RAMA UNIVERSITY

www.ramauniversity.ac.in

FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

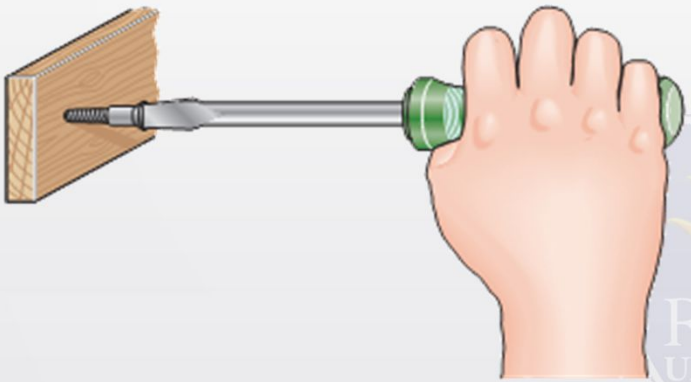
Lecture 7

Topic: Torsion and Hooke's Law in shear

Instructor:

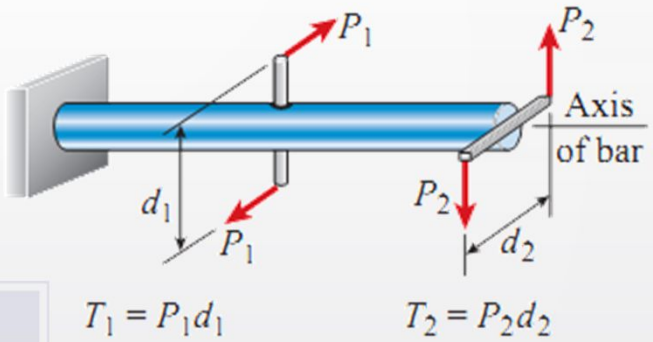
Aditya Veer Gautam

Torsion



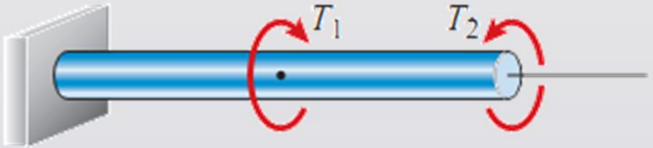
(a)

(b)



(a)

(b)



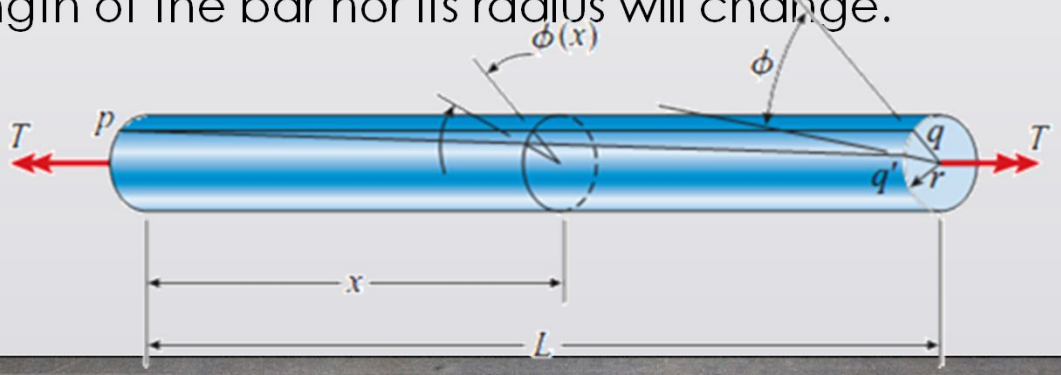
(c)

Torsion

- Moments that produce twisting of a bar, such as the moments $\mathbf{T_1}$ and $\mathbf{T_2}$, are called **torques or twisting moments**.
- Cylindrical members that are subjected to torques and transmit power through rotation are called **shafts**
- The moment of a couple may be represented by a vector in the form of a double-headed arrow. The arrow is perpendicular to the plane containing the couple, and therefore in this case both arrows are parallel to the axis of the bar.
  - An alternative representation of a moment is a curved arrow acting in the direction of rotation
- The direction (or sense) of the moment is indicated by the right-hand rule for moment vectors—namely, using your right hand, let your fingers curl in the direction of the moment, and then your thumb will point in the direction of the vector

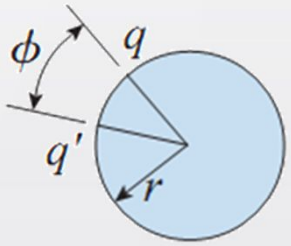
Torsional Deformation of Circular Bars

- ✓ Considering a prismatic bar of circular cross section twisted by torques T acting at the ends .
- ✓ Since every cross section of the bar is identical, and since every cross section is subjected to the same internal torque T, we say that the bar is in **pure torsion**.
- ✓ From symmetry, it can be proved that cross sections of the bar do not change in shape as they rotate about the longitudinal axis. In other words, all cross sections remain plane and circular and all radii remain straight.
- ✓ The angle of rotation between one end of the bar and the other is small, neither the length of the bar nor its radius will change.



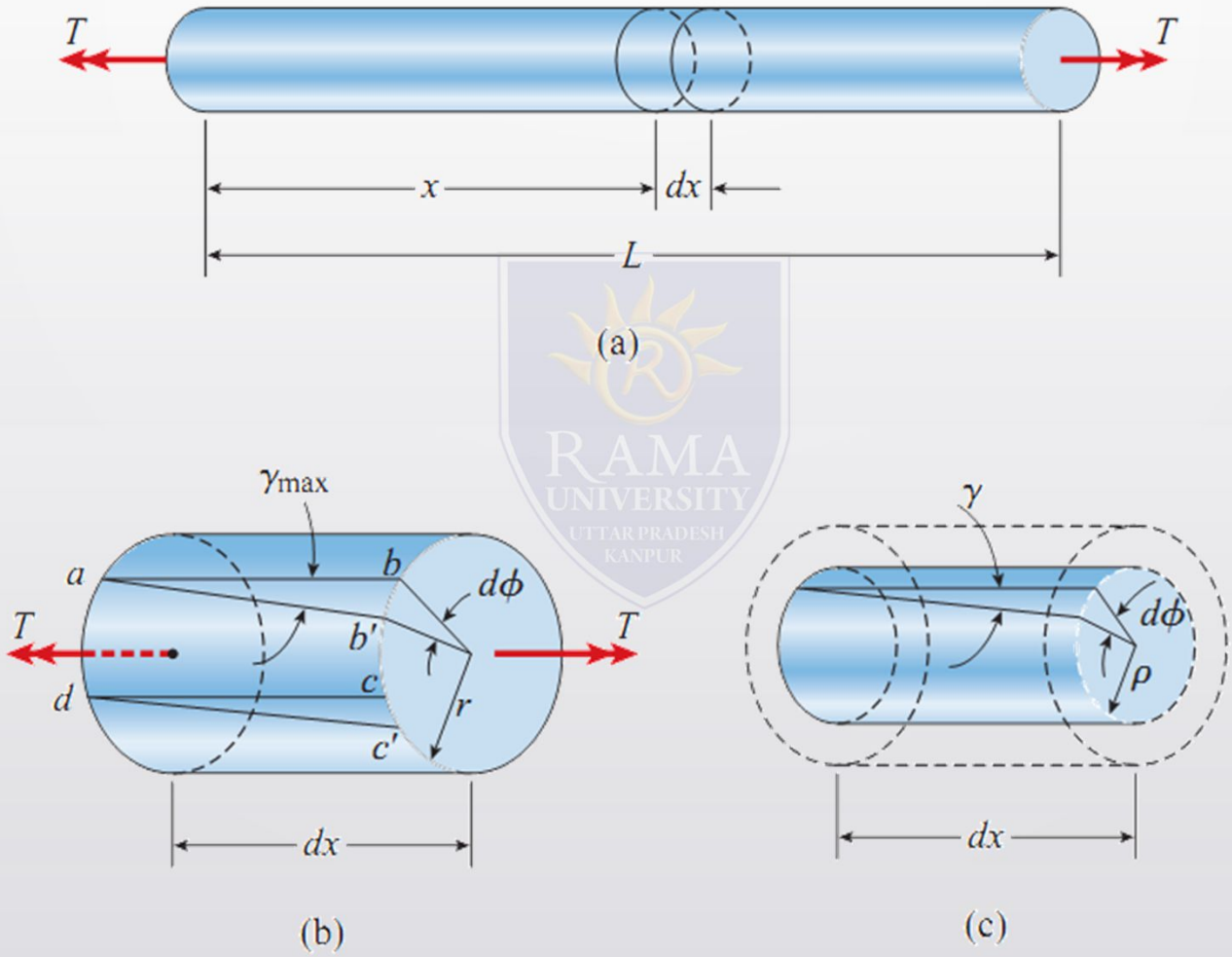
Torsional Deformation of Circular Bars

- Under the action of the torque T, the right-hand end will rotate (with respect to the left-hand end) through a small angle $\boldsymbol{\phi}$, known as the angle of twist (or angle of rotation).
- A straight longitudinal line *pq* on the surface of the bar will become a helical curve *pq*, where q is the position of point q after the end cross section has rotated through the angle $\boldsymbol{\phi}$



- The angle of twist changes along the axis of the bar, and at intermediate cross sections it will have a value $\boldsymbol{\phi}(\boldsymbol{x})$ that is between zero at the left-hand end and f at the right-hand end.
- If every cross section of the bar has the same radius and is subjected to the same torque (pure torsion), the angle $\boldsymbol{\phi}(\boldsymbol{x})$ will vary linearly between the ends

Shear Strains at the Outer Surface



Shear Strains at the Outer & inner Surface

- During twisting of the bar, the right-hand cross section rotates with respect to the left-hand cross section through a small angle of twist $d\phi$
  - The magnitude of the ***shear strain at the outer surface*** of the bar , is equal to the decrease in the angle at point ***a***, that is, the decrease in angle ***bab'***

$$\gamma_{max} = \frac{bb'}{ab} \qquad where\ \boldsymbol{\gamma}\ is\ measured\ in\ \boldsymbol{radian}$$

$$\gamma_{max} = \frac{rd\phi}{dx} = r\theta$$

  - Where ***θ rate of twist***, or the angle of twist per unit length. Its value doesn't change inside the element. ***For pure torsion only***

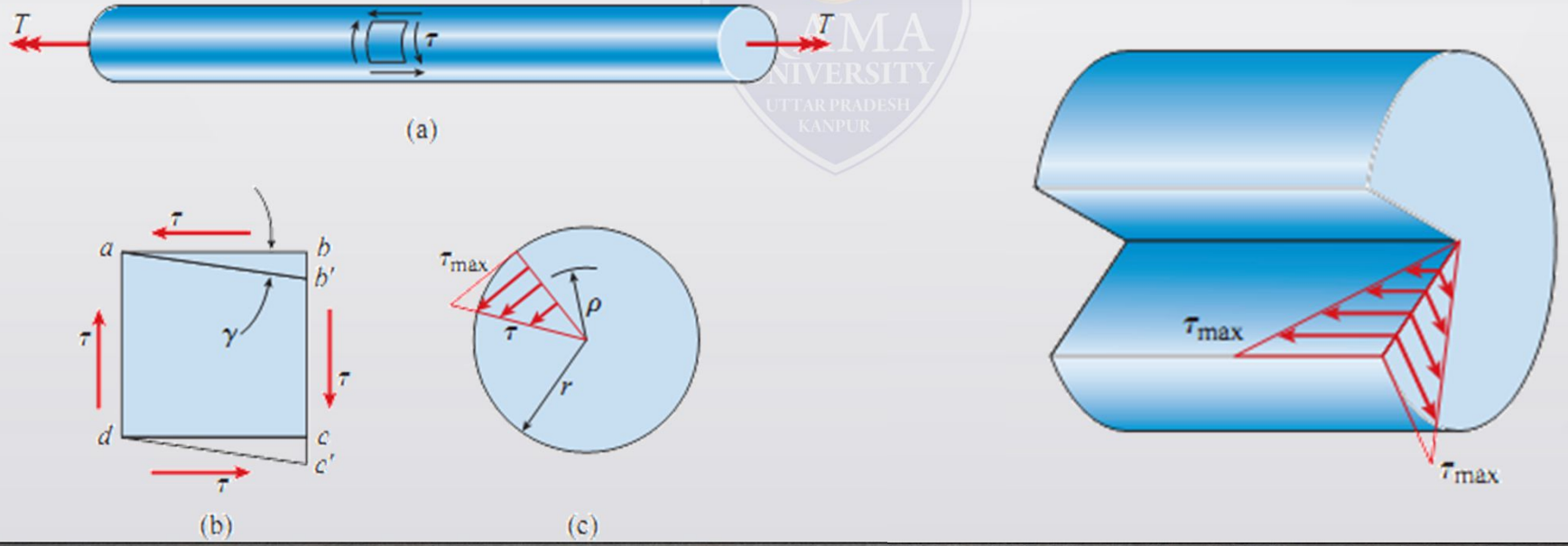
$$\gamma_{max} = \frac{r\phi}{L}$$

  - Similarly element situated on the surface of an ***interior cylinder of radius ρ***

$$\gamma = \rho\theta = \frac{\rho}{r}\gamma_{max}$$

Hooke's Law for Shear Stress

- ✓ The shear stresses acting on a cross-sectional plane are accompanied by shear stresses of the same magnitude acting on longitudinal planes.
- ✓ The shear stresses vary linearly with the distance from the centre of the bar, as illustrated by the triangular stress diagram



Hooke's Law for Shear Stress

$$\tau = G\gamma = G\rho\theta = \frac{\rho}{r}\tau_{max}$$

G *is shear modulus of elasticity*

γ *is shear stain in radians*

$$\tau_{max} = G\gamma_{max} = Gr\theta$$

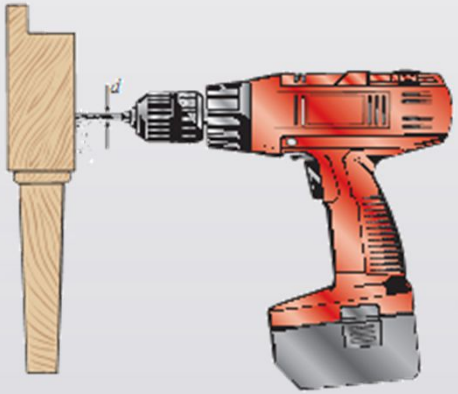
Review Questions

Q1 A plastic bar of diameter d = 56 mm is to be twisted by torques T until the angle of rotation between the ends of the bar is 4.0°. If the allowable shear strain in the plastic is 0.012 rad, what is the minimum permissible length of the bar?

Q2 When drilling a hole in a table leg, a furniture maker uses a hand-operated drill (see figure) with a bit of diameter d = 4.0 mm.

(a) If the resisting torque supplied by the table leg is equal to 0.3 Nm, what is the maximum shear stress in the drill bit?

(b) If the shear modulus of elasticity of the steel is G = 75 GPa, what is the rate of twist of the drill bit (degrees per meter)?

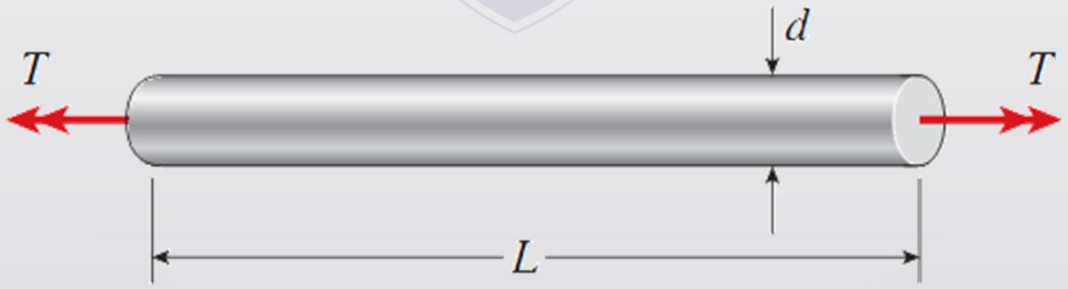


Review Question

3.2-1 A copper rod of length $L = 460$ mm is to be twisted by torques T (see figure) until the angle of rotation between the ends of the rod is 3.0°.

(a) If the allowable shear strain in the copper is 0.0006 rad, what is the maximum permissible diameter of the rod?

(b) If the rod diameter is 12.5 mm, what is the minimum permissible length of the rod?

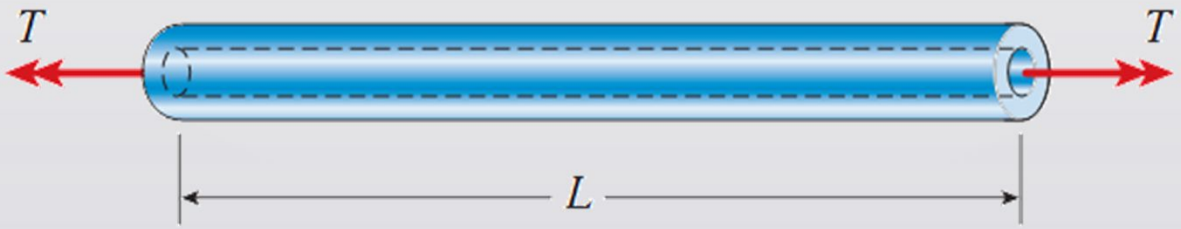


Review Question

3.2-3 A circular aluminum tube subjected to pure torsion by torques T (see figure) has an outer radius r_2 equal to 1.5 times the inner radius r_1.

(a) If the maximum shear strain in the tube is measured as 400×10^{-6} rad, what is the shear strain γ_1 at the inner surface?

(b) If the maximum allowable rate of twist is 0.125 degrees per meter and the maximum shear strain is to be kept at 400×10^{-6} rad by adjusting the torque T, what is the minimum required outer radius $(r_2)_{min}$?



Review Question

3.2-4 A circular steel tube of length $L = 1.0$ m is loaded in torsion by torques T (see figure).

(a) If the inner radius of the tube is $r_1 = 45$ mm and the measured angle of twist between the ends is 0.5°, what is the shear strain γ_1 (in radians) at the inner surface?

(b) If the maximum allowable shear strain is 0.0004 rad and the angle of twist is to be kept at 0.45° by adjusting the torque T, what is the maximum permissible outer radius $(r_2)_{max}$?