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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 37

Topic: Theories of Failure I

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Failure

- ✓ Failure can mean a part has separated into two or more pieces; has become permanently distorted, thus ruining its geometry; has had its reliability downgraded; or has had its function compromised, whatever the reason. A designer speaking of failure
- ✓ there is no universal theory of failure for the general case of material properties and stress state. Instead, over the years several hypotheses have been formulated and tested, leading to today's accepted practices.re can mean any or all of these possibilities.
- ✓ Structural metal behavior is typically classified as being ductile or brittle, although under special situations, a material normally considered ductile can fail in a brittle manner

Theories of Failure

The generally accepted theories are

Ductile materials (yield criteria)

- **Maximum shear stress (MSS)**
- **Distortion energy (DE)**
- Ductile Coulomb-Mohr (DCM)

Brittle materials (fracture criteria)

- Maximum normal stress (MNS)
- Brittle Coulomb-Mohr (BCM)
- Modified Mohr (MM)

Maximum-Shear-Stress Theory for Ductile Materials

- *The maximum-shear-stress theory predicts that yielding begins whenever the maximum shear stress in any element equals or exceeds the maximum shear stress in a tension-test specimen of the same material when that specimen begins to yield.*
- The MSS theory is also referred to as the **Tresca or Guest** theory.
- For a general state of stress, three principal stresses can be determined and ordered such that

$$\sigma_1 \geq \sigma_2 \geq \sigma_3$$

- The maximum shear stress is then

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \geq \frac{S_y}{2}$$

Where $S_y = yield\ strength$

Maximum-Shear-Stress Theory for Ductile Materials

- Plane stress problems are very common where one of the principal stresses is zero, and the other two are, σ_A and σ_B . Assuming that $\sigma_A \geq \sigma_B$, there are three cases to consider for plane stress
- Case 1: $\sigma_A \geq \sigma_B \geq 0$. For this case, $\sigma_1 = \sigma_A$ and $\sigma_3 = 0$. Equation reduces to a yield condition of

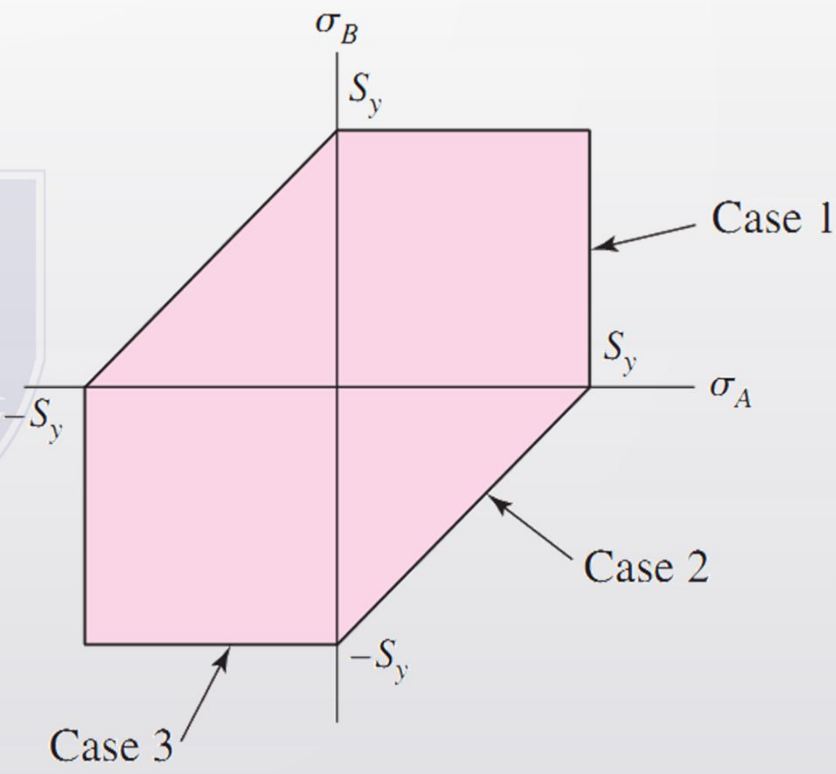
$$\sigma_A \geq S_y$$

- Case 2: $\sigma_A \geq 0 \geq \sigma_B$. Here, $\sigma_1 = \sigma_A$ and $\sigma_3 = \sigma_B$,

$$\sigma_A - \sigma_B \geq S_y$$

- Case 3: $0 \geq \sigma_A \geq \sigma_B$. For this case, $\sigma_1 = 0$ and $\sigma_3 = \sigma_B$

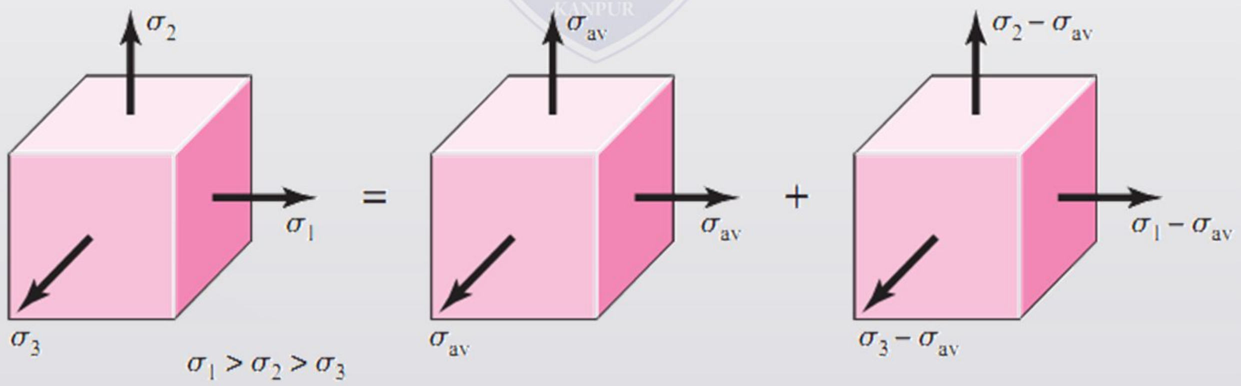
$$\sigma_B \leq -S_y$$



Distortion-Energy Theory for Ductile Materials

- ✓ *The distortion-energy theory predicts that yielding occurs when the distortion strain energy per unit volume reaches or exceeds the distortion strain energy per unit volume for yield in simple tension or compression of the same material.*

- ✓ The distortion-energy (DE) theory originated from the observation that **ductile materials stressed hydrostatically** exhibited yield strengths greatly in excess of the values given by the simple tension test. Therefore it was postulated that yielding was not a simple tensile or compressive phenomenon at all, but, rather, that it was related some-how to the angular distortion of the stressed element



(*a*) Triaxial stresses (*b*) Hydrostatic component (*c*) Distortional component

Distortion-Energy Theory for Ductile Materials

$$\sigma_{av} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

$$u = \tfrac{1}{2}[\epsilon_1\sigma_1 + \epsilon_2\sigma_2 + \epsilon_3\sigma_3]$$

$$u = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

$$u_v = \frac{3\sigma_{av}^2}{2E}(1 - 2\nu)$$

$$u_v = \frac{1 - 2\nu}{6E}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\sigma_1\sigma_2 + 2\sigma_2\sigma_3 + 2\sigma_3\sigma_1\right)$$

$$u_d = u - u_v = \frac{1 + \nu}{3E}\left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}\right]$$

Distortion-Energy Theory for Ductile Materials

- For the simple tensile test, at yield, $\sigma_1 = S_y$ and $\sigma_2 = \sigma_3 = 0$, and the distortion energy is

$$u_d = \frac{1+\nu}{3E} S_y^2$$

$$\left[\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}\right]^{1/2} \geq S_y \qquad \sigma' \geq S_y$$

- This effective stress is usually called the **von Mises stress, σ'**

$$\sigma' = \left[\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}\right]^{1/2}$$

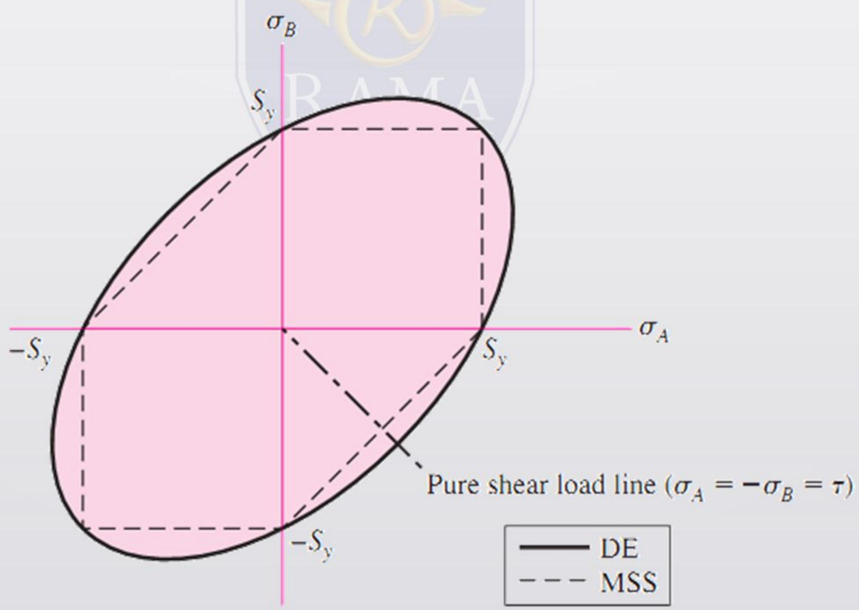
- For plane stress, let σ_A and σ_B be the two nonzero principal stresses.

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}$$

Distortion-Energy Theory for Ductile Materials

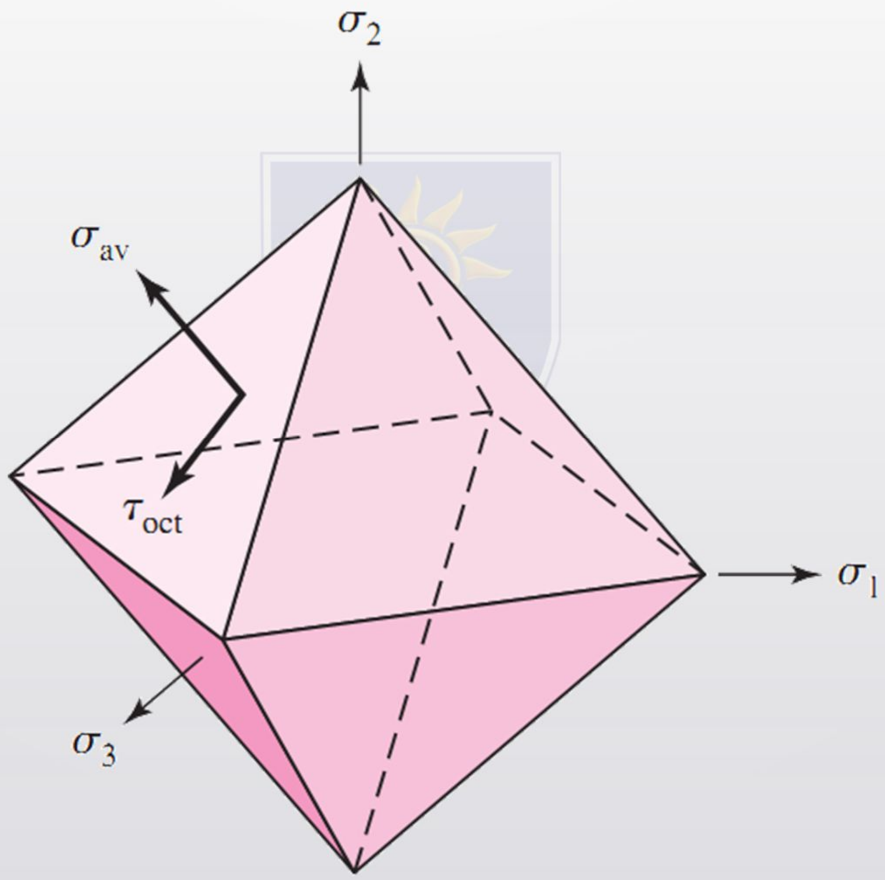
✓ The distortion-energy theory is also called:

- The von Mises or von Mises–Hencky theory
- The shear-energy theory
- The octahedral-shear-stress theory



Distortion-Energy Theory for Ductile Materials : 3D

✓ The octahedral-shear-stress theory



Octahedral-shear-stress Theory

- ✓ Using xyz components of three-dimensional stress, the von Mises stress can be written as

$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)\right]^{1/2}$$

- ✓ Consider an isolated element in which the normal stresses on each surface are equal to the hydrostatic stress σ_{av}. There are eight surfaces symmetric to the principal directions that contain this stress. The shear stresses on these surfaces are equal and are called the octahedral shear stresses Through coordinate transformations the octahedral shear stress is given by

$$\tau_{oct} = \frac{1}{3}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

Octahedral-shear-stress Theory

- ✓ Under the name of the octahedral-shear-stress theory, failure is assumed to occur when-ever the octahedral shear stress for any stress state equals or exceeds the octahedral shear stress for the simple tension-test specimen at failure.
- ✓ When, for the general stress case,

$$\left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]^{1/2} \geq S_y$$

- ✓ which is identical to distortion-energy theory equation, hence verifying that the maximum-octahedral-shear-stress theory is equivalent to the distortion-energy theory.