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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 36

Topic: Helical Springs

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Deflection of springs by energy method

✓ Shaft under constant torque

$$Solid\ u = \frac{\tau^2}{4G}(\text{Volume})$$

Let d = Diameter of spring wire

p = Pitch of the helical spring

n = Number of coils

R = Mean radius of spring coil

W = Axial load on spring

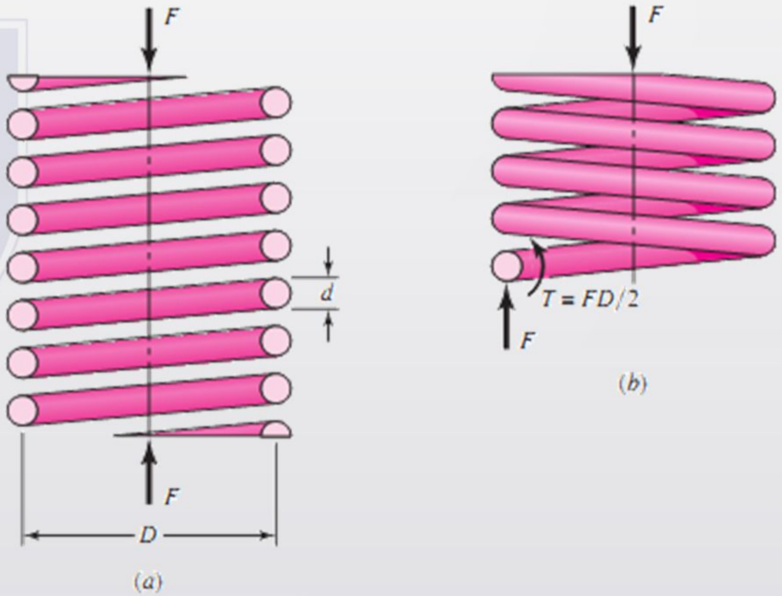
C = Modulus of rigidity

τ = Max. shear stress induced in the wire

θ = Angle of twist in spring wire, and

δ = Deflection of spring due to axial load

l = Length of wire.



$$\tau_{max} = \frac{Tr}{J} + \frac{F}{A}$$

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Deflection of springs by energy method

Now twisting moment on the wire,

$$T = W \times R$$

But twisting moment is also given by

$$T = \frac{\pi}{16} \tau d^3$$

Equating equations (*i*) and (*ii*), we get

$$W \times R = \frac{\pi}{16} \tau d^3 \quad \text{or} \quad \tau = \frac{16W \times R}{\pi d^3}$$

Expression for deflection of spring

Now length of one coil $= \pi D$ or $2\pi R$

$\therefore$ Total length of the wire $=$ Length of one coil $\times$ No. of coils or $l = 2\pi R \times n$.

As the every section of the wire is subjected to torsion, hence the strain energy stored by the spring due to torsion is given by equation (16.20).

$\therefore$ Strain energy stored by the spring,

$$U = \frac{\tau^2}{4C} \text{ . Volume} = \frac{\tau^2}{4C} \text{ . Volume}$$

$$= \left(\frac{16W.R}{\pi d^3}\right)^2 \times \frac{1}{4C} \times \left(\frac{\pi}{4} d^2 \times 2\pi R.n\right)$$

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Deflection of springs by energy method

$$\left(\because \quad \tau = \frac{16WR}{\pi d^3} \text{ and Volume} = \frac{\pi}{4} d^2 \times \text{Total length of wire}\right)$$

$$= \frac{32W^2R^2}{Cd^4} .R.n = \frac{32W^2R^3 .n}{Cd^4}$$

Work done on the spring = Average load × Deflection

$$= \frac{1}{2} W \times \delta$$

Equating the work done on spring to the energy stored, we get

$$\frac{1}{2} W.\delta = \frac{32W^2R^3 .n}{Cd^4}$$

$$\therefore \qquad \delta = \frac{64WR^3n}{Cd^4}$$

Expression for stiffness of spring

The stiffness of spring,

$$s = \text{Load per unit deflection}$$

$$= \frac{W}{\delta} = \frac{W}{\frac{64 . WR^3 .n}{Cd^4}} = \frac{Cd^4}{64 . R^3 .n}$$

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Question

Problem 16.35. *A closely coiled helical spring is to carry a load of 500 N. Its mean coil diameter is to be 10 times that of the wire diameter. Calculate these diameters if the maximum shear stress in the material of the spring is to be 80 N/mm².*

Sol. Given :

Load on spring, $W = 500$ N

Max. shear stress, $\tau = 80$ N/mm²

Let d = Diameter of wire

D = Mean diameter of coil

$\therefore$ $D = 10\,d.$

Using equation (16.24), $\tau = \dfrac{16WR}{\pi d^3}$

$$80 = \frac{16 \times 500 \times \left(\frac{D}{2}\right)}{\pi d^3}$$

$$= \frac{8000 \times \left(\frac{10d}{2}\right)}{\pi d^3}$$

$$80 \times \pi d^3 = 8000 \times 5d$$

$$d^2 = \frac{8000 \times 5}{80 \times \pi} = 159.25$$

$$d = \sqrt{159.15} = 12.6 \text{ mm} = \mathbf{1.26 \text{ cm}}. \quad \textbf{Ans.}$$

$$D = 10 \times d = 10 \times 1.26 = \mathbf{12.6 \text{ cm}}. \quad \textbf{Ans.}$$

Question

Problem 16.43. *A closely coiled helical spring made of 10 mm diameter steel wire has 15 coils of 100 mm mean diameter. The spring is subjected to an axial load of 100 N. Calculate :*

(*i*) *The maximum shear stress induced,*

(*ii*) *The deflection, and*

(*iii*) *Stiffness of the spring.*

Take modulus of rigidity, $C = 8.16 \times 10^4 \text{ N/mm}^2$.

Sol. Given :

Dia. of wire, $d = 10$ mm

Number of coils, $n = 15$

Mean dia. of coil, $D = 100$ mm

$\therefore$ Mean radius of coil, $R = \dfrac{100}{2} = 50$ mm

Axial load, $W = 100$ N

Modulus of rigidity, $C = 8.16 \times 10^4 \text{ N/mm}^2$.

Question

(*i*) *Maximum shear stress induced*

Using equation (16.24), $\tau = \frac{16WR}{\pi d^3} = \frac{16 \times 100 \times 50}{\pi \times 10^3}$ $=$ **24.46 N/mm². Ans.**

(*ii*) *The deflection* (δ)

Using equation (16.26),

$$\delta = \frac{64W \times R^3 \times n}{C \times d^4} = \frac{64 \times 100 \times 50^3 \times 15}{8.16 \times 10^4 \times 10^4}$$

$=$ **14.7 mm. Ans.**

(*iii*) *Stiffness of the spring*

$$\text{Stiffness} = \frac{\text{Load on spring}}{\text{Deflection of spring}}$$

$$= \frac{\text{Load on spring}}{\text{Deflection of spring}} = \frac{100}{14.7} = \textbf{6.802 N/mm.} \quad \textbf{Ans.}$$

Question

Problem 16.42. *Two close-coiled concentric helical springs of the same length, are wound out of the same wire, circular in cross-section and supports a compressive load 'P'. The inner spring consists of 20 turns of mean diameter 16 cm and the outer spring has 18 turns of mean diameter 20 cm. Calculate the maximum stress produced in each spring if the diameter of wire = 1 cm and P = 1000 N.* (AMIE, Summer 1989)

Sol. Given :

Total load supported, $P = 1000$ N

Both the springs are of the same length of the same material and having same dia. of wire. Hence values of L, C and 'd' will be same.

For inner spring

No. of turns, $n_i = 20$

Mean dia., $D_i = 16 \text{ cm} = 160 \text{ mm}$ $\therefore$ $R_i = \frac{160}{2} = 80 \text{ mm}$

Dia. of wire, $d_i = 1 \text{ cm} = 10 \text{ mm}$

Question

For outer spring

No. of turns, $n_0 = 18$

Mean dia., $D_0 = 20 \text{ cm} = 200 \text{ mm} \quad \therefore \quad R_0 = \frac{200}{2} = 100 \text{ mm}$

Dia. of wire, $d_0 = 1 \text{ cm} = 10 \text{ mm}$

Let W_i = Load carried by inner spring

W_0 = Load carried by outer spring

τ_i = Max. shear stress produced in inner spring

τ_0 = Max. shear stress produced in outer spring.

Now $W_i + W_0$ = Total load carried = 1000 ...(*i*)

Since there are two *close-coiled concentric* helical springs, hence deflection of both the springs will be same.

$\therefore \quad \delta_0 = \delta_i$ where δ_0 = Deflection of outer spring

δ_i = Deflection of inner spring.

Question

The deflection of close-coiled spring is given by equation (16.26) as

$$\delta = \frac{64W \times R^3 \times n}{C \times d^4}$$

Hence for outer spring, we have

$$\delta_0 = \frac{64W_0 \times R_0^{\ 3} \times n_0}{C \times d_0^{\ 4}} = \frac{64W_0 \times 100^3 \times 18}{C \times 10^4} \qquad (\because \quad R_0 = 100, d_0 = 10)$$

Similarly for inner spring, we have

$$\delta_i = \frac{64W_i \times R_i \times n_i}{C \times d_i^{\ 4}} = \frac{64W_i \times 80^3 \times 20}{C \times 10^4}$$

(Material of wires is same. Hence value of C will be same.)

But $\qquad \delta_0 = \delta_i$

$$\therefore \quad \frac{64W_0 \times 100^3 \times 18}{C \times 10^4} = \frac{64W_i \times 80^3 \times 20}{C \times 10^4}$$

$$W_0 \times 100^3 \times 18 = W_i \times 80^3 \times 20$$

Question

$$W_0 = \frac{W_i \times 80^3 \times 20}{100^3 \times 18} = 0.569\ W_i.$$

Substituting the value of W_0 in equation (i), we get

$$W_i + 0.569\ W_i = 1000 \quad \text{or} \quad 1.569\ W_i = 1000$$

$$\therefore \qquad W_i = \frac{1000}{1.569} = 637.3\ \text{N}.$$

But from equation (i), $W_i + W_0 = 1000$

$$\therefore \qquad W_0 = 1000 - W_i = 1000 - 637.3 = 362.7\ \text{N}.$$

The maximum shear stress produced is given by equation (16.24) as

$$\tau = \frac{16\ WR}{\pi d^3}$$

For outer spring, the maximum shear stress produced is given by,

$$\tau_0 = \frac{64 W_0 \times R_0}{\pi d_0{}^3} = \frac{16 \times 362.7 \times 100}{\pi \times 10^3}$$

$$= \mathbf{184.72\ N/mm^2.} \quad \textbf{Ans.}$$

Similarly for inner spring, the maximum shear stress produced is given by,

$$\tau_i = \frac{16 \times W_i \times R_i}{\pi \times d_i{}^3} = \frac{16 \times 637.3 \times 80}{\pi \times 10^3}$$

$$= \mathbf{259.66\ N/mm^2.} \quad \textbf{Ans.}$$