



**FACULTY OF ENGINEERING AND  
TECHNOLOGY**

**Department of Mechanical Engineering**



# MEPS102:Strength of Material

## Lecture 1

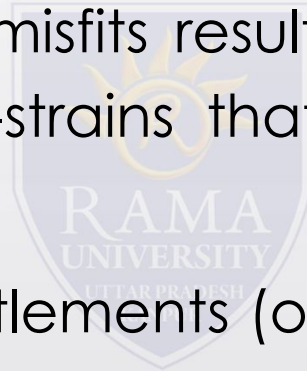
**Topic:Thermal Effects, Misfit  
and Pre-strain**

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## • THERMAL EFFECTS, MISFITS, AND PRESTRAINS

- ✓ External loads are not the only sources of stresses and strains in a structure.
- ✓ Other sources include thermal effects arising from temperature changes, misfits resulting from imperfections in construction, and pre-strains that are produced by initial deformations.
- ✓ Still other causes are settlements (or movements) of supports, inertial loads resulting from accelerating motion, and natural phenomenon such as earthquakes.



# • Thermal Effects

- ✓ Changes in temperature produce expansion or contraction of structural materials, resulting in thermal strains and thermal stresses
  - ✓ Lets take a block of material is unrestrained and therefore free to expand.
  - ✓ When the block is heated, every element of the material undergoes thermal strains in all directions, and consequently the dimensions of the block increase.
  - ✓ If we take corner A as a fixed reference point and let side AB maintain its original alignment, the block will have the shape shown by the dashed lines



# • Thermal Effects

- ✓ For most structural materials, thermal strain  $\epsilon_T$  is proportional to the temperature change  $\Delta T$

$$\epsilon_T = \alpha(\Delta T)$$

Where

$\alpha =$  coefficient of thermal expansion

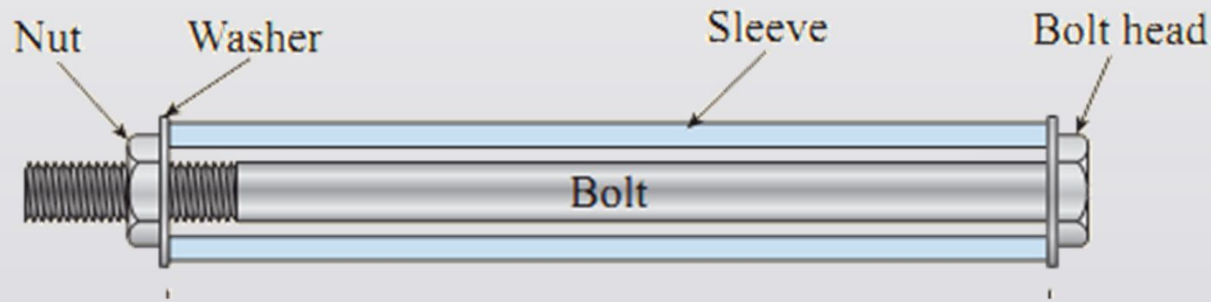
Therefore

$$\sigma = E\alpha(\Delta T)$$

- ✓ When an object rests on a frictionless surface or hangs in open space. In such cases **no stresses** are produced by a uniform temperature change throughout the object
- ✓ Nonuniform temperature changes may produce internal stresses.
- ✓ Normally structures have supports that prevent free expansion and contraction, in which case thermal stresses will develop even when the temperature change is uniform throughout the structure.

## • Thermal Effects: Example

- ✓ A sleeve in the form of a circular tube of length  $L$  is placed around a bolt and fitted between washers at each end. The nut is then turned until it is just snug. The sleeve and bolt are made of different materials and have different cross-sectional areas. (Assume that the coefficient of thermal expansion  $\alpha_s$  of the sleeve is greater than the coefficient  $\alpha_B$  of the bolt.)
- ✓ (a) If the temperature of the entire assembly is raised by an amount  $T$ , what stresses  $\alpha_s$  and  $\alpha_B$  are developed in the sleeve and bolt, respectively?
- ✓ (b) What is the increase  $d$  in the length  $L$  of the sleeve and bolt?

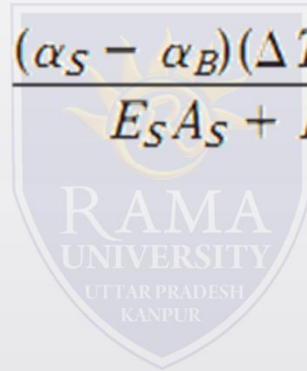




# • Answers

$$\sigma_S = \frac{P_S}{A_S} = \frac{(\alpha_S - \alpha_B)(\Delta T)E_S E_B A_B}{E_S A_S + E_B A_B}$$

$$\sigma_B = \frac{P_B}{A_B} = \frac{(\alpha_S - \alpha_B)(\Delta T)E_S A_S E_B}{E_S A_S + E_B A_B}$$



$$\delta = \frac{(\alpha_S E_S A_S + \alpha_B E_B A_B)(\Delta T)L}{E_S A_S + E_B A_B}$$

## • Misfits and Pre-Strains

- ✓ Suppose that a member of a structure is manufactured with its length slightly different from its prescribed length. Then the member will not fit into the structure in its intended manner, and the geometry of the structure will be different from what was planned. We refer to situations of this kind as misfits.
- ✓ Sometimes misfits are intentionally created in order to introduce strains into the structure at the time it is built. Because these strains exist before any loads are applied to the structure, they are called pre-strains.

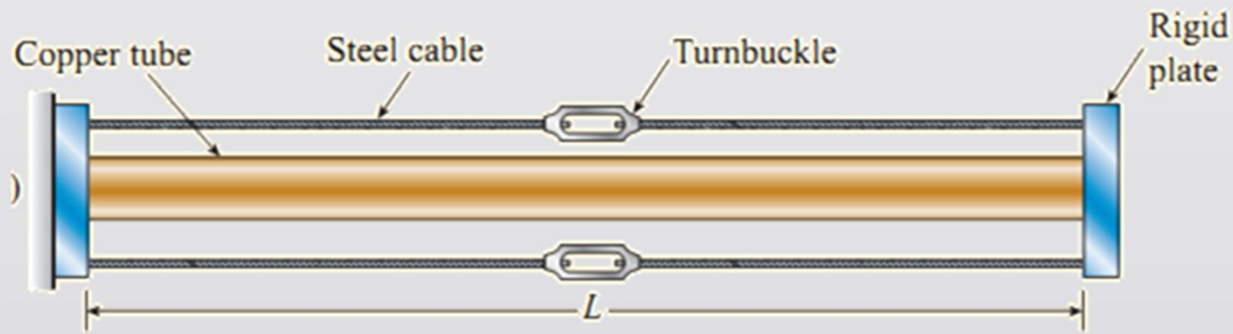


## • Pre-Strain

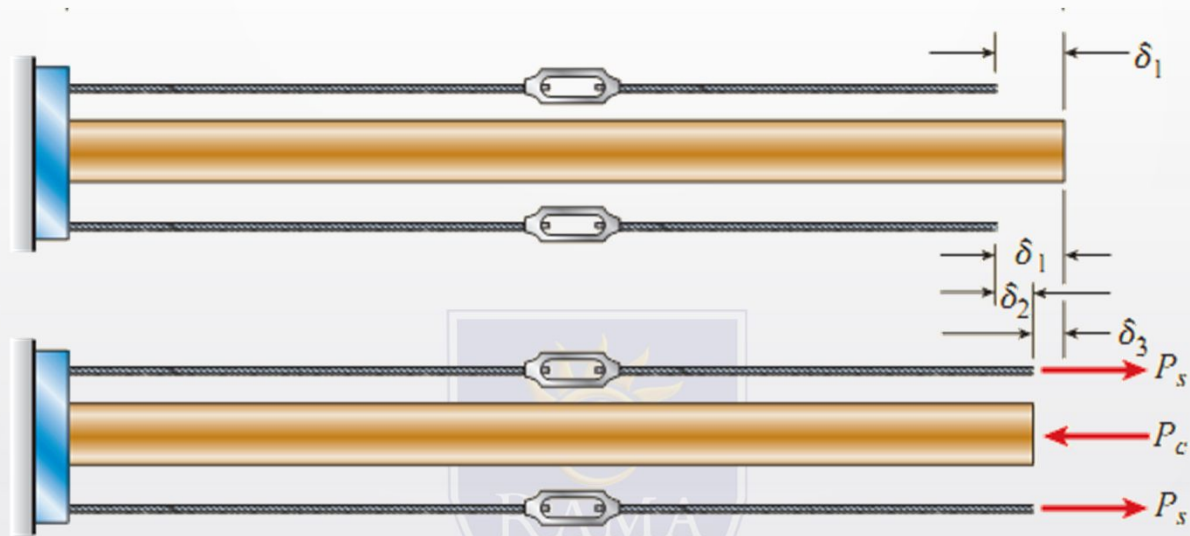
✓ The mechanical assembly shown consists of a copper tube, a rigid end plate, and two steel cables with turnbuckles. The slack is removed from the cables by rotating the turnbuckles until the assembly is snug but with no initial stresses. (Further tightening of the turnbuckles will produce a prestressed condition in which the cables are in tension and the tube is in compression.)

(a) Determine the forces in the tube and cables when the turnbuckles are tightened by  $n$  turns.

(b) Determine the shortening of the tube.



# • Pre-Strain



$$P_s = \frac{2npE_c A_c E_s A_s}{L(E_c A_c + 2E_s A_s)} \quad P_c = \frac{4npE_c A_c E_s A_s}{L(E_c A_c + 2E_s A_s)}$$

$$\delta_3 = \frac{P_c L}{E_c A_c} = \frac{4npE_s A_s}{E_c A_c + 2E_s A_s}$$

# Questions

**2.5-1** The rails of a railroad track are welded together at their ends (to form continuous rails and thus eliminate the clacking sound of the wheels) when the temperature is  $10^{\circ}\text{C}$ .

What compressive stress  $\sigma$  is produced in the rails when they are heated by the sun to  $52^{\circ}\text{C}$  if the coefficient of thermal expansion  $\alpha = 12 \times 10^{-6}/^{\circ}\text{C}$  and the modulus of elasticity  $E = 200 \text{ GPa}$ ?



## Questions

**2.5-2** An aluminum pipe has a length of 60 m at a temperature of  $10^{\circ}\text{C}$ . An adjacent steel pipe at the same temperature is 5 mm longer than the aluminum pipe.

At what temperature (degrees Celsius) will the aluminum pipe be 15 mm longer than the steel pipe? (Assume that the coefficients of thermal expansion of aluminum and steel are  $\alpha_a = 23 \times 10^{-6}/^{\circ}\text{C}$  and  $\alpha_s = 12 \times 10^{-6}/^{\circ}\text{C}$ , respectively.)

# Questions

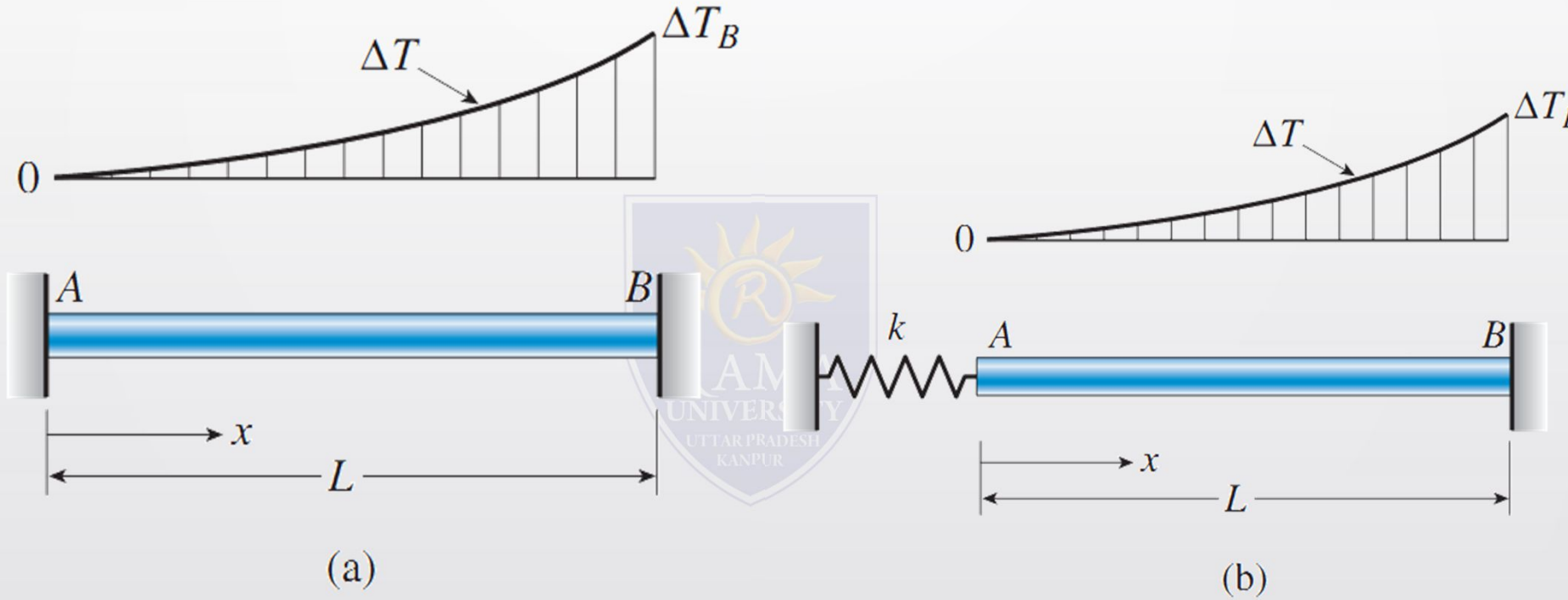
**2.5-5** A bar  $AB$  of length  $L$  is held between rigid supports and heated nonuniformly in such a manner that the temperature increase  $\Delta T$  at distance  $x$  from end  $A$  is given by the expression  $\Delta T = \Delta T_B x^3/L^3$ , where  $\Delta T_B$  is the increase in temperature at end  $B$  of the bar (see figure part a).

(a) Derive a formula for the compressive stress  $\sigma_c$  in the bar. (Assume that the material has modulus of elasticity  $E$  and coefficient of thermal expansion  $\alpha$ ).

(b) Now modify the formula in part (a) if the rigid support at  $A$  is replaced by an elastic support at  $A$  having a spring constant  $k$  (see figure part b). Assume that only bar  $AB$  is subject to the temperature increase.

**Figure on next slide**

# Questions



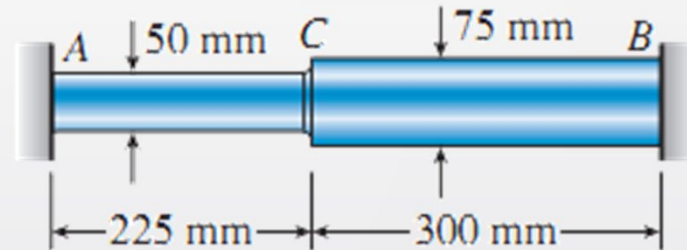


# Questions

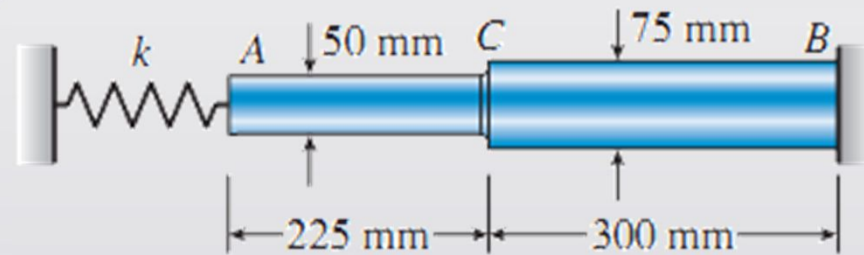
**2.5-6** A plastic bar  $ACB$  having two different solid circular cross sections is held between rigid supports as shown in the figure. The diameters in the left- and right-hand parts are 50 mm and 75 mm, respectively. The corresponding lengths are 225 mm and 300 mm. Also, the modulus of elasticity  $E$  is 6.0 GPa, and the coefficient of thermal expansion  $\alpha$  is  $100 \times 10^{-6}/^\circ\text{C}$ . The bar is subjected to a uniform temperature increase of  $30^\circ\text{C}$ .

(a) Calculate the following quantities: (1) the compressive force  $N$  in the bar; (2) the maximum compressive stress  $\sigma_c$ ; and (3) the displacement  $\delta_C$  of point  $C$ .

(b) Repeat part (a) if the rigid support at  $A$  is replaced by an elastic support having spring constant  $k = 50 \text{ MN/m}$  (see figure part b; assume that only the bar  $ACB$  is subject to the temperature increase).



(a)

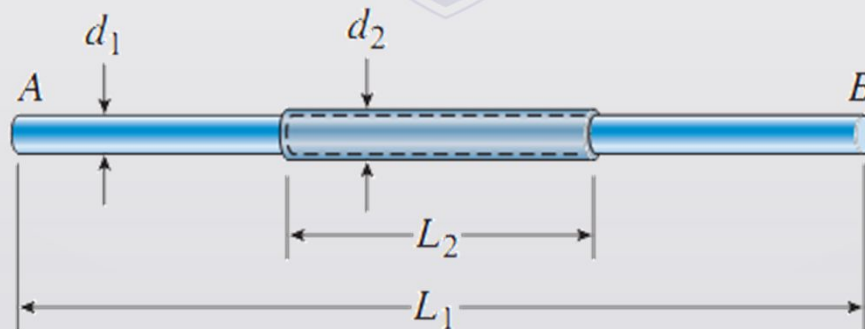


(b)

# Questions

**2.5-7** A circular steel rod  $AB$  (diameter  $d_1 = 15$  mm, length  $L_1 = 1100$  mm) has a bronze sleeve (outer diameter  $d_2 = 21$  mm, length  $L_2 = 400$  mm) shrunk onto it so that the two parts are securely bonded (see figure).

Calculate the total elongation  $\delta$  of the steel bar due to a temperature rise  $\Delta T = 350^\circ\text{C}$ . (Material properties are as follows: for steel,  $E_s = 210$  GPa and  $\alpha_s = 12 \times 10^{-6}/^\circ\text{C}$ ; for bronze,  $E_b = 110$  GPa and  $\alpha_b = 20 \times 10^{-6}/^\circ\text{C}$ .)



# Questions

**2.5-17** Wires  $B$  and  $C$  are attached to a support at the left-hand end and to a pin-supported rigid bar at the right-hand end (see figure). Each wire has cross-sectional area  $A = 19.3 \text{ mm}^2$  and modulus of elasticity  $E = 210 \text{ GPa}$ . When the bar is in a vertical position, the length of each wire is  $L = 2.032 \text{ m}$ . However, before being attached to the bar, the length of wire  $B$  was  $2.031 \text{ m}$  and wire  $C$  was  $2.030 \text{ m}$ .

Find the tensile forces  $T_B$  and  $T_C$  in the wires under the action of a force  $P = 3.115 \text{ kN}$  acting at the upper end of the bar.

