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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 5

Topic: Elastic constants

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Independent Elastic Constants

- ✓ *Generally isotropic material 81 independent elastic constants*
  - ✓ *If stress symmetry is use, then 54*
  - ✓ *If strain symmetry is use, then 36*
- ✓ *Anisotropic or allotropic material has 21 independent elastic constants*
- ✓ *Monoclinic material (one plane of material symmetry) has 13*
- ✓ *Orthotropic material (orthogonal plane of symmetry) has 9*
- ✓ *Lamina (transversely isotropic) has 5*
- ✓ ***Isotropic has 2 independent elastic constant***

Isotropic Constants

1. Modulus of Elasticity (E)
    1. The modulus of elasticity is the slope of stress-strain diagram in the linearly elastic region
    2. It represent stiffness of material

2. Poisson's ratio $(\nu)\mathrm{or}\,\frac{1}{\mathrm{m}}$

The lateral strain ϵ' at any point in a bar is proportional to the axial strain ϵ at that same point if the material is linearly elastic.

$$\frac{1}{m} = \nu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\epsilon'}{\epsilon}$$

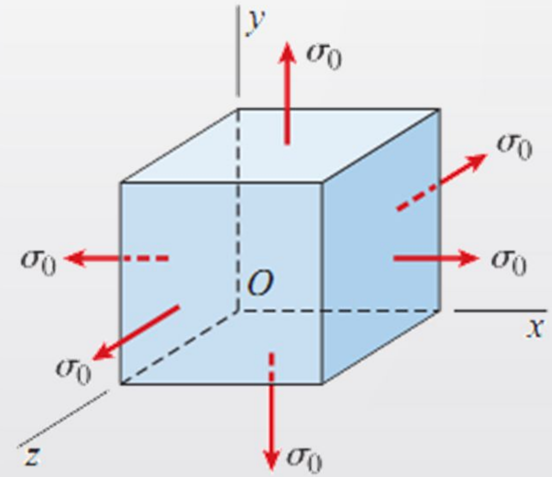
3. Modulus of Rigidity (G) or shear modulus of elasticity

4. Bulk Modulus (K)

Next slide

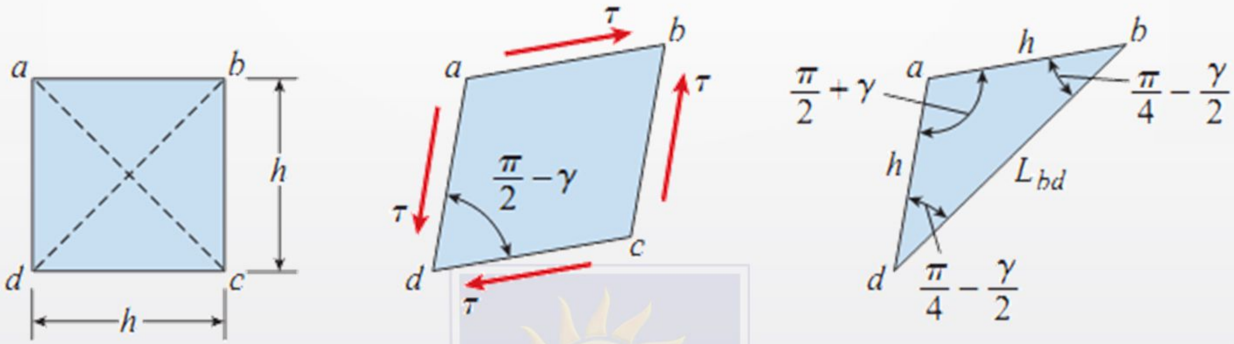
Bulk Modulus of Elasticity

- ✓ A special type of triaxial stress, called spherical stress, occurs whenever all three normal stresses are equal
- ✓ Under these stress conditions, any plane cut through the element will be subjected to the same normal stress σ_0 and will be free of shear stress. Thus, we have equal normal stresses in every direction and no shear stresses anywhere in the material.



$$Bulk\ Modulus\ K = \frac{Direct\ stress}{Volumetric\ Strain} = \frac{\sigma_0}{\epsilon_v}$$

Relation Between E and G



$$L_{bd} = \sqrt{2}\, h(1 + \epsilon_{\max})$$

$$L_{bd}^2 = h^2 + h^2 - 2h^2 \cos\left(\frac{\pi}{2} + \gamma\right)$$

$$(1 + \epsilon_{\max})^2 = 1 - \cos\left(\frac{\pi}{2} + \gamma\right)$$

$$1 + 2\epsilon_{\max} + \epsilon^2_{\max} = 1 + \sin\gamma$$

$$\because \cos(\pi/2 + \gamma) = -\sin\gamma,$$

$$\epsilon_{\max} = \frac{\gamma}{2}$$

$$\epsilon_{\max} = \frac{\tau}{E} + \frac{\nu\tau}{E} = \frac{\tau}{E}(1 + \nu)$$

$$G = \frac{E}{2(1+\nu)}$$

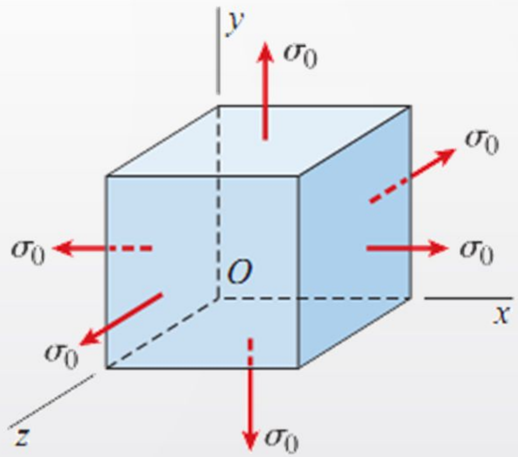
Relation Between E and K

Unit Volume Change

$$\epsilon_x + \epsilon_y + \epsilon_z = e$$

From Hooke's Law we can say that

$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$



Under Spherical Stress condition

$$\epsilon_x = \epsilon_y = \epsilon_z = \epsilon_0$$

$$\sigma_x = \sigma_y = \sigma_z = \sigma_0$$

Therefore

$$e = 3\epsilon_0 = 3\sigma_0\left(\frac{1-2\nu}{E}\right) = \frac{\sigma_0}{K}$$

Hence

$$K = \frac{E}{3(1-2\nu)}$$

Other relations

Relation Between E ,K and G

The relation can be derive by E-G equation and E-K equation by eliminating ν

$$\frac{9}{E} = \frac{1}{K} + \frac{3}{G}$$

Relation between ν ,K and G

The relation can be derive by E-G equation and E-K equation by eliminating E

$$\nu = \frac{3K - 2G}{6K + 2G}$$

MCQ

Q1 For a linearly elastic, isotropic and homogeneous material, the number of elastic constants required to relate stress and strain is:

(a) Two (b) Three (c) Four (d) Six

Q2 E, G, K and ν represent the elastic modulus, shear modulus, bulk modulus and Poisson's ratio respectively of a linearly elastic, isotropic and homogeneous material. To express the stress-strain relations completely for this material, at least

(a) E, G and Ă must be known (b) E, K and Ă must be known

(c) Any two of the four must be known (d) All the four must be known

Q3 Young's modulus of elasticity and Poisson's ratio of a material are 1.25×10^5 MPa and 0.34 respectively. The modulus of rigidity of the material is:

(a) 0.4025×10^5 Mpa (b) 0.4664×10^5 Mpa

(c) 0.8375×10^5 MPa (d) 0.9469×10^5 MPa

MCQ

Q4 If E, G and K denote Young's modulus, Modulus of rigidity and Bulk Modulus, respectively, for an elastic material, then which one of the following can be possibly true?

(a) G = 2K (b) G = E (c) K = E (d) G = K = E

Q5 If a material had a modulus of elasticity of 2.1×10^6 kgf/cm^2 and a modulus of rigidity of 0.8×10^6 kgf/cm^2 then the approximate value of the Poisson's ratio of the material would be

(a) 0.26 (b) 0.31 (c) 0.47 (d) 0.5

Q6 Consider the following statements:

1. Two-dimensional stresses applied to a thin plate in its own plane represent the plane stress condition.

2. Under plane stress condition, the strain in the direction perpendicular to the plane is zero.

3. Normal and shear stresses may occur simultaneously on a plane.

Which of the above statements is /are correct?

(a) 1 only (b) 1 and 2 (c) 2 and 3 (d) 1 and 3