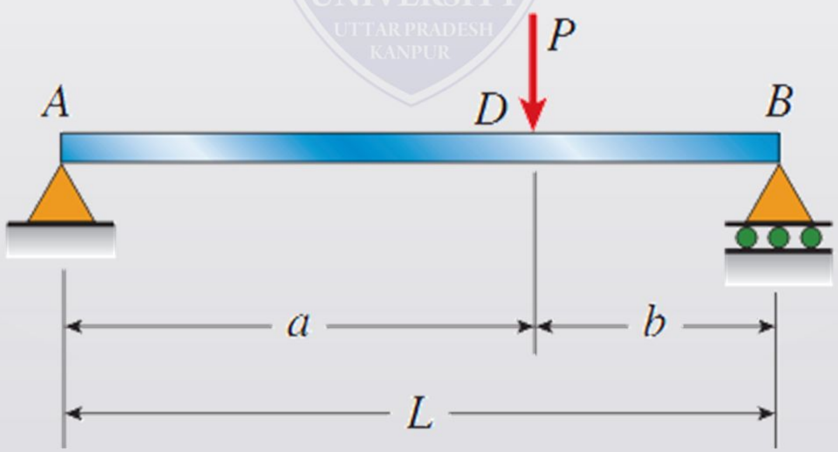
9.9-1 A simple beam *AB* of length L is loaded at the left-hand end by a couple of moment M_0 (see figure).

Determine the angle of rotation θ_A at support *A*. (Obtain the solution by determining the strain energy of the beam and then using Castigliano's theorem.)

Question

9.9-2 The simple beam shown in the figure supports a concentrated load P acting at distance a from the left-hand support and distance b from the right-hand support.

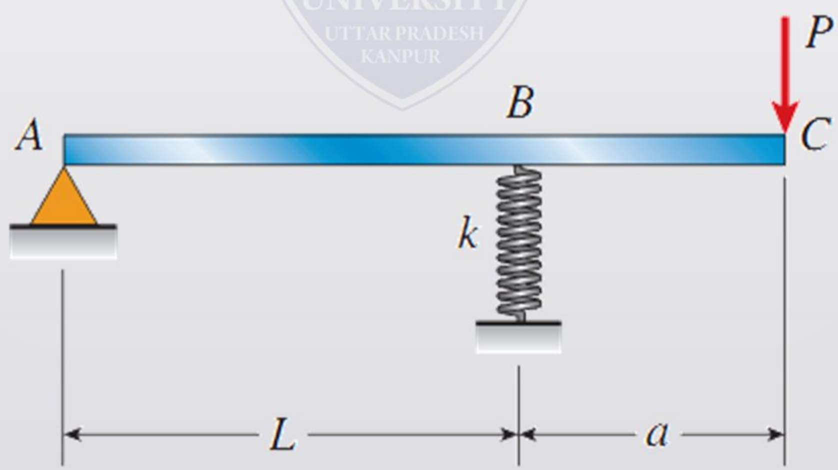
Determine the deflection δ_D at point D where the load is applied. (Obtain the solution by determining the strain energy of the beam and then using Castigliano's theorem.)



Question

9.9-11 An overhanging beam ABC rests on a simple support at A and a spring support at B (see figure). A concentrated load P acts at the end of the overhang. Span AB has length L, the overhang has length a, and the spring has stiffness k.

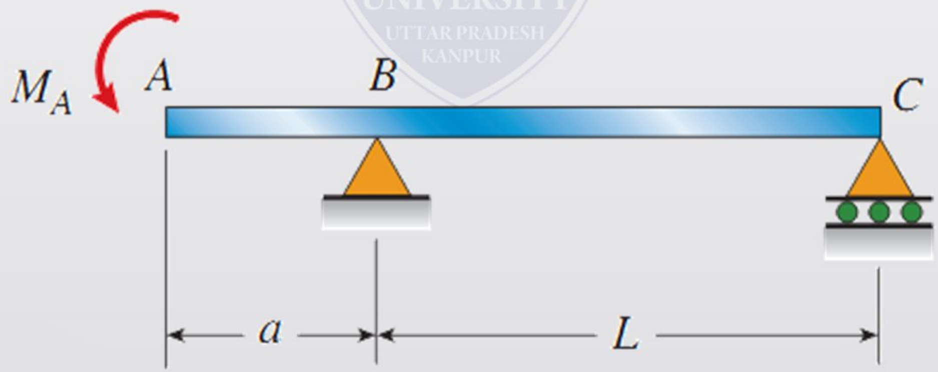
Determine the downward displacement δ_C of the end of the overhang. (Obtain the solution by using the modified form of Castigliano's theorem.)



Question

9.9-10 An overhanging beam ABC is subjected to a couple M_A at the free end (see figure). The lengths of the overhang and the main span are a and L, respectively.

Determine the angle of rotation θ_A and deflection δ_A at end A. (Obtain the solution by using the modified form of Castigliano's theorem.)



Question

9.9-7 The cantilever beam *ACB* shown in the figure is subjected to a uniform load of intensity q acting between points A and C.

Determine the angle of rotation θ_A at the free end A. (Obtain the solution by using the modified form of Castigliano's theorem.)